퍼지 및 지능시스템학회 논문지 2003, Vol. 13, No. 6, pp. 729-736

Chaotic behavior analysis in the mobile robot of embedding some chaotic equation with obstacle

Youngchul Bae, Juwan Kim, Yigon Kim

Division of electronic communication and electrical engineering of Yosu National University

Abstract

In this paper, we propose that the chaotic behavior analysis in the mobile robot of embedding some chaotic such as Chua's equation, Arnold equation with obstacle. In order to analysis of chaotic behavior in the mobile robot, we apply not only qualitative analysis such as time-series, embedding phase plane, but also quantitative analysis such as Lyapunov exponent in the mobile robot with obstacle. We consider that there are two type of obstacle, one is fixed obstacle and the other is VDP obstacle which have an unstable limit cycle. In the VDP obstacles case, we only assume that all obstacles in the chaos trajectory surface in which robot workspace has an unstable limit cycle with Van der Pol equation.

Key words : Chaos, Arnold equation, mobile robot, Lyapunov Exponent, Chua's equation

1. Introduction

Chaos theory has been drawing a great deal of attention in the scientific community for almost two decades. Remarkable research efforts have been spent in recent years, trying to export concepts from Physics and Mathematics into the real world engineering applications. Applications of chaos are being actively studied in such areas as chaos control [1-2], chaos synchronization and secure/crypto communication [3-7], Chemistry [8], Biology [9], and robots and their related themes [10].

Recently, Nakamura, Y. et al [10] proposed a chaotic mobile robot, where a mobile robot is equipped with a controller that ensures chaotic motion and the dynamics of the mobile robot is represented by Arnold equation. They applied obstacle with chaotic trajectory, but they have not mentioned about the chaotic behavior except Lyapunov exponent.

In this paper, we propose that the chaotic behavior analysis in the mobile robot, in which the Arnold equation and Chua's equation are embedded, and which have obstacle. In order to analysis of chaotic behavior in the mobile robot, we apply not only qualitative analysis such as time-series, embedding phase plane, but also quantitative analysis such as Lyapunov exponent in the mobile robot with obstacle. In order to avoid obstacles, we assume that all obstacles in the chaos trajectory surface have an unstable limit cycle of Van der Pol equation. When chaos robots meet obstacles among the arbitrary wondering chaos trajectory with chaos circuit equation such as Chua's equation, Arnlod equation, obstacles pull out the chaos robots out of chaos trajectory because obstacles have unstable limit cycle with Van der Pol equation.

접수일자 : 2003년 8월 27일
완료일자 : 2003년 11월 17일

2. Chaotic Mobile Robot embedding Chaos Equation

2.1 Mobile Robot

As the mathematical model of mobile robots, we assume a two- wheeled mobile robot as shown in Fig. 1.

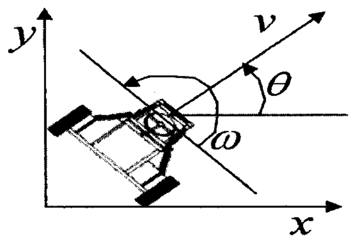


Fig. 1. Two-wheeled mobile robot

Let the linear velocity of the robot v [m/s] and angular velocity ω[rad/s] be the input to the system. The state equation of the four-wheeled mobile robot is written as follows:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \quad (1)$$

where (x,y) is the position of the robot and θ is the angle of the robot.

2.2 Some Chaos Equations

In order to generate chaotic motions for the mobile robot, we apply some chaos equations such as an Arnold equation or Chua's equation.

1) Arnold equation [10]

We define the Arnold equation as follows:

$$\begin{aligned} \dot{x}_1 &= A\sin x_3 + C\cos x_2 \\ \dot{x}_2 &= B\sin x_1 + A\cos x_3 \\ \dot{x}_3 &= C\sin x_2 + B\cos x_1 \end{aligned} \quad (2)$$

where A, B, C are constants. The Arnold equation describes a steady solution to the three-dimensional (3D) Euler equation

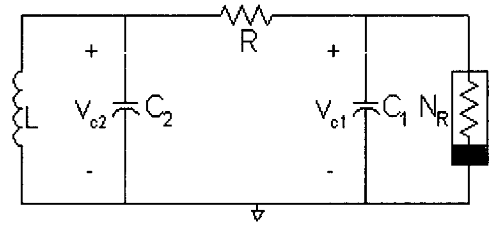
$$\frac{\partial v_i}{\partial t} + \sum_{k=1}^{3} v_k \frac{\partial v_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + f_i \quad (3)$$

$$\sum_{i=1}^{3} \frac{\partial v_i}{\partial x_i} = 0 \quad (4)$$

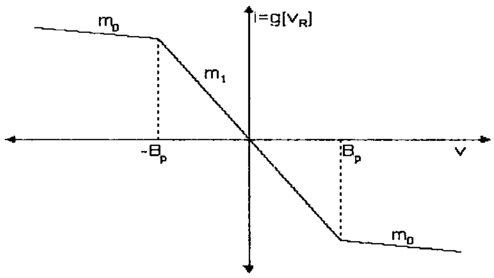
which express the behaviors of noncompressive perfect fluids on a 3D torus space. (x_1, x_2, x_3) and (v_1, v_2, v_3) denote the position and velocity of particle and p, and (f_1, f_2, f_3) and ρ denote the pressure, external force, and density, respectively. It is known that the Arnold equation shows periodic motion when one of the constant, for example C, is 0 or small and shows chaotic motion when C is large[14].

2) Chua's Circuit Equation (2-Double Scroll)

Chua's circuit is one of the simplest physical models that has been widely investigated by mathematical, numerical and experimental methods. One of the main attractions of Chua's circuit is that it can be easily built with less than a dozen standard circuit components, and has often been referred to as the "poor man's chaos generator." Since the Chua's circuit is endowed with an



(a)



(b)

Fig. 2. (a) Chua circuit, (b) nonlinear resistor

unusually rich repertoire of nonlinear dynamical phenomena, it has become a universal paradigm for chaos. The Chua's circuit and their nonlinear resister are shown on Fig. 2(a), 2(b) respectively.

We can derive the state equation of Chua's circuit following as from Fig. 2(a) and 2(b) and then we also can get the phase plane looks like Fig. 3

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - g(x_1)) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \end{aligned} \quad (5)$$

where

$$g(x) = m_{2n-1}x + \frac{1}{2}\sum_{k=1}^{2n-1}(m_{k-1} - m_k)(|x + c_k| - |x - c_k|) .$$

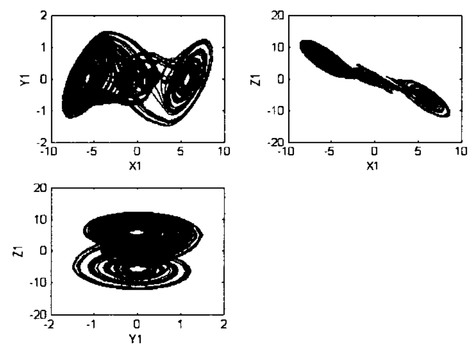


Fig. 3. Phase plane of Chua's circuit

2.3 Embedding of Chaos circuit in the Robot

In order to embed the chaos equation into the mobile robot, we define and use the Arnold equation and Chua's circuit equation as follows.

1) Arnold equation

We define and use the following state variables:

$$\begin{aligned} \dot{x}_1 &= D\dot{y} + C\cos x_2 \\ \dot{x}_2 &= D\dot{x} + B\sin x_1 \\ \dot{x}_3 &= \theta \end{aligned} \quad (6)$$

where B, C, and D are constant.

Substituting (1) into (2), we obtain a state equation on $\dot{x}_1$, $\dot{x}_2$, and $\dot{x}_3$ as follows:

$$\begin{aligned} \dot{x}_1 &= Dv + C\cos x_2 \\ \dot{x}_2 &= Dv + B\sin x_1 \\ \dot{x}_3 &= \omega \end{aligned} \quad (7)$$

We now design the inputs as follows [10]:

$$\begin{aligned} v &= A/D \\ \omega &= C\sin x_2 + B\cos x_1 \end{aligned} \quad (8)$$

Finally, we can get the state equation of the mobile robot as follows:

$$\begin{aligned} \dot{x}_1 &= A\sin x_3 + C\cos x_2 \\ \dot{x}_2 &= B\sin x_1 + A\cos x_3 \\ \dot{x}_3 &= C\sin x_2 + B\cos x_1 \\ \dot{x} &= V\cos x_3 \\ \dot{y} &= V\sin x_3 \end{aligned} \quad (9)$$

Equation (9) includes the Arnold equation. Fig. 4 shows the phase plane of the gradients of the mobile robot of Arnold equation in x−y plane and in 3D plane respectively.

In the Nakamura et al.[10], they used phase plane components such as $(x,y),(y,z),(z,x)$ in the equation (9), but we used gradients of each variables such as, $\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}, \frac{\partial z}{\partial t}$ for convenience in computation of chaotic path of the mobile robot .

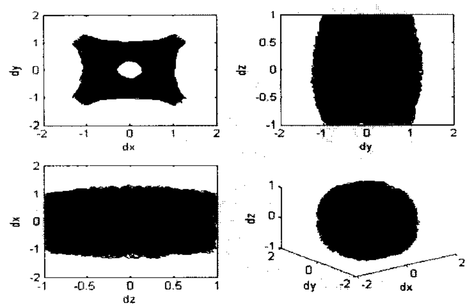


Fig. 4. Phase plane of gradient ($\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}, \frac{\partial z}{\partial t}$) of Arnold equation in x−y plane and in 3D (v=1, A=1, B=0.5, C=0.5)

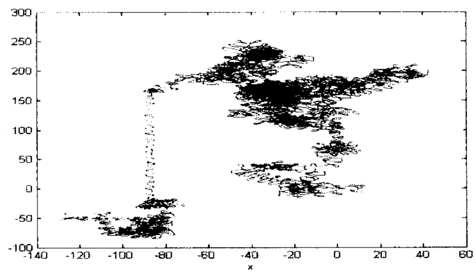


Fig. 5. Trajectory of the mobile robot of Arnold equation, when there is no boundary.

Fig. 5 shows the trajectory of mobile robot of Arnold equation, when there is no boundary. In Fig. 5, we recognize intuitionally that the mobile robot trajectories with the Arnold equation have a chaotic motion.

2) Chua's Equation

Using the methods explained in equations (6)−(9), we can obtain equation (10) with Chua's equation embedded in the mobile robot.

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - g(x_1)) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \\ \dot{x} &= V\cos x_3 \\ \dot{y} &= V\sin x_3 \end{aligned} \quad (10)$$

Using equation (10), we obtain the embedding chaos robot trajectories with Chua's equation. Fig. 6 shows the phase plane of the gradients of Chua's equation, which is used for the computational convenience as in the Chua's equation.

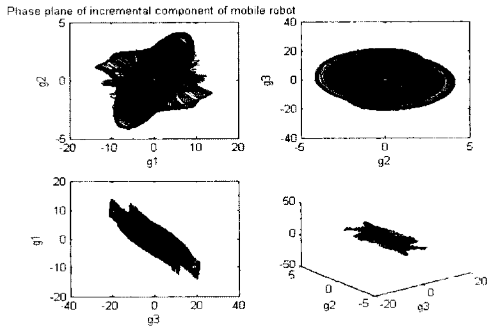


Fig. 6. Phase portrait of gradient vector($\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}, \frac{\partial z}{\partial t}$) in Chua's circuit.

Fig. 7 shows the trajectory of mobile robot of Chua's equation, when there is no boundary. In Fig. 7, we recognize intuitionally that the mobile robot trajectories with the Chua's equation have a chaotic motion.

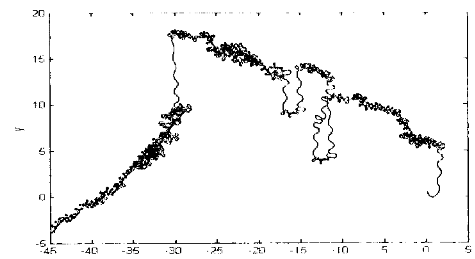


Fig. 7. Trajectory of the mobile robot of Chua's equation, when there is no boundary

2.4 Mirror Mapping.

Basically, equation (9) and (10) are assumed that the mobile robot moves in a smooth state space without boundary. However, real robot movies in space with boundary like walls or surfaces of obstacles. To avoid a boundary or obstacle, we consider mirror mapping when the robot approach walls or obstacles using the Eq. (11) and (12). Whenever the robot approaches a wall or obstacle, we calculated the robot new position

$$A = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \quad (11)$$

$$A = 1/1+m \begin{pmatrix} 1-m^2 & 2m \\ 2m & -1+m^2 \end{pmatrix} \quad (12)$$

We can use equation (11) when slope is infinitive such as $\theta = 90$ and also use equation (12) when slope is not infinitive. In Fig. 8, we can see the mirror mapping diagram.

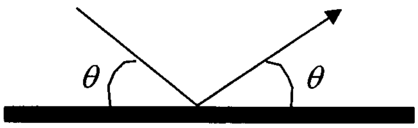


Fig. 8. Mirror mapping

3. The Chaotic Behavior of embedding Chaos Robot with obstacle avoidance behavior

We investigated by numerical analysis whether the mobile robot with the proposed controller actually behaves in a chaotic manner. In order to computer simulation, we applied mirror mapping and have shown it in Fig. 9. The parameters and initial conditions are used as follows:

Arnold equation case

Coefficients:

v= 1[m/s], A= 0.5[1/s], B=0.25[1/s], C=0.25[1/s]

Initial conditions:

$$x_1 = 4, \quad x_2 = 3.5, \quad x_3 = 0, \quad x = 0, \quad y = 0$$

Chua's equation case

Coefficients:

$$\alpha = 9, \quad \beta = 14.286$$

$$m_0 = -\frac{1}{7}, \quad m_1 = \frac{2}{7}, m_2 = -\frac{4}{7}, \quad m_3 = m_1$$

$$c_1 = 1, \quad c_2 = 2.15, \quad c_3 = 3.6$$

Initial conditions:

$$x_1 = 4, \quad x_2 = 3.5, \quad x_3 = 0, \quad x = 0, \quad y = 0$$

3.1 Fixed obstacle

In this section, we will study avoidance behavior of a chaos trajectory with obstacle mapping, relying on the Arnold equation and Chua's equation respectively.

Fig. 9 and 10 show that a chaos robot trajectories to which mirror mapping was applied in the outer wall and in the inner obstacles as well using Eq. (11) and (12), relying on Arnold equation (9) and Chua's equation (10). The chaos robot has two fixed obstacles, and we can confirm that the robot adequately avoided the fixed obstacles in the Arnold and Chua's chaos robot trajectories.

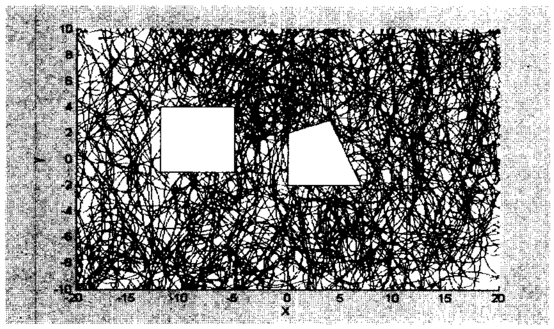


Fig. 9. Arnold equation trajectories of chaos robot with obstacle

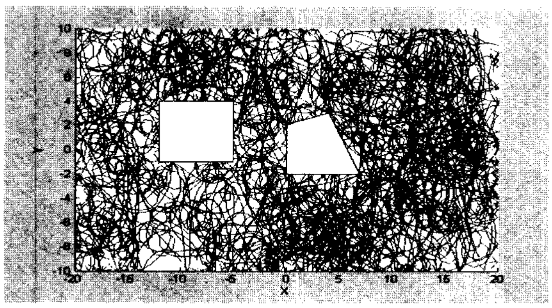


Fig.10. Chua's equation trajectories of chaos robot with obstacles

3.2 VDP equation as a obstacle

In this section, we will discuss the mobile robot's avoidance of Van der Pol(VDP) equation obstacles. We assume the obstacle has a VDP equation with an unstable limit cycle, because in this condition, the mobile robot can not move close to the obstacle and the obstacle is avoided.

1) VDP equation as an obstacle

In order to represent an obstacle of the mobile robot, we employ the VDP, which is written as follows:

$$\dot{x} = y$$
$$\dot{y} = (1 - y^2)y - x \quad (13)$$

From equation (13), we can get the following limit cycle as shown in Fig. 11.

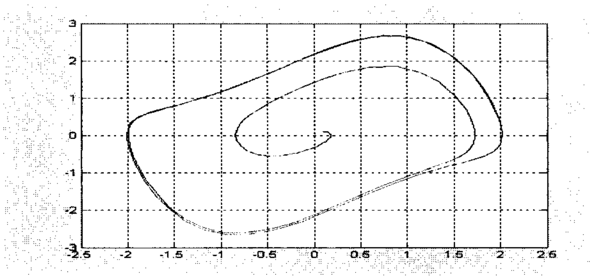


Fig. 11. Limit cycle of VDP

2) Magnitude of Distracting force from the obstacle

We consider the magnitude of distracting force from the obstacle as follows:

$$D = \frac{0.325}{(0.2D_k + 1)e^{3(0.2D_k - 1)}} \quad (14)$$

where D_k is the distance between each effective obstacle and the mobile robot.

We can also calculate the VDP obstacle direction vector as follows:

$$\begin{bmatrix} \bar{x}_k \\ \bar{y}_k \end{bmatrix} = \begin{bmatrix} x_o - y \\ 0.5(1 - (y_o - y)^2)(y_o - y) - (x_o - x) \end{bmatrix} \quad (15)$$

where (x_o, y_o) are the coordinates of the center point of each obstacle. Then we can calculate the magnitude of the VDP direction vector (L), the magnitude of the moving vector of the virtual robot (I) and the enlarged coordinates (I/2L) of the magnitude of the virtual robot in VDP$(x_k^{'}, y_k^{'})$ as follows:

$$L = \sqrt[2]{(\bar{x}_{vdp}^{\ 2} + \bar{y}_{vdp}^{\ 2})}$$
$$I = \sqrt{(x_r^2 + y_r^2)} \quad (16)$$
$$x_k^{'} = \frac{\bar{x}_k}{L}\frac{I}{2}, y_k^{'} = \frac{y_k^{'}}{L}\frac{I}{2}$$

Finally, we can get the Total Distraction Vector (TDV) as shown by the following equation.

$$\begin{bmatrix} \dfrac{\sum_k^n ((1 - \frac{D_k}{D_0})\bar{x} + \frac{D_k}{D_0}\bar{x}_k)}{n} \\ \dfrac{\sum_k^n ((1 - \frac{D_k}{D_0})\bar{y} + \frac{D_k}{D_0}\bar{y}_k)}{n} \end{bmatrix} \quad (17)$$

Using equations (14)−(17), we can calculate the avoidance method of the obstacle in the Arnold equation and Chua's equation trajectories with one or more VDP obstacles.

In Fig. 12, the computer simulation result shows that the chaos robot has two robots and a total of 5 VDP obstacles, including two VDP obstacles at the origin in the Arnold equation trajectories. We can see that the robot sufficiently avoided the obstacles in the Arnold equation trajectories.

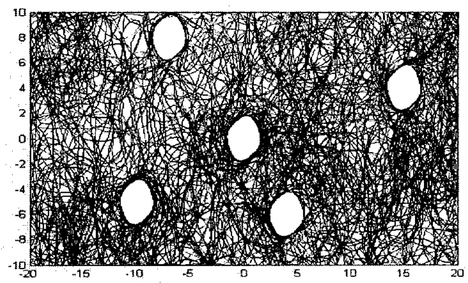


Fig. 12. Computer simulation result of obstacle avoidance with 2 robots and 5 obstacles in Arnold equation trajectories.

In Fig. 13, the computer simulation result shows that the chaos robot surface has two robots and total of 5 VDP obstacles, including 2 VDP obstacles at the origin in the Chua's equation trajectory. We can see that the robot sufficiently avoided the obstacles in the Chua's equation trajectory.

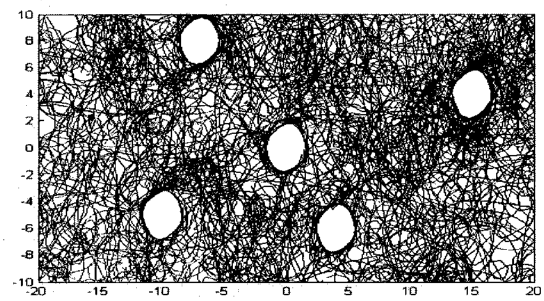


Fig. 13. Computer simulation result of obstacle avoidance with 2 robots and 5 obstacles in Chua's equation trajectory.

4. Chaotic behavior analysis in the Mobile Robot

To analysis of chaotic behavior in the mobile robot, we investigated the chaotic characteristics from the mobile robot trajectories data. Firstly, we applied embedding method as a qualitative analysis and then we get the reconstruction phase plane from the one dimensional mobile robot trajectories data. Second, we calculated Lyapunov exponent as quantitative analysis.

4.1 Embedding method

In order to reconstruct phase plane from data of robot's single variable, we applied an embedding method proposed by Takens [12]. The embedding method is referring to the process in which a representation of the attractor can be constructed from a set of scalar time-series. The form of such reconstructed state is given as follows:

$$X_t = [x(t), x(t+\tau), \ldots\ldots, x(t+(m-1)\tau] \qquad (18)$$

Where $x(t)$ is a robot trajectory data, τ is a delay time, and m is an embedding dimension. It is significant factor to get reasonable embedding phase plane. In chaos mobile case, we choose τ is 400 using an auto-correlation time and m is chosen 5 because nearest false neighbor disappears in that dimension. Fig. 14 and Fig. 15 shows time -series of embedding Arnold equation chaos robot from equation (9) and Chua's equation chaos robot from equation ion (10) receptively.

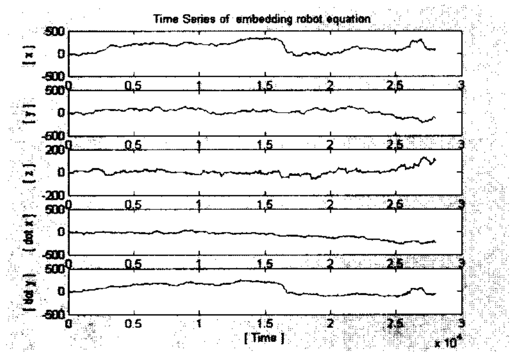


Fig. 14. Arnold chaos robot time-series

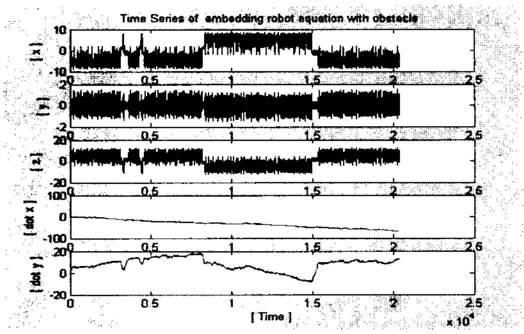
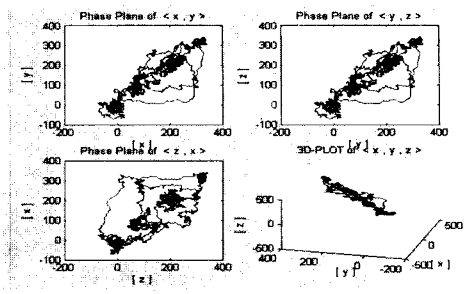


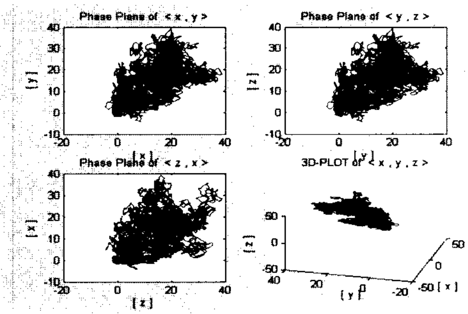
Fig.15. Chua's chaos robot time-series

4.2 Qualitative Analysis

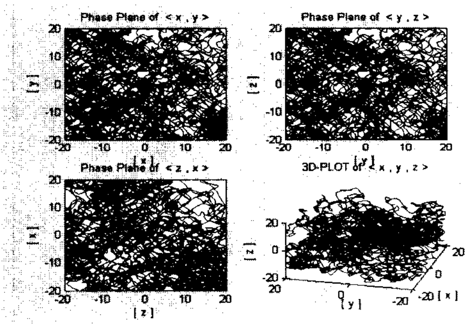
With reconstructed state, the qualitative chaotic degree of chaotic robot path is analyzed in this section using embedding phase plane. Fig. 16 shows phase plane of these embedding state which are originally robot paths when robot has a (a) no obstacle, (b) fixed obstacle, and (c) VDP obstacle from the Arnold embedding chaos robot. Fig. 17 shows phase plane of these embedding state which are originally robot paths when robot has a (a) no obstacle, (b) fixed obstacle, and (c) VDP obstacle from the Chua's embedding chaos robot.



(a)

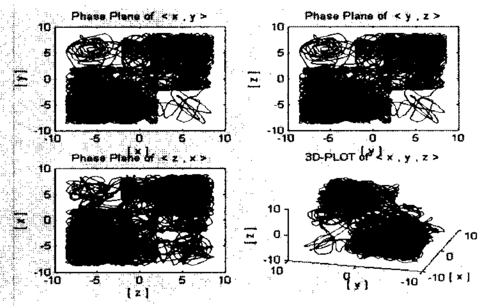


(b)

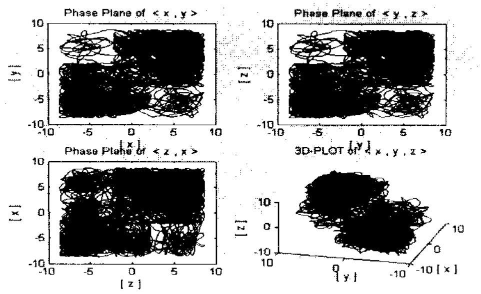


(c)

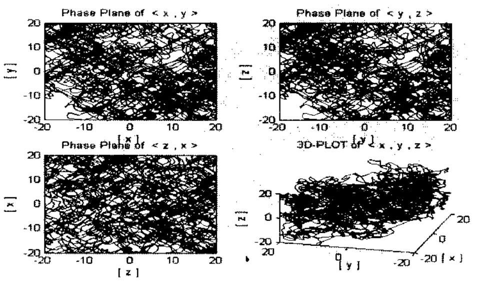
Fig. 16. Reconstructed phase plane (a) no obstacle, (b) fixed obstacle, and (c) VDP obstacle.



(a)



(b)



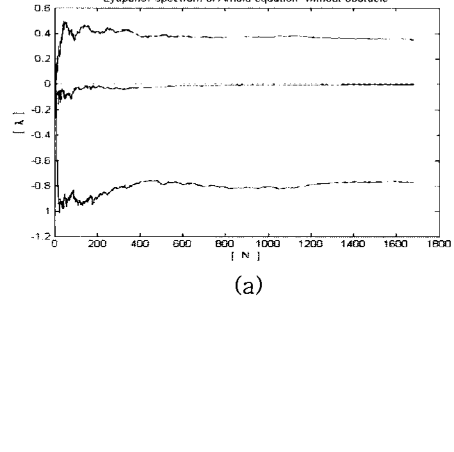
(c)

Fig. 17. Reconstructed phase plane (a) no obstacle, (b) fixed obstacle, and (c) VDP obstacle.

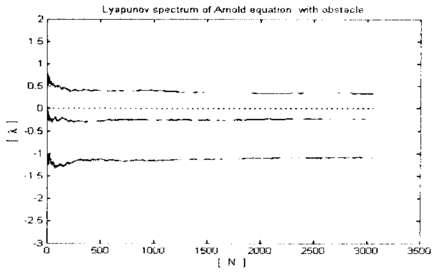
In Fig. 16 and 17, we can recognize that reconstructed robot path from one dimensional mobile trajectories are very complicated signal seems like chaos signal. We can also confirmed that when the robots has a obstacle, reconstructed phase planes are more complicated compare with no obstacle.

4.3. Quantiative Analysis

In this section, we evaluate Lyapunov spectrum [13] in the mobile robot as a quantitative chaos analysis and shows Arnold chaos robot in Fig. 18 and also shows Chua's chaos robot in Fig. 19. Generally speaking, when the largest Lyapunov exponent more than zero we can say chaotic motion and less than or equal zero, we say periodic motion. In Fig. 18 and 19, we can confirm that reconstructed phase planes are chaotic motion because there are largest Lyapunov exponents more than zero.

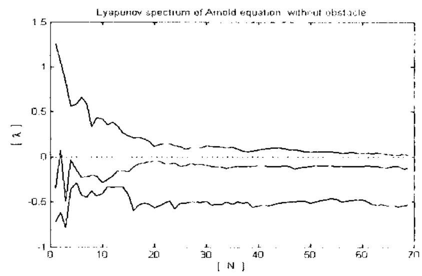


(a)

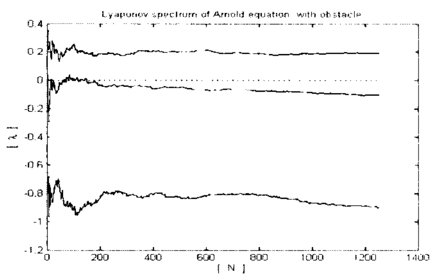


(b)

Fig. 18. Lyapunov spectrum of mobile robot (a) without obstacle, (b) with obstacle



(a)



(b)

Fig. 19. Lyapunov spectrum of mobile robot (a) without obstacle, (b) with obstacle

5. Conclusion

In this paper, we propose that the chaotic behavior analysis in the mobile robot of embedding Arnold equation and Chua's equation with obstacle. In order to analysis of chaotic behavior in the mobile robot, we apply not only qualitative analysis such as time−series, embedding phase plane, but also quantitative analysis such as Lyapunov exponent in the mobile robot with obstacle. In the obstacle, we only assume that all obstacles in the chaos trajectory surface in which robot workspace has an unstable limit cycle with Van der Pol equation.

Reference

[1] E. Ott, C.Grebogi, and J.A York," Controlling Chaos", Phys. Rev.Lett., vol. 64, no.1196−1199, 1990.

[2] T. Shinbrot, C.Grebogi, E.Ott, and J.A.Yorke, " Using small perturbations to control chaos", Nature, vol. 363, pp. 411-417, 1993.
[3] M. Itoh, H. Murakami and L. O. Chua, "Communication System Via Chaotic Modulations" IEICE. Trans. Fundmentals. vol.E77-A, no. , pp.1000-1005, 1994.
[4] L. O. Chua, M. Itoh, L. Kocarev, and K. Eckert, "Chaos Synchronization in Chua's Circuit" J. Circuit. Systems and computers, vol. 3, no. 1, pp. 93-108, 1993.
[5] M. Itoh, K. Komeyama, A. Ikeda and L. O. Chua, " Chaos Synchronization in Coupled Chua Circuits", IEICE. NLP. 92-51. pp. 33-40. 1992.
[6] K. M. Short, " Unmasking a modulated chaotic communications scheme", Int. J. Bifurcation and Chaos, vol. 6, no. 2, pp. 367-375, 1996.
[7] L. Kocarev, " Chaos-based cryptography: A brief overview," IEEE, Vol. pp. 7-21. 2001.
[8] M. Bertram and A. S. Mikhailov, "Pattern formation on the edge of chaos: Mathematical modeling of CO oxidation on a Pt(110) surface under global delayed feedback", Phys. Rev. E 67, pp. 036208, 2003.
[9] K. Krantz, f. H. Yousse, R.W, Newcomb, "Medical usage of an expert system for recognizing chaos", Engineering in Medicine and Biology Society, 1988. Proceedings of the Annual International Conference of the IEEE , 4-7, pp. 1303 -1304,1988 .
[10, Y] Nakamura, A. , Sekiguchi. "The chaotic mobile robot", IEEE Transactions on Robotics and Automation , Volume: 17 Issue: 6 , pp. 898 -904, 2001.
[12] F. Takens, " Detecting strange attractors in turbulence in Dynamical systems and turbulence", Lecture Notes in Mathematics, 898, pp. 363-381, Springer, 1998.
[13] J.P. Eckmann, S.O.Kamphorst, D.Ruelle, S.Ciliberto,"Liapunov exponents from time series", Phys. Rev. A34(6), pp. 4971-4979, 1986.
[14] Okamoto and H. Fujii, Nonlinear Dynamics, Iwanami Lectures of Applied Mathematics, Iwanami, Tokyo, 1995, vol. 14.
[15] Y. Bae, J. Kim, Y.Kim, " The obstacle collision avoidance methods in the chaotic mobile robot", 2003 ISIS, pp.591-594, 2003.
[16] Y. Bae, J. Kim, Y.Kim, " Chaotic behavior analysis in the mobile robot: the case of Chua's equation" , Proceeding of KFIS Fall Conference 2003, vol. 13, no. 2, pp.5-8, 2003.
[17] Y. Bae, J. Kim, Y.Kim, " Chaotic behavior analysis in the mobile robot: the case of Arnold equation" , Proceeding of KFIS Fall Conference 2003, vol. 13, no. 2, pp110-113, 2003.

저 자 소 개

Youngchul Bae received his B.S degree, M.S and Ph. D. degrees in Electrical Engineering from Kwangwoon University in 1984, 1986 and 1997, respectively. From 1986 to 1991, he joined at KEPCO, where he worked as Technical Staff. From 1991 to 1997, he joined Korea Institute of Science and Technology Information (KISTI), where he worked as Senior Research. In 1997, he joined the Division of Electron Communication and Electrical Engineering, Yosu National University, Korea, where he is presently a professor. His research interest is in the area of Chaos Nonlinear Dynamics that includes Chaos Synchronization, Chaos Secure Communication, Chaos Crypto Communication, Chaos Control and Chaos Robot etc.
E-mail : ycbae@yosu.ac.kr

Juwan Kim : He was born in Yosu, South Korea, on October 21, 1972. He received the B.E degree in Electronic Engineering from Sunchon Naiotnal University in 1998 and works currently under M.E course in Yosu National University, since 2001. He interests in chaos synchronization, secure communication, chaos robot.

Yigon Kim received the MS degree in avionic electric engineering from Hankuk aviation university, Seoul Korea, in 1986 and 1988 respectively. He received Ph.D. degree in electrical engineering from Chonnam National university in 1993. He performed research at Tokyo Institute of Technology by research member in 1991 and at Iowa State University by visiting professor in 2000-2001. He is professor in the School of ECC at Yosu National University. His interests fuzzy-Neuro Modeling and its application to diagnosis and control industrial systems.
E-mail : yigon@yosu.ac.kr