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# A Study on Wave Observation System with GPS Arrayed Buoys by using MUSIC Method

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**Abstract** : *The long-period gravity wave, the wave period from some ten seconds to some minutes, induces not only the big sway of a ship moored and berthed in the harbor due to the horizontal long-distance motion of a water but also strong exfoliated flow and vortices near the harbor entrance. They cause serious problems on the safety navigation of vessels entering and leaving the harbor, but this gravity wave has not been searched sufficiently yet. Then it is quite important to reveal the characteristics of this long-period gravity wave and to solve various problems induced by this wave. The long-period gravity wave measurement system with arrayed buoys installed the kinematic GPS was already proposed, which provides the precise propagating direction of the long-period gravity wave. In this paper, the observation results of the wave measurement system are shown by the MUSIC method. And the propagating wave direction was estimated precisely enough in comparison with other results used other method.*

**Key words** : *Long-period gravity wave, MUSIC spectrum, Wave direction measurement system, Kinematic GPS, Arrayed buoys*

## 1. Introduction

Designing a harbor, various problems caused by a general gravity wave (the wave period of about 0.1-30 seconds) are considered to be solved well already in the field of coastal and harbor engineering. However, in spite of the fact that the long-period gravity wave (the period from some ten seconds to some minutes) also causes serious problems, this gravity wave has not been searched and solved sufficiently yet.

Since a long-period gravity wave is thought to be propagated from the open sea to coastal waters and its wave energy has a small propagating and reflection decrement, most parts of wave energy are kept and it may grow in the case of resonance with the harbor inherent period. Therefore, it induces dangerous swaying movements of a moored or anchored ship in a harbor, and also causes various problems concerning with the mooring line's strength and cargo handling works.

Furthermore, it induces the strong exfoliated flow and vortices near the harbor mouth which bury a harbor by drifting the sand at the bottom of the sea, it may cause serious and security problems on the safety navigation of vessels entering and leaving the harbor at a narrow area near the harbor entrance (Sasa, 1997).

Therefore it's quite important to reveal the characteristics of this long-period gravity wave for solving various problems induced by this wave, and there were already various trials to estimate a wave direction by radar (Ishida, 1998) or three directional movements measured precisely by the kinematic GPS. Especially the wave observation system by the arrayed buoys in double triangular forms with the kinematic GPS was proposed and evaluated already by some simulation results. And also the estimated accuracy of the wave direction and the allowable limit of the span distance between buoys was discussed also on a condition that five buoys were used (Fujii, 2002 a ; Fujii, 2002b ; Fujii, 2002c ; Fujii, 2003).

In this paper, we explain the observation environments of the above wave measurement system in which five buoys were arrayed in double triangular forms with the kinematic GPS, and show the MUSIC (the MUltiple SIgnal Classification) method is applied to estimate the wave direction. This method estimates the wave direction by using the eigenvector and eigenvalues of the correlation matrix, and is known as one of the precise methods for estimating the wave direction (上坂, 2000, 芦野, 1997, 菊間, 1998). And then the propagating wave direction derived by this measurement system was estimated precisely enough.

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## 2. Wave Observation System

### 2.1 System Configuration

The proposed wave observation system measures three-dimensional movements of each observational buoy relative to the fixed reference station by the kinematic GPS. Because the distances between the reference station and each buoy are about several kilometers at most, the measurement error of the kinematic GPS can be estimated less than a few cm.

The data of each observational buoy are sent to the reference station with radio, then the data from each observational buoy are used to estimate an existence and frequency of the long-period gravity wave. Finally, the wave direction at the frequency of the long-period wave is estimated. On this occasion, the number of observational buoys depends on the number of expected waves. The configuration of the proposed system is shown in Fig.1.

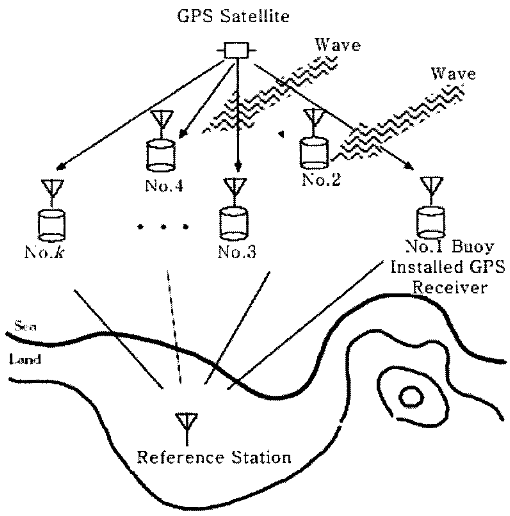


Fig. 1 Configuration of the proposed wave observational buoy system

### 2.2 Estimation Method of Wave Direction

1) MUSIC Method

The descriptive principle of MUSIC method is described below.

Several observational buoys, the number of buoys is selected properly, are aligned, and the kinematic GPS receiver installed at each buoy measures the three-dimensional buoy's movements. The basic array geometry and the situation of a wave propagating are shown in Fig.2.

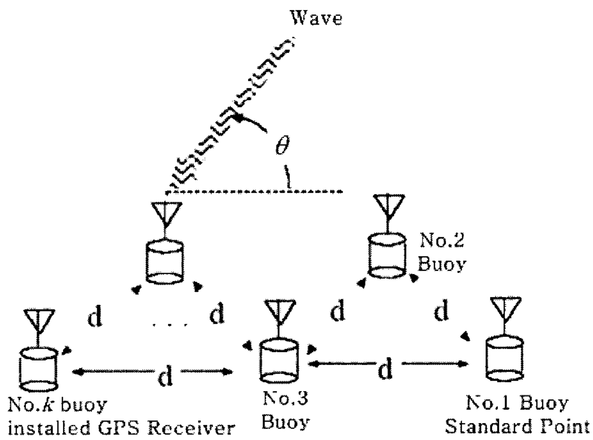


Fig. 2 Buoy alignment and wave propagation

When $L$ waves, $F_l(t)(l=1,\cdots,L)$, come at each angle $\theta_l$, a phase difference between two buoys is obtained from the span distance between each buoy. When No.1 observational buoy is selected as the standard buoy, the phase vector of the $k$ observational buoy is expressed as follows,

$$V_l = [1,\ \exp j\frac{2\pi}{\lambda}(\frac{d}{2}\cos\theta_l + \frac{\sqrt{3}d}{2}\sin\theta_l), \cdots,$$
$$\exp j\frac{2\pi}{\lambda}(\frac{d(k-1)}{2}\cos\theta_l + \frac{\sqrt{3}d1+(-1)^k}{4}\sin\theta_l)]^T \quad (1)$$
$$\equiv a(\theta_l)$$

where $d$ is the each span distance between buoys, $\theta_l$ is the incident angle of the wave $F_l$, $\lambda$ and $K$ are the wavelength and the number of observational buoys respectively.

Therefore, when the wave propagates into the observational buoy area, the input vector $X(t)$ (the height data) of the wave at each observational buoy is,

$$X(t) = [x_1(t), x_2(t), \cdots, x_k(t)]^T \quad (2)$$
$$= AF(t) + N(t) \quad (3)$$

where $F(t)$ is an arrival wave vector, $N(t)$ is a noise vector of each observational buoy. And the matrix $A$ called direction vector is obtained as follows,

$$A = [a(\theta_l), a(\theta_2), \cdots, a(\theta_L)] \quad (4)$$

Furthermore, the correlation matrix $R_{xx}$ of the input vector $X(t)$ is given,

$$R_{xx} = E[X(t)X^H(t)] \quad (H\text{:complex conjugate transposition})$$
$$= ASA^H + \sigma^2 I \quad (5)$$

where $E$ means the statistical expectation and $\sigma^2$ is the noise of each observational buoy. The $S$ called as an signal (or source) correlation matrix is defined as eq.(6).

$$S = E[F(t)F^H(t)] \tag{6}$$

If all waves are supposed to be uncorrelated each other, the above source correlation matrix $S$ can be expressed,

$$S = \begin{bmatrix} P_1 & 0 & \cdots & 0 \\ 0 & P_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & P_L \end{bmatrix} \tag{7}$$

where $P_l \equiv E[\,|\,F_l(t)^2\,|\,]\ (l=1,\cdots,L)$ is the input power spectrum of propagating wave at each buoy. Because the correlation matrix $R_{xx}$ is a Hermitian matrix with a rank $L$, the number of eigenvalues greater than the noise power $\sigma^2$ is equal to $L$(the number of the estimated waves).

Therefore, the directional spectrum of the wave ( $P_M$ is hereafter called the MUSIC spectrum) is given in the next equation by the MUSIC method if the number of waves can be decided,

$$P_M = \frac{a^H(\theta)a(\theta)}{a^H(\theta)E_N E_N^H a(\theta)} \tag{8}$$

where $E_N$ is the row vector that consists of eigenvectors $e_i(i=L+1,\cdots,K)$ on condition that an eigenvalue becomes $\sigma^2$. Therefore $E_N$ is expressed as follows,

$$E_N = [e_{L+1},\cdots,e_K] \tag{9}$$

where $K$ is the number of buoys. In addition, $a(\theta)$ is called the mode vector and is obtained from the next equation of the weight vector $W$ in which $\theta$ is changed to investigate a maximum peak of wave energy,

$$\begin{aligned} W = [1,\ &\exp j\frac{2\pi}{\lambda}(\frac{d}{2}\cos\theta + \frac{\sqrt{3}d}{2}\sin\theta),\cdots, \\ &\exp j\frac{2\pi}{\lambda}(\frac{d(k-1)}{2}\cos\theta + \frac{\sqrt{3}d1+(-1)^k}{4}\sin\theta)]^T \\ \equiv\ &a(\theta) \end{aligned} \tag{10}$$

2) Spatial Smoothing for Coherent Waves

By the reason that a coherent wave degrades the rank of the source correlation matrix $S$, the direction of the long-period gravity wave could not be estimated well. By using the spatial smoothing method, the rank $S$ can be recovered to the full rank. The spatial smoothing method is described below.

When the given $K$ (the number of arrays) arrays aligned in triangular form are subdivided into $N$ sub-arrays, the input vector and the correlation matrix of the $n$-th sub-array can be expressed as eq.(11) and eq.(12).

$$X_n(t) = [x_{2n-1}(t), x_{2n}(t), x_{2n+1}(t)]^T\ (n=1,2,\cdots,N) \tag{11}$$

$$R^n_{xx} = E[X_n(t)X^H_n(t)] \quad (n=1,2,\cdots,N) \tag{12}$$

The average of sub-array correlation matrix is

$$\overline{R}^n{}_{xx} = \frac{1}{N}\sum_{n=1}^{N} R^n_{xx} \tag{13}$$

$$= A\overline{S}A^H + \sigma^2 I \tag{14}$$

Here, if the number of sub-arrays is greater than the number of waves, the average source correlation matrix $\overline{S}$ recovers to the full rank. As a consequence of the above reason, the propagating directions of coherent waves can be estimated as well as the directions of non-coherent waves.

## 3. Observation and Analysis

### 3.1 Observation Conditions

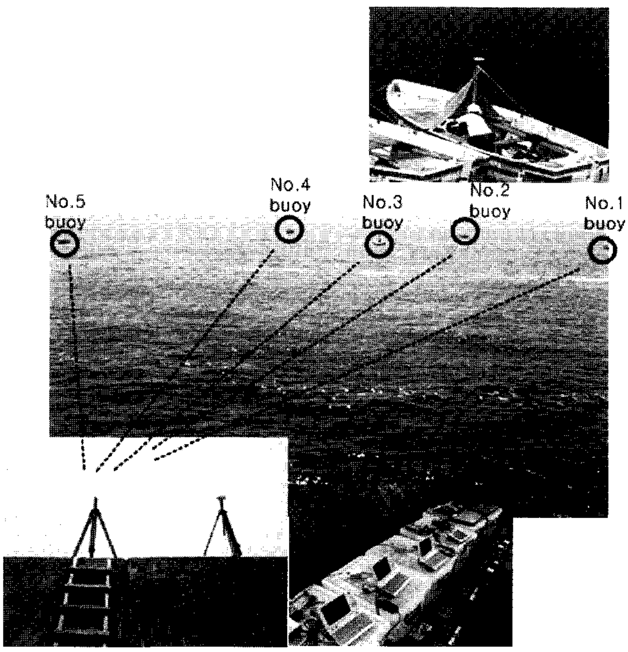


Fig. 3 Outline of observation condition

In this study, real observation data about three directional movements of buoys, which was obtained in the Bay of Osaka adjacent to the Fukae Campus of Kobe University, were used to estimate the propagating wave direction. And instead of real buoys, five small boats installed the kinematic GPS were used. Fig.3 shows the outline of this

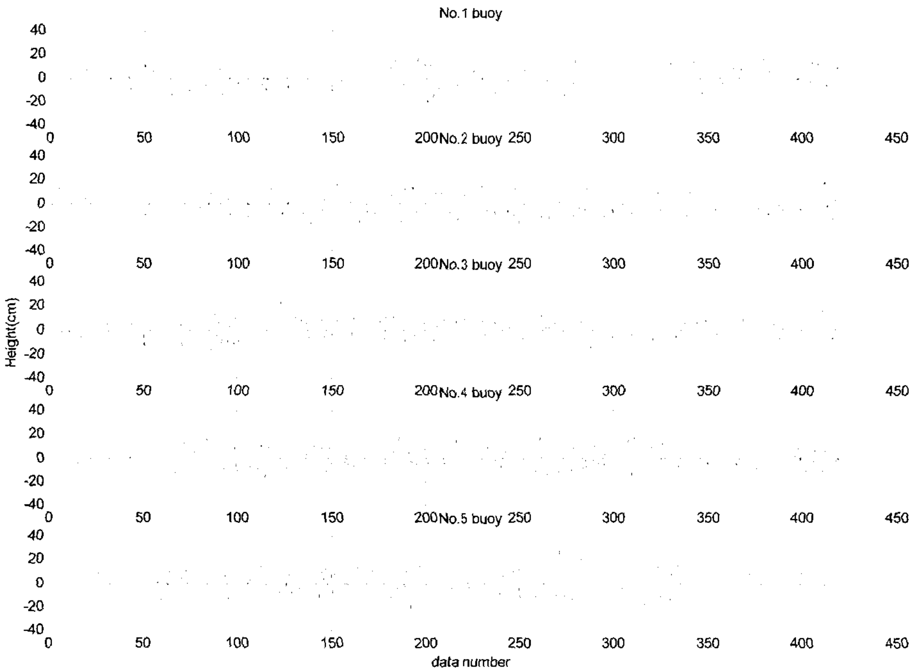


Fig. 4 Height data obtained from five buoys by the kinematic GPS

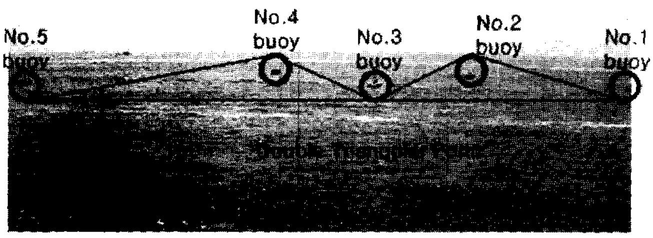


Fig. 5 Configuration of observation system without spatial smoothing method

observation condition including the small boat and the fixed reference station. The detailed observation conditions are described as follows.

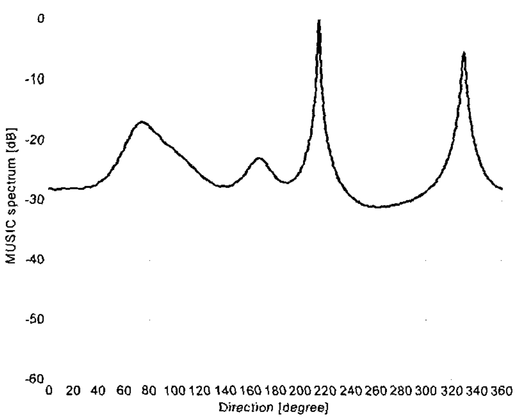


Fig. 6 Result of observation system without spatial smoothing method

(1) Observation was done on September 5 in 2002 because of the fact that a gravity wave was expected to come due to frequent typhoons around observation area.

(2) The distance of each buoy was about 50m, and three kinds of data (longitude, latitude and height data) were measured from each buoy.

(3) The data sampling interval was one second, and 420 sampled data having the most stable movements were used.

(4) Wave frequency having the most long period after FFT was 0.08Hz.

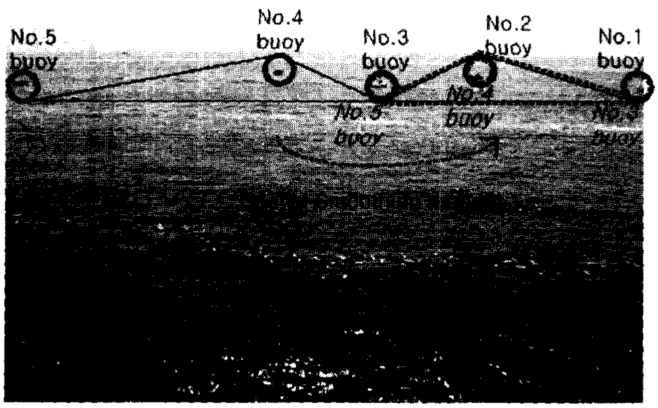


Fig. 7 Configuration of observation system with spatial smoothing method

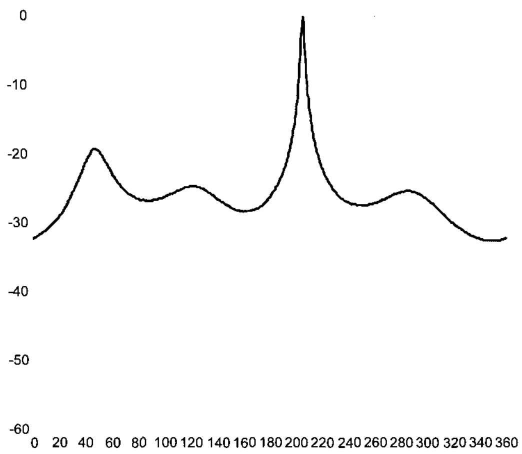


Fig. 8 Result of observation system with spatial smoothing method

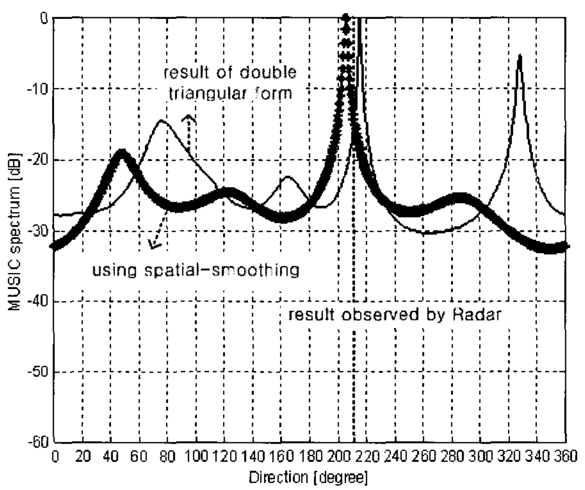


Fig. 9 Comparison of two results of the proposed system with that of Radar

In this study, the MUSIC method uses only the height data to estimate the wave direction. Fig.4 shows the height data obtained from five buoys by the kinematic GPS. All buoys were moved slightly while observing, but it was known already by the simulation results that their movements can be negligible.

### 3.2 Result of Analysis

Fig.5 shows the configuration of the observation system using five buoys in double triangular form without spatial smoothing method. Because the number of observational buoys is decided by that of expected waves, four wave directions could be estimated in this observation system. Fig.6 shows the MUSIC spectrum defined as $10\times \log_{10}(P_M/P_{Max})$ at each direction, where $P_{Max}$ is the maximum value of $P_M$ . This observation result shows that there are four waves at 70, 168, 212 and 328 degrees respectively.

Fig.7 shows the configuration of the observation system with spatial smoothing method. Five buoys were subdivided into two sub-arrays (consisted of No.1-2-3 and No.3-4-5 arrays). Fig.8 indicates the MUSIC spectrum by this method, and shows that there are two waves at 50 and 208 degrees.

In order to show the effectiveness of the proposed observation system, two results of this observation were compared with that observed by a Radar. In Fig.9, a bold line and a solid line show the MUSIC spectrum with and without spatial smoothing method respectively, and a dot line indicates the direction of the wave obtained by a Radar.

The direction of the wave observed by a Radar was about 210 degree, and Fig.9 shows that the results of our proposed observation system is in quite good coincidence with the result observed by Radar. However, by a FFT of the height data, we obtained a 12.5sec wave as the most long period wave, it is thought that the long-period gravity wave didn't come at this time.

## 4. Conclusion

In this paper, we explained the wave measurement system in which five buoys were arrayed in double triangular forms with the kinematic GPS, and showed the MUSIC method for finding the wave directions with a spatial smoothing.

From the result of double triangular form, it was thought that the coherent wave was not in this experiment, and by comparing two results of this observation with that observed by a Radar, we found that the results of our proposed observation system is in quite good coincidence with that of Radar.

In this observation, only 420 data obtained during seven minutes from five buoys were used to estimate the wave direction, but if we would increase buoy numbers and observation time, it is considered to be earned more good result.

Unfortunately we could not measure any waves more than having 12.5sec period in this experiment, but it is thought that this system is adjustable to estimate the long-period gravity wave. In addition some general waves, comparatively having short period, also can be estimated by shortening the sampling period in this wave measurement system, if it is necessary.

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