

# Application of Composite Grid Method for the Simulation of Oscillating Body

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**Abstract :** *The main objective of this study is to estimate the hydrodynamic forces and to investigate the nonlinear behaviors of fluid motion around the oscillating body on or below a free surface. We have developed a composite grid method to solve the radiation problems. This method is applied to numerical computation of the radiation forces generated by the oscillating body. The numerical results obtained by the present method are compared with the experimental data and a linear potential theory. As a result, we can confirm the accuracy of the present method. Finally, we have evaluated the effect of viscosity on the hydrodynamic forces acting on the oscillating body.*

**Key words :** *Composite grid method, Heave radiation force, Viscous damping, Submerged-plate*

## 1. Introduction

The free surface effect can strongly influence the added mass and damping coefficient values as a function of the frequency when the submerged body oscillates near a free surface (Chung, 1977). Also the flow is highly nonlinear even at very small amplitude of oscillation, when the body is slightly submerged. The purpose of this study is to predict and to understand the hydrodynamic forces and their nonlinear behaviors of fluid motion around the oscillating body on or below the free surface. To achieve this objective, we have developed a composite grid method for the solution of the radiation problem. In case of the radiation problem, it is difficult to deal with the relative motion between the moving body and free surface. Thus we divide the domain into two different grids; one is a moving grid system and the other is a fixed grid system. The moving grid is employed the body fitted coordinate system. The advantages of this approach are that the complex domain is dealt with more easily and it can be used to follow the moving body.

This numerical method is applied to calculation of the radiation forces generated by the oscillating body near the free surface. The numerical results obtained by the present method are compared with the experimental data and a linear potential theory, and we discuss the effect of nonlinear and viscous damping on the hydrodynamic forces acting on the oscillating body.

## 2. Composite Grid System

### 2.1 Composite Grid Method

The composite grid method has some attractive features. It allows without additional difficulty calculation of flows around bodies which move relative to the environment or each other. Each grid is attached to one reference frame, including some which move with the bodies. In such a case, the overlap region changes with time and has to be determined together with the interpolation factors after time step. The grid has to be recalculated after time step in any problem containing a moving body. The only additional effort is the interpolation from one reference frame to the other at the interfaces. Grids of this kind are called *Chimera* grids in the literature (Ferziger and Peric, 1999).

The composite grid consists of two different grid systems, one is a moving grid system and the other is a fixed grid system. The moving grid system is applied for the oscillating body near a free surface to deal with the relative motion between the moving body and free surface, while the fixed grid system covers the surrounding of moving grid and whole of the computational domain. The flow information is transferred from one grid to another by an interpolation procedure.

The pressures are computed simultaneously on the entire flow field until convergence, while the momentum equations are solved independently on each sub-domain. Schwarz had proposed an alternating solution procedure for elliptic function problems (Hinatsu, 1991). Therefore the Schwarz iteration is used to calculate the pressure equation over the

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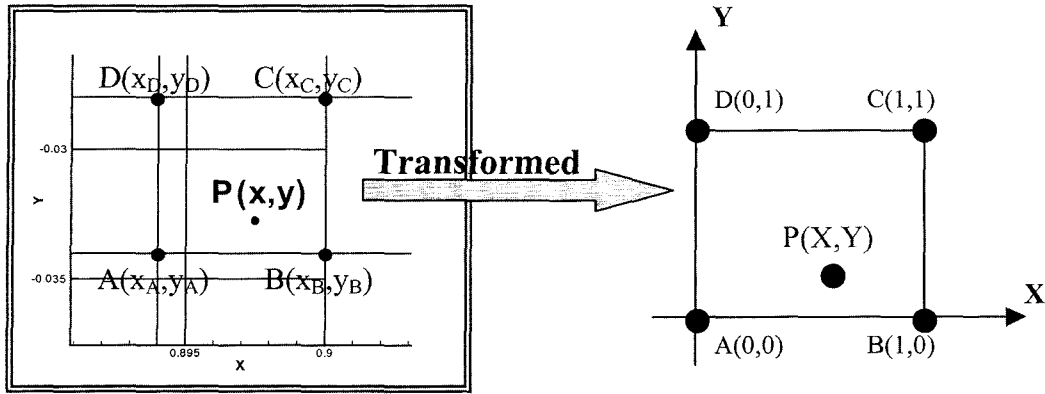


Fig. 1 Transformation of physical coordinate for Newton-Raphson interpolation

composite grid.

In order to use the Schwarz iteration, we need the interior boundary value which lays in the overlap region and is obtained by interpolating from the other grid.

## 2.2 Interpolation Method

The Newton-Raphson interpolation method is employed at the different grids to transmit the flow data from one grid to another.

The overlap regions change with time and the flagging of all the grid points of both grids is performed after each time step. In the overlap region, boundary conditions for one grid are obtained by interpolating from the other grid.

The interpolated data can be expressed by

$$\phi = (1-X)(1-Y)\phi_A + X(1-Y)\phi_B + XY\phi_C + (1-X)Y\phi_D \quad (1)$$

where  $\phi$  is the interpolated flow data,  $\phi_A, \phi_B, \phi_C$  and  $\phi_D$  are the value at the corner A, B, C and D of the cell, respectively.  $(X, Y)$  is the local coordinate of the interpolation point in the transformed system obtained by solving the following equations:

$$x = (1-X)(1-Y)x_A + X(1-Y)x_B + XYx_C + (1-X)Yx_D \quad (2)$$

$$y = (1-X)(1-Y)y_A + X(1-Y)y_B + XYy_C + (1-X)Yy_D \quad (3)$$

where  $(x, y)$  is the physical coordinate of the interpolated point.

Fig.1 shows the transformation of the physical coordinate to the local coordinate for using the Newton-Raphson interpolation method.

## 3. Numerical Procedure

The governing equations are the Navier-Stokes equation and the continuity equation for 2-dimensional,

incompressible and viscous fluid. They can be written as

$$\nabla \cdot \vec{U} = 0, \quad (4)$$

$$\frac{\partial \vec{U}}{\partial t} + \nabla \cdot (\vec{U} - \vec{V})\vec{U} = -\frac{\nabla P}{\rho} + \nu \nabla^2 \vec{U} - g \quad (5)$$

where  $\rho$  is the fluid density,  $\vec{U} = (u, w)$  is the fluid velocity vector,  $\vec{V}$  is the moving velocity of grid,  $P$  is the pressure,  $\nabla$  is the gradient operator,  $\nu$  is the kinematic viscosity, and  $g$  is the gravitational acceleration. By comparing with the fixed grid system, the velocity of moving grid is included in the convective terms (Demirdzic, 1990). The same equations (4) and (5) are used for the fixed and moving grids with the exception that the moving grid velocity  $\vec{V}$  becomes zero on the fixed grid. The computational procedure is similar to the modified TUMMAC- $V_{uv}$  method, which incorporate the subgrid-scale (SGS) turbulent model (Lee, 1990).

The velocity components are advanced explicitly and the pressure is obtained by solving a Poisson equation simultaneously using the Schwarz iterative method on the entire domain. The momentum equations are solved independently on each sub-domain. Interpolation on the overlapping grids is computed by the Newton-Raphson method. Lee et al. (2003) has presented the detailed description of the Schwarz iteration and Newton-Raphson interpolation method.

Zero-normal gradient conditions are given for the velocity and pressure at the bottom and outflow boundaries of the computational domain. We assume the axis symmetry condition on the center of entire domain.

The body is forced to heave in the form of

$$z(t) = z_a \sin(\omega t) \quad (6)$$

where  $z_a$  is the amplitude of oscillation. On the body surface, no flux and no slip conditions are imposed by

$$u = 0 \quad ; \quad w = z_a \omega \cos(\omega t) \tag{7}$$

The body is set into motion from a quiescent state, that is, the velocity and free surface elevation are zero at initial time  $t = 0$ .

### 4. Application of the Composite Grid Method

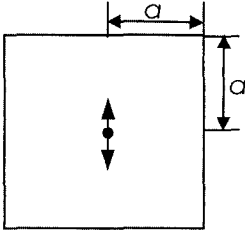
#### 4.1 Validation of the Composite Grid Method

Numerical calculation was made to test and validate the accuracy of the results of composite grid method. The performance of the present method has been validated by comparing the results with those from single-grid method and those from experiments. The first confirmation of the accuracy is made for two-dimensional rectangular body in infinite fluid. For infinite fluid, the added mass coefficients are constant and are independent of the oscillation frequencies for small amplitudes. Table 1 shows that the added mass coefficient was in good agreement with theoretical results obtained by other researchers for infinite fluid.

Furthermore, another calculation to check the accuracy of the CFD code results was performed for the hydrodynamic forces acting on a surface piercing rectangular body ( $L \times B \times d = 1.16m \times 0.4m \times 0.2m$ ). Fig.2 shows very good comparison for the radiation wave amplitude ( $\zeta_H$ ), added mass ( $M_H$ ) and damping force coefficients ( $N_H$ ) obtained by the present code, experiment and computation based on the potential theory (Takaki, 1984). Here,  $\lambda$  denotes the wave length of radiation wave. It can be seen that the results obtained by the present CFD code agree well with the experimental ones. The accuracy of these results demonstrates and

confirms that the present numerical method is able to successfully solve the hydrodynamic forces on submerged plate oscillating near a free surface.

Table 1 Added mass coefficient for two-dimensional rectangular body in infinite fluid

	$M_H$
	
Frank(1967)	$4.909 \rho a^2$
Newman(1977)	$4.754 \rho a^2$
Present Result	$4.801 \rho a^2$

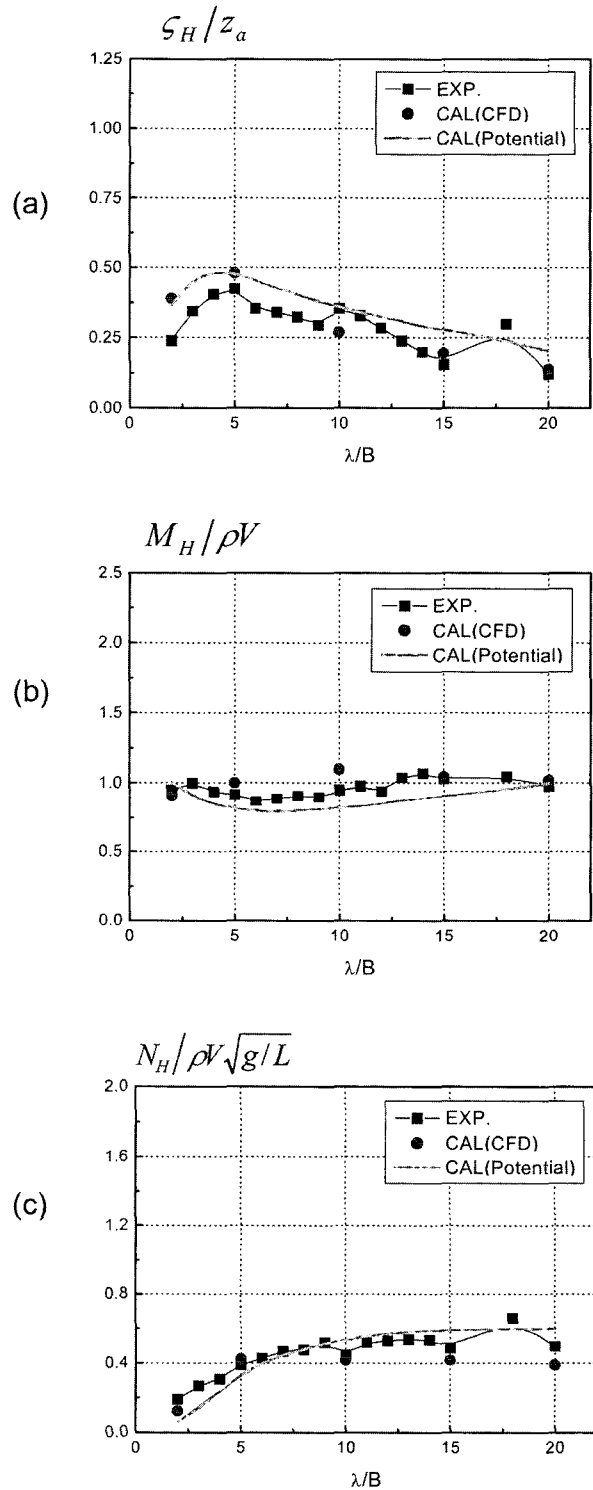
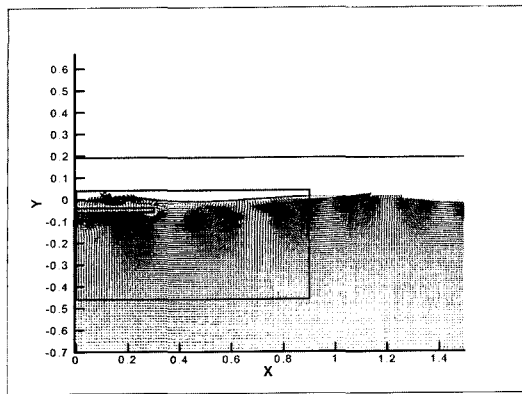
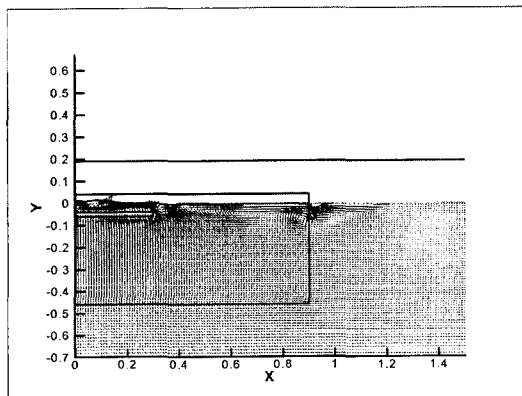


Fig. 2 Comparison of (a) radiation wave amplitude, (b) added mass and (c) damping force coefficient for the surface piercing rectangular body



(a)



(b)

Fig. 3 Velocity vector field for  $d=40\text{mm}$ ,  $z_a=10\text{mm}$  of (a)  $T=0.8\text{sec.}$  and (b)  $T=2.2\text{sec.}$

#### 4.2 Application for the Submerged-Plate

The forced heaving tests have been performed to investigate the characteristics of hydrodynamic forces on the submerged plate in the 2 dimensional wave tank of Hiroshima University, which dimensions are 42m in length, 1.2m in width and 2m in depth. The forced oscillation experiments have been carried out using the 1/50 scale model under the different test conditions;  $d = 20, 40$  and  $60\text{mm}$ ,  $Za = 10, 20$  and  $30\text{mm}$ ,  $T = 0.8\text{-}2.6$  seconds. We also have performed the numerical simulation based on the composite grid method under the above assumptions.

Fig.3 shows the velocity vector field of the submerged plate corresponding to the short oscillation period ( $T = 0.8$  sec.) and long oscillation period ( $T = 2.2$  sec.),  $d = 40\text{mm}$ , and  $Za = 10\text{mm}$ . Here, the inner grid is moving with the submerged plate. It can be observed that the flow incoming from the end of the submerged plate is splashed above the plate. Upon splashing, the surface flow is joined to the outgoing wave. It has been shown that the free surface is connected smoothly between the moving grid and fixed grid system, and the radiation waves are properly propagated to

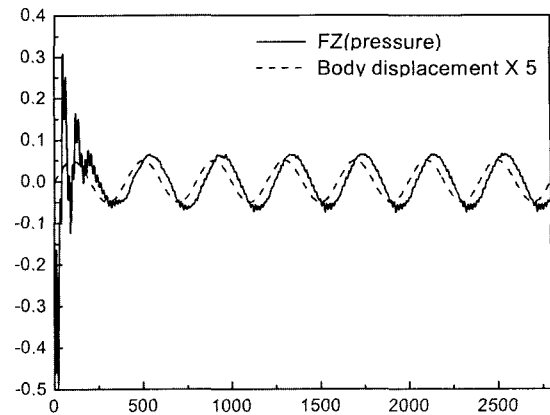


Fig. 4 Time history of the hydrodynamic force for  $T=0.8\text{sec.}$ ,  $d=40\text{mm}$ ,  $Za=10\text{mm}$

outward direction. At the long oscillation period, it can be seen that the vortex ring generated at the edge of the submerged plate is transported to the outward direction underneath the free surface. According to this fact, it is considered that the generation of vortex is affecting to the damping forces on the moving body.

Fig.4 shows the time history of the hydrodynamic force acting on the submerged plate corresponding to  $T = 0.8$  sec.,  $d = 40\text{mm}$ , and  $Za = 10\text{mm}$ . In the figure, a sine curve denoting body displacement is also plotted in order to depict the phase difference between the body motion and the heave hydrodynamic force. It can be seen that steady-state is reached within about two periods of oscillation. As shown in Fig.4, negative large value is found at the start of the simulation. The body motion is started impulsively at  $t = 0$  in a viscous fluid. However, this negatively large value seems to appear only for a short duration of time.

Figs.5 and 6 show the added mass ( $M_H$ ) and damping force coefficients ( $N_H$ ) corresponding to the different amplitude( $Za$ ) = 10 and 30mm at same submergence depth( $d = 40\text{mm}$ ). In order to investigate the effect of viscosity in the radiation problem, we compare the results of numerical simulation (CFD), linear potential theory and experiments in Figs. 5 and 6. By comparing these results, the results due to the numerical simulation based on the composite grid method agree well with the experimental ones regardless of the amplitude and oscillation periods. On the other hand, the results due to the linear potential theory agree with those of others at the short oscillation periods. But we notice that the linear theory results considerably deviate from others at the long oscillation periods. In general, it is noted that the differences between the linear theory and experiments increase at the large amplitude of

oscillation

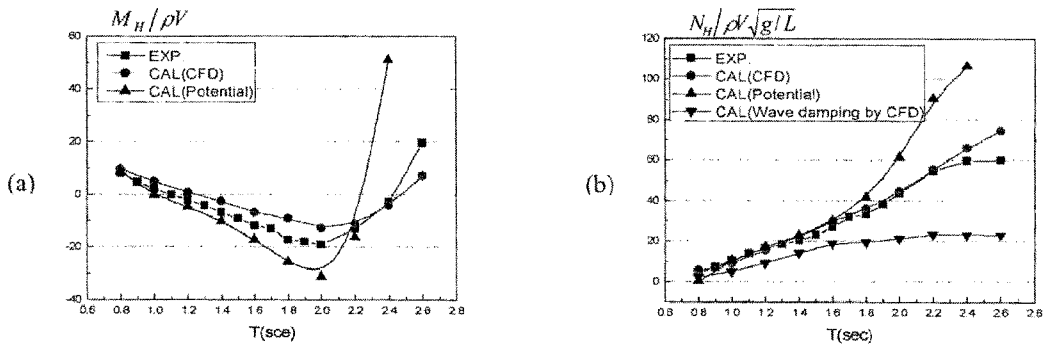


Fig. 5 Comparison of (a) added mass and (b) damping force coefficient with experimental values in the condition of  $d=0.04m$ ,  $Za=0.01m$

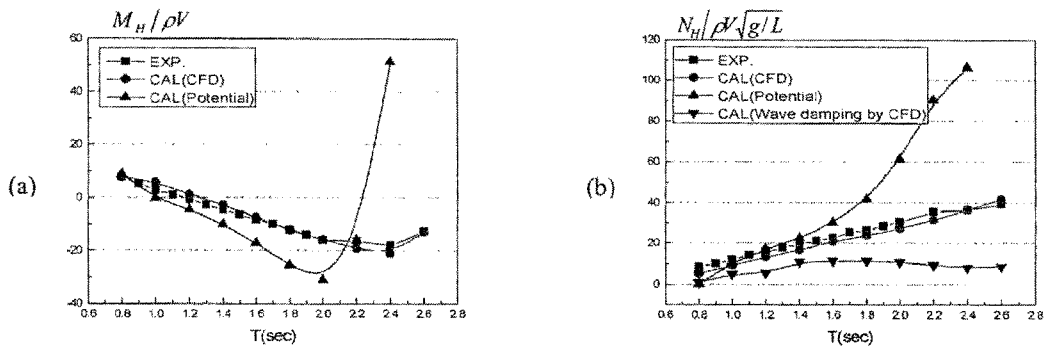


Fig. 6 Comparison of (a) added mass and (b) damping force coefficient with experimental values in the condition of  $d=0.04m$ ,  $Za=0.03m$

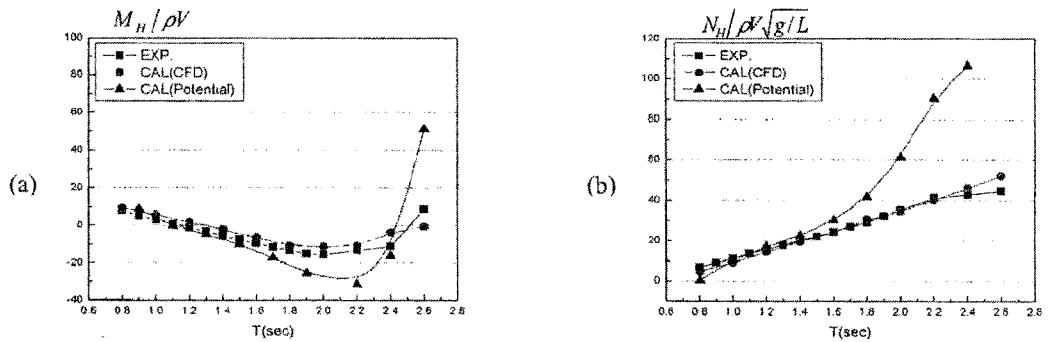


Fig. 7 Comparison of (a) added mass and (b) damping force coefficient with experimental values in the condition of  $d=0.04m$ ,  $Za=0.02m$

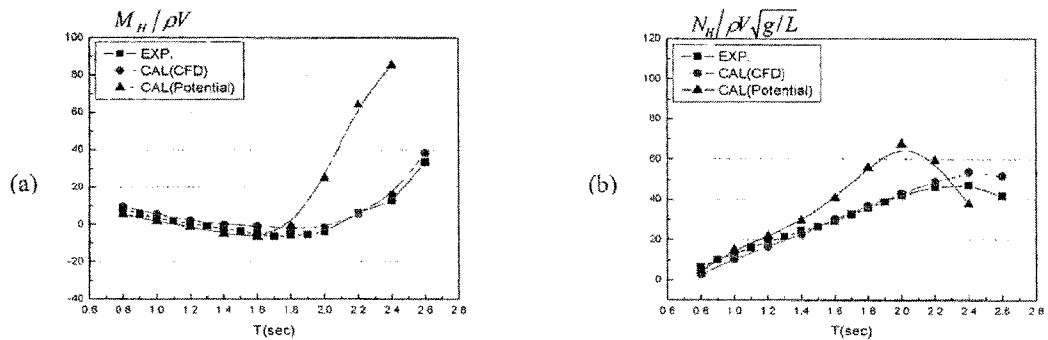


Fig. 8 Comparison of (a) added mass and (b) damping force coefficient with experimental values in the condition of  $d=0.06m$ ,  $Za=0.02m$

It can be observed that the components of viscous damping force are larger than those of wave damping force at the long oscillation periods. In summary, these results seem to confirm that the effect of viscosity on the hydrodynamic force is significant at the long oscillation periods.

It is noted that the added mass takes negative value for intermediate oscillation periods. Visual observations of the surface flows showed interferences of incoming flows of surface fluid over the top surface of the model. The heave oscillation of the submerged plate during downstroking at shallow submergence depth instantly created a depression on the surface of water above the submerged plate. This periodically depressed surface was filled instantly by incoming flows from both directions causing surface splashes at the surface on the top centerline of the model. Upon splashing, the flows appeared to be reflected in the outgoing directions, thus generating the higher frequency component in addition to the model frequency. Upstroking motion displaced water above the top surface of the model in the outgoing directions. The splashing or interference of the flows above the heaving model incoming from opposing directions appeared to generate nonsinusoidal outgoing waves. The splashing must have accompanied downward flows acting on the model. It is speculated that these interferences may have exerted external forces implicitly created by and acting on the heaving model to cause the negative added mass in the heave coefficient curve.

The added mass ( $M_H$ ) and damping force coefficients ( $N_H$ ) are shown in Figs.7 and 8 for the different submergence depth ( $d$ ) = 40 and 60mm at same amplitude ( $Za = 20$ mm). From Figs.7 and 8, we find that the deviations of linear theory results become larger with decrease of submergence depth. This means that the radiation force become highly nonlinear at the long oscillation periods due to the increase of viscous forces, and the free surface effect increases when the submerged plate come close to the free surface. Also we can observe the negative added mass for the intermediate oscillation periods at the shallow submergence depth ( $d= 40$ mm). On the other hand, at the deep submergence depth ( $d = 60$ mm) the negative added mass become smaller due to the decrease of free surface effect. Finally, we compare the results of damping forces between the shallow( $d= 40$ mm) and deep( $d = 60$ mm) submergence depth. As fore mentioned, we notice that the effect of viscosity on the hydrodynamic force is rather small at the short oscillation periods but quite significant at the long oscillation periods.

## 5. Conclusion

We have developed the numerical method for solution of the hydrodynamic forces on the oscillating body on or below a free surface using the composite grid method. Then we have applied this method for numerical computation of the radiation forces generated by the oscillating body. Through the computation by the present method, the following conclusions can be obtained:

- (1) The simulation results based on the composite grid method show a good agreement with the experimental ones. This confirms that the present numerical method is reliable and accurate to solve the nonlinear viscous flow problems.
- (2) The results obtained by experiment and numerical calculation for the submerged plate oscillating near a free surface show the occurrence of negative added mass at the intermediate oscillation periods.
- (3) The effect of viscosity on the hydrodynamic force is evaluated by comparing the results with those from linear theory and those from CFD. It is noted that the viscosity effect on the hydrodynamic force is rather small at the short oscillation periods, but quite significant at the long oscillation periods due to the vortical force.

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