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로그분포모형을 이용한 토양수분특성 추정

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Estimation of Water Retention Characteristics Using Lognormal Distribution Model

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요 약 문

황과 Powers(2003)는 입도분포와 공극크기분포에 로그분포함수를 적용하여, 입도분포로부터 토양수분특성을 직접 추정하는 간단한 모형을 개발하였다. 본 연구의 목적은 황과 Powers(2003)가 개발한 모형의 추정능력이 토성에 의해 영향을 받는가를 밝히는 것이다. 연구결과, 모형은 토성에 의해 영향을 받았고, 특히 토양내 세립질 분율이 커질수록 모형의 추정능력은 감소하였다. 또한 입도와 공극크기사이의 관계를 비선형으로 가정한 비선형모형이 선형모형보다 토성에 관계없이 그 추정능력이 크게 나타났다.

주제어 : 토양수분특성, 입도분포, 로그분포모형, 토성

ABSTRACT

Hwang and Powers (2003) developed a simple model for estimating water retention characteristic (WRC) directly from particle-size distribution (PSD) data, by applying a lognormal distribution law to both PSD and pore-size distribution. The objective of this work was to determine if the performance of the model developed by Hwang and Powers (2003) would be affected by soil texture. The results of this research proved that the performance of the model was indeed affected by soil texture. In particular, its performance diminished with increases in the fine particle fractions. Also, the nonlinear model, which assumes a nonlinear relation between particle-size and pore-size, performed better than the linear model, regardless of soil texture classes.

Key words : Water retention characteristics, Particle-size distribution, Lognormal distribution model, Soil texture.

1. Introduction

Water retention characteristic (WRC) is a basic soil property necessary for the study of plant-available water, infiltration, drainage, and contaminant transport. However, the high variability and the complexity of soil make direct determination of the soil water retention property costly, time-consuming, and subject to significant sources of error, especially in the hydrologic survey of a large area. Furthermore, direct measurement may not be suitable at the screening stage where detailed hydrologic survey is not needed. Therefore, an alternative method is to estimate this property indirectly by using more readily available information, such as particle-size distribution (PSD), bulk density, organic matter content, and porosity[1]. Numerous attempts have been made to relate the PSD and other properties to soil water retention data[2],[3]. However, most of the methods developed are based on statistical techniques. The applicability and accuracy of those models are limited for various reasons[4],[5].

A different approach is to develop models to predict the

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WRC from the PSD using mathematical expressions. Considerable efforts have been made in this research area[6,7]. Especially, Hwang and Powers[7] developed a simple model for estimating the WRC directly from the PSD, by applying a lognormal distribution law to the PSD and pore-size distribution and then combining with the WRC function developed by Kosugi[8]. They applied the model for sandy soils alone and found out that it performed reasonably well for estimating the WRC. They argued that the nonlinear model, which is based on a nonlinear relationship between particle-size and pore-size, performed better than the linear model. The objective of this work is to determine if the performance of the model developed by Hwang and Powers[7] can be affected by soil texture.

2. Hwang and Powers[7] Model

On the basis of the assumption that particle diameter (d) and pore radius (r) are lognormally distributed[9,10], the cumulative distribution functions, $F(\ln d)$ and $G(\ln r)$, are expressed as follows:

$$F(\ln d)=F_n\left(\frac{\ln d-\ln d_m}{\sigma_d}\right) \tag{1}$$

$$G(\ln r)=F_n\left(\frac{\ln r-\ln r_m}{\sigma_r}\right) \tag{2}$$

where $F_n(x)$ is the cumulative normal distribution function, d_m and r_m are the geometric mean particle diameter and pore radius, and σ_d and σ_r the geometric standard deviations. Kosugi[8] developed a lognormal distribution model for soil water retention on the basis of the lognormal pore-size distribution. The resulting water retention function, S_e ($\ln h$), is:

$$S_e(\ln h)=F_n\left(\frac{\ln h_m-\ln h}{\sigma_h}\right) \tag{3a}$$

$$S_e=(\theta-\theta_r)/(\theta_s-\theta_r) \tag{3b}$$

where $\ln h_m$ and σ_h are mean and standard deviation of $\ln h$, respectively, S_e is an effective saturation, and θ_r is a residual water content. The h_m is a capillary pressure head at S_e=0.5. Kosugi[8] provides details of the derivation of equation (3). The basis of the Hwang and Powers[7] model is that equations (1) through (3a) have the same form (i.e. cumulative lognormal distribution) and two parameters (i.e., geometric mean and standard deviation) determining its shape. On the basis of two assumptions, i.e., (1) the relationship between d and r is linear ($d = \gamma r$) or nonlinear ($d = ur^v$) and (2) the relationship between h and r is defined by the Young-Laplace equation ($h = 0.149/r$), the resulting model expressions are as follows:

For the linear relationship between d and r:

$$\ln h_m = \ln 0.149 + \ln \gamma - \ln d_m \tag{4a}$$

$$\sigma_h = \sigma_d \tag{4b}$$

For the nonlinear relationship between d and r:

$$\ln h_m = \ln 0.149 - (\ln d_m - \ln u)/v \tag{5a}$$

$$\sigma_h = \sigma_d/v \tag{5b}$$

where γ is a packing parameter of the soil, u and v are parameters related to the packing and the respective shapes of the particles and the corresponding pores.

The only parameters that need to be estimated for predicting the WRC are γ for the linear relationship, and u and v for the nonlinear one. Herein, equation (4) is named as the linear (L) model and equation (5) as the nonlinear (NL) model. Hwang and Powers[7] provide details for their model development.

The model developed by Hwang and Powers[7] does not describe hysteresis because of the assumption that the WRC represents only a single pore-size distribution. The lognormal pore-size distribution derived from the lognormal PSD can estimate the drainage curve. However, to account for hysteresis, we require the ability to model the wetting curve as well. It can be done by obtaining a distribution for the maximum opening of a pore. The scanning curves can be predicted by scaling the draining and wetting curves appropriately. This approach should be explored in the future.

3. Research Methods

Experimental WRC and PSD data were obtained from the Unsaturated Soil Hydraulic Database (UNSODA)[11]. The UNSODA contains measured PSD, soil water retention, hydraulic conductivity, and water diffusivity data as well as pedological information of some 790-soil samples from around world. The 159 undisturbed soil samples with more than four points on both the PSD and on the drying branch of the WRC measured in the laboratory were chosen from the UNSODA database. The soils chosen were classified by five aggregated United States Department of Agriculture (USDA) classes[12] (Table 1). The parameters in the L and NL models (i.e., γ or u and v) were calculated for each texture by using equations (4) or (5).

Parameters $\ln d_m$ and σ_d for each soil were estimated by fitting equation (1) to experimental PSD data using iterative nonlinear regression procedure that is provided in the Microsoft Excel software[13]. Parameters $\ln h_m$ and σ_h for a soil were estimated by fitting equation (3) to experimental water retention data. Parameters (i.e., γ or u and v) in equations (4) and (5) were optimized for each texture class, using minimization of the following objective function.

$$O(\mathbf{b})=W_h\sum_{i=1}^{N}\{\ln h_{m,i}-\overline{\ln h_{m,i}}(\mathbf{b})\}^2+W_\sigma\sum_{i=1}^{N}\{\sigma_{h,i}-\overline{\sigma_{h,i}}(\mathbf{b})\}^2 \quad (6)$$

where and $\overline{\ln h_{m,i}}$ are observed and predicted logarithm of capillary pressure head at S_e=0.5, and $\overline{\sigma_{h,i}}$ are observed and predicted standard deviation of ln h, **b** is the trial parameter vector (i.e., γ or u and v), N is the number of soils for each texture, W_h and W_σ are the weighting factors. The goodness-of-fit of the L or NL models was quantified with the root mean squared error (RMSE) for each texture class:

$$RMSE=\sqrt{\frac{O(\mathbf{b})}{2N}} \quad (7)$$

4. Results and Discussion

Average and standard deviation for estimated ln d_m and σ_d values, which together describe the cumulative PSD function of equation (1) for a soil, are presented in Table 2. As expected, ln d_m decreased with finer textured soils. The σ_d increased with finer textured soils because inclusion of fine particles made the PSD wider (Table 2). Note that standard deviation of estimated ln d_m and σ_d values was higher for moderately fine-textured soil. Average and standard deviation of the r^2 values for fitting equation (1) to experimental PSD data for a soil are also presented in Table 2. The lowest average and highest standard deviation values of r^2 were obtained in moderately fine-textured soils, whereas coarse- and fine-textured soils showed higher average and smaller standard deviation of r^2 values. This result indicates that the PSD of soils dominated by sand or clay fractions follows exactly the lognormal distribution. As a result of fitting equation (3) to experimental WRC data, average values of estimated ln h_m and σ_h generally increased with finer textured soils because of smaller pore size and wider pore-size distribution (Table 2). Standard deviation of estimated σ_h values was higher for moderately fine-textured soil. The lowest average and highest standard deviation values of r^2 were obtained again in moderately fine-textured soils (Table 2). It was interesting to find out that standard deviations of ln h_m and σ_h values were higher than those of ln d_m and σ_d values, respectively (Table 2). This result indicates that the

Table 1. Soils selected from the UNSODA soil hydraulic database used by this study

Texture Classes1)	No. of soils	UNSODA No.
Coarse	60	1010, 1012, 1013, 1020, 1021, 1023, 1024, 1030, 1031, 1041, 1043, 1051, 1052, 1053, 1054, 1060, 1062, 1063, 1070, 1072, 1073, 1075, 1090, 1100, 1111, 1110, 1141, 1142, 1160, 2101, 2102, 2104, 2105, 2110, 2342, 3131, 3132, 3144, 3330, 3332, 3340, 4000, 4011, 4020, 4021, 4051, 4060, 4061, 4062, 4130, 4132, 4140, 4142, 4150, 4151, 4340, 4341, 4521, 4651, 4661
Moderately coarse	19	1091, 1120, 1131, 1380, 3010, 3291, 3300, 3310, 3311, 3320, 3323, 4100, 4111, 4112, 4161, 4162, 4170, 4172, 4571
Medium coarse	50	1211, 1281, 1282, 1340, 1342, 1350, 1352, 1370, 1490, 2000, 2002, 2010, 2011, 2321, 2351, 2530, 3090, 3240, 3242, 3250, 3253, 3260, 3262, 3264, 3293, 3302, 4031, 4032, 4033, 4040, 4042, 4043, 4071, 4080, 4081, 4090, 4092, 4101, 4181, 4182, 4183, 4531, 4540, 4570, 4572, 4573, 4575, 4590, 4671, 4673
Moderately Fine	22	1092, 1103, 1104, 1113, 1122, 1166, 1192, 1194, 1213, 1362, 1372, 1382, 3101, 3103, 3111, 3113, 3241, 3251, 3292, 3312, 4450, 4601
Fine	8	1360, 1383, 1400, 2360, 2362, 3030, 4121, 4681
Total	159	

1) Soils used in this work were classified by five aggregated USDA classes (Soil Survey Division Staff, 1993).

Table 2. Fitting results for the PSDs and WRCs for 159 soils used by this study

Texture Class	No. of Soils	PSD			WRC		
		ln d_m (cm in d_m)	σ_d	r^2	ln h_m (cm in h_m)	σ_h	r^2
Coarse	60	−3.794±0.5421)	0.728±0.205	0.992±0.009	3.859±0.765	1.084±0.481	0.988±0.012
Moderately coarse	19	−5.059±0.482	1.415±0.215	0.972±0.020	6.217±0.752	2.110±0.383	0.991±0.006
Medium coarse	50	−6.378±0.617	1.493±0.577	0.969±0.024	6.447±1.096	2.333±0.699	0.986±0.021
Moderately fine	22	−6.528±1.324	2.168±0.589	0.941±0.042	6.196±1.625	2.395±1.009	0.974±0.051
Fine	8	−8.514±0.543	2.396±0.424	0.990±0.008	8.643±2.044	2.916±0.774	0.989±0.015

1) Average and standard deviation values.

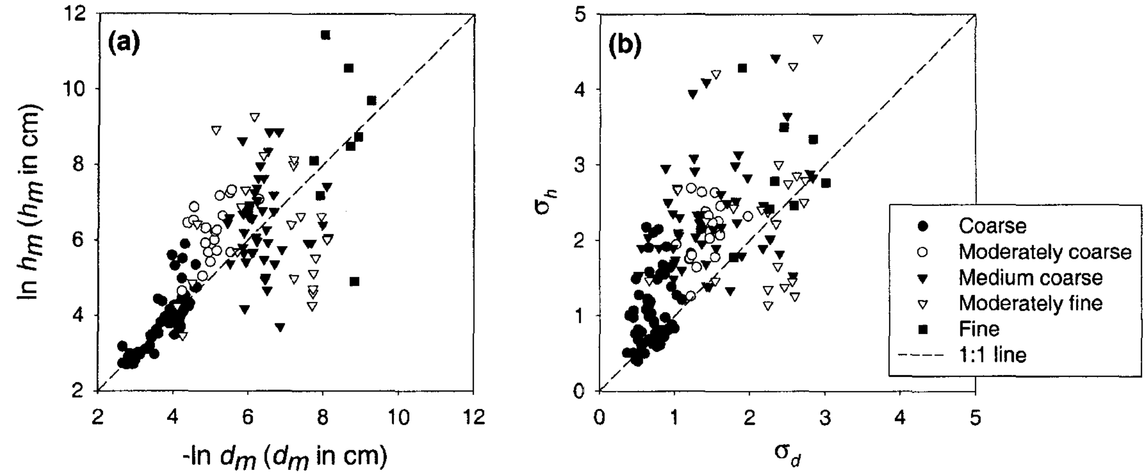

Fig. 1. Scatter plot of (a) $-\ln d_m$ vs. $\ln h_m$ and (b) σ_d vs. σ_h pairs.

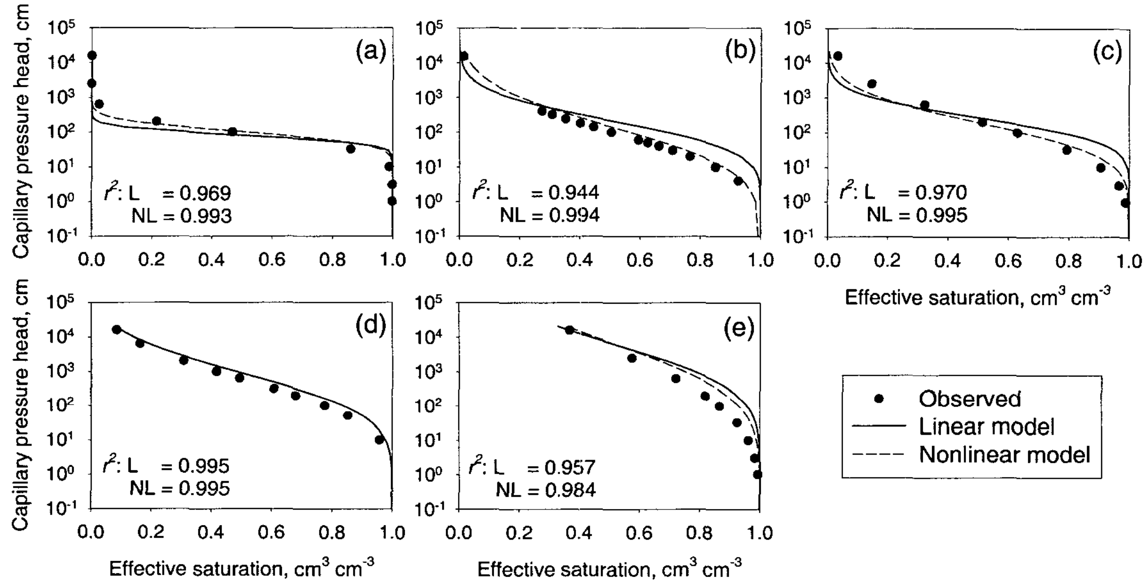

Fig. 2. Examples of observed vs. predicted WRCs for each texture using the L and NL models: (a) coarse-textured soil (UNSODA No. 4061), (b) moderately coarse-textured soil (UNSODA No. 1091), (c) medium coarse-textured soil (UNSODA No. 4031), (d) moderately fine-textured soil (UNSODA No. 1362), and (e) fine-textured soil (UNSODA No. 4121).

PSDs within each texture class were more homogeneous than the corresponding WRCs because the WRC in undisturbed soils might be affected by many field heterogeneities (e.g., root channels, microcracks, or macropores).

Values for $\ln h_m$ vs. $-\ln d_m$ pair for each soil sample are presented in Fig. 1a. In the region with $\ln h_m$ and $-\ln d_m$ values between 2 and 4, most data points were concentrated on the 1:1 line. However, in the region with larger $\ln h_m$ value (>4), the data points were dispersed along the 1:1 line. This result indicates that soils including finer particles showed more heterogeneous WRCs due to field conditions. Parameter σ is the standard deviation of the lognormal distribution and controls the magnitude of changes in F ($\ln d$), G ($\ln r$), or S_e ($\ln h$) around the inflection point. The value of σ_d (or σ_h) is small for a soil with a narrow PSD (or pore-size distribution) and is large for a soil that has a wide PSD (or pore-size distribution). Therefore, σ_h value will increase with the increase of σ_d. Values for σ_d vs. σ_h pair for each soil sample are presented in Fig. 1b. In most regions, the σ_h values were greater than σ_d values, indicating that the L model (equation 4b) may not represent exactly the slope of the WRC.

In order to estimate the parameters (i.e., γ for the L model, and u, v for the NL model), equations (4) and (5)

Table 3. Parameter values from model fitting and RMSE values as goodness-of-fit

Texture Class	No. soils	Linear		Nonlinear		
		γ	RMSE	u	v	RMSE
Coarse	60	7.16	1.455	1.238	0.70	1.181
Moderately coarse	19	21.35	1.684	1.617	0.68	0.942
Medium coarse	50	7.19	2.401	0.731	0.73	2.087
Moderately fine	22	4.81	2.420	4.617	1.00	2.420
Fine	8	7.64	2.452	1.516	0.85	2.271

were optimized to the estimated ln h_m vs. ln d_m and σ_d vs. σ_h pairs for each texture class, using equation (6). The resulting parameter values and RMSEs are presented in Table 3. Typical examples of predicted and experimental WRCs for a range of textures are presented in Fig. 2. The NL model performed better for most texture classes. For moderately fine-textured soils, the curves from the L and NL model were too close to be differentiated. This trend was confirmed by the RMSE values (Table 3). The RMSE values of the NL model were smaller than those of the L model for most texture classes, whereas they were the same for moderately fine-texture soils (Table 3). Relatively poor performance of the models for moderately fine-textured soils results partly from poorer fitting for both PSDs and WRCs in that texture class (see r^2 values in Table 2).

The RMSE values of the L model increased with finer-textured soils (Table 3). This trend was generally the same for the NL model, except for moderately fine-textured soils. Poorer performance of the models with increase of fine particle fractions is not surprising. The L and NL models are based on an assumed correspondence between particle- and pore-size distribution, and more specifically on the assumption of water retention within pores by capillary action. However, the clay fraction retains water primarily by surface sorption, rather than by capillary action, so that the L and NL models would not be appropriate as a model to predict the contribution of the clay fraction to the WRCs.

5. Summary and Conclusions

Hwang and Powers[7] developed a simple model for estimating the WRC directly from PSD data, by applying a lognormal distribution law to the PSD and pore-size distribution. They applied the model for sandy soils alone and found out that it performed reasonably well for estimating the WRC, and the nonlinear model performed better than the linear model. The objective of this work was to determine if the performance of the model developed by Hwang and Powers[7] is affected by soil texture. Important conclusions deduced from this work are as follows.

(1) The performance of Hwang and Powers[7] model was affected by soil texture. Its performance was poorer with increase of fine particle fractions. Its reason seems to result partly from the surface sorption behavior of clay.

(2) The NL model performed better than the L model, regardless of soil texture classes.

(3) Further research needs to focus on the role of clay fraction, and to include its role into new model development.

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