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Bootstrapping Log Periodogram Regression[1)]

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Abstract

In this paper, we consider a modified bootstrap scheme for inference of the GPH estimator and establish the sup-norm consistency of the proposed bootstrapping.

Keywords : Long Memory, Log Periodogram, Bootstrapping

1. Introduction

Time series whose autocorrelation function decays slowly to zero at a polynomial rate as the lag increases are referred to as long memory time series. Following Brockwell and Davis (1991), we state that a weakly stationary process has long memory if its autocorrelation function $\rho(k)$ has a hyperbolic decay

$$\rho(k) \sim k^{2d-1} \text{ as } k \to \infty,$$

where $\sim$ denotes that the ratio of left- and right-hand side tends to 1 and $d < 0.5$. It is well-known that the spectral density function, the Fourier transform of the autocorrelation function, of long memory time series satisfies, for $-0.5 < d < 0.5$,

$$f(\lambda) \sim |\lambda|^{-2d} \text{ as } \lambda \to 0,$$

where $\lambda \in [-\pi, \pi]$ stands for the frequency. Thus, for long memory processes, it can be shown that $\sum_{k=0}^{\infty} |\rho(k)| = \infty$ and $f(\lambda) \to \infty$ as $\lambda \to 0$ when $0 < d < 0.5$. In practice, if the sample autocorrelation function is not large in magnitude but decays slowly, then the time series may have long memory.

The fractionally differenced ARIMA (p, d, q)(FARIMA (p, d, q)) model has been typically employed to represent long memory time series. Let L be the backshift operator and let Γ be the gamma function. Then, $(1-L)^d$ is the fractional differencing operator defined as

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$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)L^k}{\Gamma(-d)\Gamma(k+1)},$$

and FARIMA (p, d, q) takes the form as

$$\Phi(L)(1-L)^d X_t = \Theta(L)\varepsilon_t,$$

where ε_t is a white noise with mean 0 and variance σ^2. The stochastic process X_t is both stationary and invertible if all roots $\Phi(L)$and $\Theta(L)$ lie outside the unit circle and $|d| < 0.5$. The process is nonstationary for $d \geq 0.5$.

There are several approaches for estimating memory parameter of long memory process. Granger and Joyeux (1980) approximated this model by a high-order autoregressive process and estimated the differencing parameter by comparing variances for each different choice of d. Gaussian parametric estimates for long range dependent time series models have been rigorously justified by Fox and Taqqu (1986) and Giraitis and Surgailis (1990). However, if the parametric model is misspecified, these estimates are inconsistent. In order to estimate semiparametrically the parameter d, Geweke and Porter-Hudak (1983) (GPH) proposed the least square method in frequency domain based on a representation of the log periodogram without assuming short memory structure. Under stationary Gaussian assumption, Robinson (1995) developed asymptotic results, the consistency and the asymptotic normality, for the modified GPH estimator which trims out low frequency periodogram ordinates, as suggested by Künsch (1986). Hurvich, Deo, and Brodsky (1998) extended Robinson's results to low frequency ordinates. They established the asymptotic normality of GPH estimator under Gaussian assumption in the stationary case without any modification.

We consider bootstrapping GPH estimator which can be applied to a small size of samples. Since errors in log periodogram regression or spectral regression of long memory process are not independent and identically distributed, see Künsch (1986), Hurvich and Beltrao (1993), and Robinson (1995), the assumption for standard bootstrap in regression breaks down in the log periodogram regression model. In this paper, we propose a modified resampling method to overcome this difficulty. Let n be the number of observations from a stochastic model. Then, the modified resampling method is based on drawing subsamples of size $m < n$ from the original data. Datta (1996) provided subsampling methods to obtain the bootstrap consistency in the first-order autoregressive processes for all ranges of the autoregressive parameter. In Section 2, we establish the asymptotic validity of the bootstrapping log periodogram regression estimator and mathematical proofs are given in Section 3.

2. Main Results

The log periodogram regression of a stationary Gaussian long memory time series X_t is generated by the model

$$(1-L)^d X_t = u_t, \quad t=1,2,\ldots,$$

and its spectral density function is

$$f(\lambda) = |1-\exp(-i\lambda)|^{-2d} g(\lambda),$$

where $d \in (-0.5, 0.5)$ is the long memory parameter and g is the spectral density function of u_t. Here, g is an even, positive, bounded, and continuous function on $[-\pi, \pi]$ bounded above. The first derivative of g also is bounded away from zero with $g'(0)=0$ and the second and the third derivatives are bounded in a neighborhood of zero. The parameter d controls the long memory aspects of the process, whereas the function g determines the high frequency properties. Note that the processes u_t have been left unspecified. In fact, its spectral density function g satisfying above conditions endows u_t with a short term correlation structure which is free from any parametrically imposed constraints.

In applications, d is generally unknown and should be estimated from a data $X_1, \ldots, X_n$. Geweke and Porter-Hudak (1983) propose a semiparametric estimator, hereafter called the GPH estimator, of d based on the first m periodogram ordinates

$$I_x(\lambda_s) = \frac{1}{2\pi n} \left| \sum_{t=1}^{n} X_t e^{\lambda_s} \right|^2,$$

where $\lambda_s = 2\pi s/n$, $s=1,\ldots,m$, are fundamental frequencies, for some $m < n$. Let $C=0.577216\ldots$ be Euler's constant. Then, GPH estimator $\hat{d}$ is the least squares estimate of the slope in the regression model

$$\log(I_x(\lambda_s)) = a - 2d\,a_s + \varepsilon_s, \tag{2.1}$$

where $a = \log(g(\lambda_0)) - C$, $a_s = \log|1-\exp(-i\lambda_s)|$, and

$$\varepsilon_s = \log(g(\lambda_s)/g(0)) + \log(I_x(\lambda_s)/f(\lambda_s)) + C.$$

Here, the GPH estimators of d and a are given by

$$\hat{d} = -\frac{\sum_{s=1}^{m}(a_s - \bar{a})\log(I_x(\lambda_s))}{2\sum_{s=1}^{m}(a_s-\bar{a})^2} \tag{2.2}$$

$$\hat{a} = \frac{1}{m}\sum_{s=1}^{m}\log(I(\lambda_s)) + 2\frac{\hat{d}}{m}\sum_{s=1}^{m} a_s,$$

where $\bar{a}$ is sample mean of a_s's. Although the GPH estimator is widely used in practice, its consistency for all $d \in (-0.5, 0.5)$ has not easily been established. Robinson (1995) proved consistency and asymptotic normality for a modified estimator which regresses $\log(I_x(\lambda_s))_{s=l+1}^{m}$ on $a_{s\,s=l+1}^{m}$ where l is a lower truncation point which tends to infinity more slowly than m. However, simulations show that the modified estimator is typically outperformed in finite samples. Instead, we consider a modified bootstrap scheme for inference of the GPH estimator and investigate the sup-norm consistency of the proposed bootstrapping.

Let $\bar{e}_{(l)}=(m-l)^{-1}\sum_{s=l+1}^{m}e_s$ be the mean of residuals $\{e_s\}$, $e_s=\log(I(\lambda_s))-\hat{a}+2\hat{d}a_s$, in the log periodogram regression and let $\hat{\sigma}^2_{m(l)}=(m-l)^{-1}\sum_{s=l+1}^{m}(e_s-\bar{e}_{(l)})^2$ be the sample variance. Then, our bootstrap method for $T_n=2\sqrt{m}(\hat{d}-d)/\hat{\sigma}_{m(l)}$ is a modification of the standard bootstrap in the classical regression model. If $\varepsilon_{s\,s=1}{}^{m}$ are uncorrelated and homoscedastic with zero mean, it is well known that the bootstrap approximation to the distribution of the least squares estimates is valid, see Freedman (1981). However, $\varepsilon_{s\,s=1}{}^{m}$ in the log periodogram regression do not have the former properties. The motivation for the bootstrapping log periodogram regression is that $\varepsilon_{s\,s=l+1}{}^{m}$ in (2.1) can indeed be replaced by the $W_{s\,s=l+1}{}^{m}$, for l increasing suitably with n, without affecting the limit distribution of $\hat{d}$ under Gaussian assumptions where $W_{s\,s=l+1}{}^{m}$ are independent random variables with common distribution function $N(0,\pi^2/6)$, see Robinson (1995).

Now consider a bootstrap scheme for the system defined in (2.1). Upon observing the log periodogram $\log(I_x(\lambda_1)),\ldots,\log(I_x(\lambda_m))$, calculate the residuals $e_{s\,s=1}{}^{m}$. Let $\tilde{e}_s=e_s-\bar{e}_l$ denote the centered residual. Then we choose bootstrap samples $\varepsilon^*_{s\,s=1}{}^{m}$ from the empirical distribution $\hat{F}_{m(l)}$, the distribution with point probability mass $1/(m-l)$ on their $m-l$ observed values, of $\tilde{e}_{s\,s=l+1}{}^{m}$. We may thus regard $\varepsilon^*_{s\,s=1}{}^{m}$ as the independent and identically distributed samples from the empirical distribution of $\hat{F}_{m(l)}$, of $\tilde{e}_{s\,s=l+1}{}^{m}$. The bootstrap sample $y^*_1,\ldots,y^*_m$ can be constructed using the recursive formula $y^*_s=\hat{a}-2\hat{d}a_s+\varepsilon^*_s$.

Let $\hat{d}^*$ be the GPH estimators of d based on bootstrap samples,

$$\hat{d}^*=-\frac{\sum_{s=1}^{m}(a_s-\bar{a})y^*_s}{2\sum_{s=1}^{m}(a_s-\bar{a})^2}$$

and let

$$T^*_n=\frac{2\sqrt{m}(\hat{d}^*-\hat{d})}{\hat{\sigma}_m},$$

respectively. The conditional distribution of T^*_n constitutes a bootstrap approximation of the distribution of T_n. In the following theorem, we show that the conditional distribution of the bootstrap statistic T_n^* is asymptotically close to the distribution of the studentized form T_n of the least squares estimators $\widehat{d}$.

Theorem 2.1 If $n \to \infty$, $m \to \infty$, $m = o(n^{4/5})$, $\log^2(n) = o(m)$, and $l = m^{0.5+\delta}$ for $0 < \delta < 1/2$, then

$$\sup_x \left| P^*\left(\frac{2\sqrt{m}(\widehat{d}^* - \widehat{d})}{\widehat{\sigma}_m} \le x \right) - \Phi(x) \right| \to 0 \text{ in probability.}$$

Thus, the conditional distribution of $\sqrt{m}(\widehat{d}^* - \widehat{d})$ converges weakly to normal distribution with mean 0 and variance $\pi^2/24$ in probability. □

3. Mathematical Proofs

Lemma 3.1 If $n \to \infty$, $m \to \infty$, $m = o(n^{4/5})$, $\log^2(n) = o(m)$, and $l = m^{0.5+\delta}$ for $0 < \delta < 0.5$, then

(1) $\widehat{d} \to d$ in probability (3.1)

(2) $\widehat{a} \to a$ in probability (3.2)

(3) $\widehat{\sigma}_m \to \pi/\sqrt{6}$ in probability.

Proof.

(1) see Hurvich, Deo, and Brodsky (1998) (HDB).

(2) We write

$$\widehat{a} = \frac{1}{m} \sum_{s=1}^{m} \log(I(\lambda_s)) + 2\frac{\widehat{d}}{m} \sum_{s=1}^{m} a_s = a + T_{11} + T_{12} + T_{13},$$

where

$$T_{11} = \frac{2(\widehat{d} - d)}{m} \sum_{s=1}^{m} a_s, \quad T_{12} = \frac{1}{m} \sum_{s=1}^{l} \varepsilon_s, \text{ and } T_{13} = \frac{1}{m} \sum_{s=l+1}^{m} \varepsilon_s.$$

Then, we have

$$T_{11} = \frac{\sqrt{m}(\widehat{d} - d)}{\log(m)} \frac{m \log(m) O(\log(m))}{m^{1+1/2}} = o_p(1) O\left(\frac{(\log(m))^2}{\sqrt{m}} \right) \to 0 \text{ in probability}$$

by theorem 2 of HDB and using the fact of Hurvich and Beltrao (1994, pp. 300-301) that

$$a_s = \log(s) + O\left(\frac{m^2}{n^2} \right) = O(\log(m)) \qquad (3.3)$$

uniformly in $1 \le s \le m$. Now,

$$T_{12} = A_1 + A_2 + \frac{l}{m} C,$$

where

$$A_1 = \frac{1}{m} \sum_{s=1}^{l} \log\left(\frac{g(\lambda_s)}{g(0)}\right) \text{ and } A_2 = \frac{1}{m} \sum_{s=1}^{l} \log\left(\frac{I_x(\lambda_s)}{f(\lambda_s)}\right).$$

By the proof of Lemma 1 of HDB,

$$A_1 = \frac{g''(0)\sum_{s=1}^{l}\lambda_s^2}{2g(0)m} + o(1) = \frac{g''(0)2\pi^2\sum_{s=1}^{l}s^2}{g(0)mn^2} + o(1)$$
$$= \frac{g''(0)\pi^2 l(l+1)(2l+1)}{3g(0)mn^2} + o(1) = o(1).$$

With Jensen' inequality and Lemma 5 of HDB,

$$P(|A_2| > \delta) < \frac{1}{m\delta} \sum_{s=1}^{l} \sup E\left(\left|\log\left(\frac{I_x(\lambda_s)}{f(\lambda_s)}\right)\right|\right)$$
$$\leq \frac{1}{m\delta} \sum_{s=1}^{l} \sup E\left(\left|\log\left(\frac{I_x(\lambda_s)}{f(\lambda_s)}\right)\right|^2\right)^{1/2} \to 0.$$

Finally, let U_s be in equation (2.4) of Robinson (1995). Then, we have

$$T_{13} = \frac{1}{m} \sum_{s=l+1}^{m} U_s + \frac{2d}{m} \sum_{s=l+1}^{m} \log\left(\frac{|1-\exp(-i\lambda_s)|}{\lambda_s}\right). \tag{3.4}$$

The first part of RHS (3.4) converges to 0 in probability, due to (5.14) of Robinson (1995). By the proof of theorem 2 of HDB,

$$\log\left(\frac{|1-\exp(-i\lambda_s)|}{\lambda_s}\right) = O\left(\frac{m^2}{n^2}\right), \tag{3.5}$$

uniformly in $1 \leq s \leq m$, so that the second part is $o(1)$.

(3) Let $\overline{\varepsilon_l} = (m-l)^{-1} \sum_{s=l+1}^{m} \varepsilon_s$ and $\sigma_m^2 = (m-l)^{-1} \sum_{s=l+1}^{m} (\varepsilon_s - \overline{\varepsilon_l})^2$. Firstly, we show that $\widehat{\sigma_m} - \sigma_m \to 0$ in probability and then $\sigma_m \to \pi/\sqrt{6}$ in probability. Using lemma 2.7 of Freedman (1981).

$$(\widehat{\sigma_m} - \sigma_m)^2 \leq \frac{1}{m-l} \sum_{s=l+1}^{m} (e_s - \varepsilon_s)^2 \leq \frac{1}{m-l} \sum_{s=1}^{m} (e_s - \varepsilon_s)^2$$
$$= \frac{1}{m-l} (\hat{a} - a, -2\hat{d} + 2d) \begin{pmatrix} m & \sum_{s=1}^{m} a_s \\ \sum_{s=1}^{m} a_s & \sum_{s=1}^{m} a_s^2 \end{pmatrix} \begin{pmatrix} \hat{a} - a \\ -2\hat{d} + 2d \end{pmatrix} = T_{21} + T_{22} + T_{23},$$

where

$$T_{21} = \frac{m}{m-l}(\hat{a} - a)^2, \quad T_{22} = -4\frac{(\hat{a}-a)(\hat{d}-d)}{m-l} \sum_{s=1}^{m} a_s, \text{ and } T_{23} = 4\frac{(\hat{d}-d)^2}{m-l} \sum_{s=1}^{m} a_s^2.$$

By (3.2), it follows $T_{21}=o_p(1)$. Due to (3.1), (3.2) and (3.3),

$$T_{22}=-4\frac{O(\log(m))m}{\sqrt{m(m-l)}}(\hat{a}-a)\sqrt{m}(\hat{d}-d)=o_p(1),$$

and

$$T_{23}=\frac{O(m\log^4(m))}{(m-l)m}\frac{m(\hat{d}-d)^2}{\log^2(m)}=o_p(1).$$

We now show that $\sigma_m \rightarrow \pi/\sqrt{6}$ in probability. Write

$$\sigma_m^2=\frac{1}{m-l}\sum_{s=l+1}^{m}\varepsilon_s^2-\overline{\varepsilon_l^2}=T_{31}+T_{32},$$

where

$$T_{31}=\frac{1}{m-l}\sum_{s=l+1}^{m}\left(U_s+2d\log\left(\frac{|1-\exp(-i\lambda_s)|}{\lambda_s}\right)\right)^2,$$

and

$$T_{32}=\left(\frac{1}{m-l}\sum_{s=l+1}^{m}U_s+\frac{2d}{m-l}\sum_{s=l+1}^{m}\log\left(\frac{|1-\exp(-i\lambda_s)|}{\lambda_s}\right)\right)^2.$$

Now, $T_{31}=B_1+B_2+B_3$, where

$$B_1=\frac{1}{m-l}\sum_{s=l+1}^{m}U_s^2,\quad B_2=\frac{4d}{m-l}\sum_{s=l+1}^{m}U_s\log\left(\frac{|1-\exp(-i\lambda_s)|}{\lambda_s}\right),$$

and

$$B_3=\frac{4d^2}{m-l}\sum_{s=l+1}^{m}\left(\log\left(\frac{|1-\exp(-i\lambda_s)|}{\lambda_s}\right)\right)^2.$$

Since B_1 converges to $\pi^2/6$ in probability, by (5.8) of Robinson(1995), B_2 is $o_p(1)$, due to (3.4) and (3.5), and B_3 is $o(1)$, by (3.5). And $T_{31}=o_p(1)$ follows by (3.4) and (3.5). Therefore $\widehat{\sigma_m} \rightarrow \pi/\sqrt{6}$ in probability by Slutsky theorem and triangular inequality.

Proof of Theorem 2.1

Write

$$\sup_x\left|P^*\left(\frac{2\sqrt{m}(\widehat{d^*}-\hat{d})}{\widehat{\sigma_m}}\leq x\right)-\Phi(x)\right|$$

$$=\sup_x\left|P^*\left(-\frac{\sqrt{m}\sum_{s=1}^{m}(a_s-\bar{a})\varepsilon_s^*}{\sum_{s=1}^{m}(a_s-\bar{a})^2\hat{\sigma}_m}\leq x\right)-\Phi(x)\right|\leq T_{41}+T_{42},$$

where

$$T_{41} = \sup_x \left| P^*\left(-\frac{\sqrt{m}\sum_{s=1}^{m}(a_s - \overline{a})\varepsilon_s^*}{\sum_{s=1}^{m}(a_s-\overline{a})^2\,\hat{\sigma}_m} \le x \right) - P^*\left(-\frac{\sum_{s=1}^{m}(a_s - \overline{a})\varepsilon_s^*}{\sqrt{\sum_{s=1}^{m}(a_s-\overline{a})^2\,\hat{\sigma}_m}} \le x \right) \right|,$$

and

$$T_{42} = \sup_x \left| P^*\left(-\frac{\sum_{s=1}^{m}(a_s - \overline{a})\varepsilon_s^*}{\sqrt{\sum_{s=1}^{m}(a_s-\overline{a})^2\,\hat{\sigma}_m}} \le x \right) - \Phi(x) \right|.$$

Using the fact (see Hurvich and Beltrao, 1994, pp. 301) that $\sum_{s=1}^{m}(a_s-\overline{a})^2 = m + o(m)$, $T_{41} = o_p(1)$. We have for any $p \ge 2$ that

$$\sum_{s=1}^{m}|a_s - \overline{a}|^p = \sum_{s=1}^{l}|a_s-\overline{a}|^p + \sum_{s=1+l}^{m}|a_s-\overline{a}|^p \tag{3.6}$$
$$= m^{0.5+\delta}O(\log^p(m)) + O(m) = O(m),$$

by (A16) and (A18) of HDB. From Berry-Esseen's inequality and (3.6),

$$T_{42} \le \frac{6\sum_{s=1}^{m}(a_s-\overline{a})^3 \frac{1}{m-l}\sum_{s=l+1}^{m}(e_s-\overline{e_l})^3}{\left(\sum_{s=1}^{m}(a_s-\overline{a})^2\right)^{3/2}\widehat{\sigma_m^3}}$$
$$= \frac{O(m)\sum_{s=l+1}^{m}(e_s-\overline{e_l})^3}{O(m^{3/2})\,\widehat{\sigma_m^3}(m-l)} = \frac{\sum_{s=l+1}^{m}(e_s-\overline{e_l})^3}{(m-l)^{3/2}\,\widehat{\sigma_m^3}}\,O(1).$$

We now show that

$$\frac{1}{(m-l)^{3/2}}\sum_{s=l+1}^{m}(e_s-\overline{e_l})^3 = o_p(1).$$

Let

$$e_s = -(\hat{a}-a) - 2(\hat{d}-d)a_s + 2d\log\left(\frac{|1-\exp(-i\lambda_s)|}{\lambda_s}\right) + U_s.$$

Hence,

$$\frac{1}{(m-l)^{3/2}}\sum_{s=l+1}^{m}(e_s-\overline{e_l})^3 \le T_{51} + T_{52},$$

where

$$T_{51} = \frac{c_1}{(m-l)^{3/2}}\sum_{s=l+1}^{m}e_s^3,\quad T_{52} = \frac{c_2\,\overline{e_l}^{\,3}}{(m-l)^{3/2}},$$

and c_1 and c_2 are constants.

$$T_{51} \le D_1 + D_2 + D_3 + D_4,$$

where

$$D_1 = c_{11}\frac{|\hat{a}-a|^3}{\sqrt{m-l}},\quad D_2 = \frac{c_{12}|\hat{d}-d|^3}{(m-l)^{3/2}}\sum_{s=l+1}^{m}|a_s|^3,$$

$$D_3 = \frac{c_{13}}{\sqrt{m-l}}\left|d\log\left(\frac{|1-\exp(-i\lambda_s)|}{\lambda_s}\right)\right|^3,\ \text{and}\ D_4 = \frac{c_{14}}{(m-l)^{3/2}}\sum_{s=l+1}^{m}|U_s|^3,$$

and c_{11}, c_{12}, c_{13}, and c_{14} are constants. $D_1 = o_p(1)$ follows, by Lemma 3.1. Since $\sum_{s=l+1}^{m}|a_s|^3 = O(m\log^3(m))$ and $\hat{d}-d = o_p(1)$, D_2 converges to 0 in probability. D_3 is $o(1)$, by (3.5). And note that

$$P\left(\left|\sum_{s=l+1}^{m}|U_s|^3\right| > \delta(m-l)^{3/2}\right) < \frac{\sum_{s=l+1}^{m}E|U_s|^3}{\delta(m-l)^{3/2}} = \frac{\sum_{s=l+1}^{m}E|W_s|^3}{\delta(m-l)^{3/2}} + o(1),$$

where for all $\delta > 0$, since Robinson (1995 pp.1069-1070) has shown that the moments of $U_s/\sqrt{m}$ differ negligibly from those of the variate $W_s/\sqrt{m}$, where the W_s are i.i.d. with zero mean, finite variance and finite moments. Thus $D_4 = o_p(1)$, by the strong law of large numbers. Finally,

$$T_{52} \le \frac{1}{(m-l)^{3/2}}\left(c_{21}E_1^3 + c_{22}E_2^3 + c_{23}E_3^3 + c_{24}E_4^3\right),$$

where

$$E_1 = -\frac{m(\hat{a}-a)}{m-l},\quad E_2 = \frac{(\hat{d}-d)}{m-1}\sum_{s=l+1}^{m}a_s,\quad E_3 = \frac{d}{m-1}\log\left(\frac{|1-\exp(-i\lambda_s)|}{\lambda_s}\right),$$

and

$$E_4 = \frac{1}{m-l}\sum_{s=l+1}^{m}U_s.$$

E_1 and E_2 are $o_p(1)$, by Lemma 5.1 and E_3 and E_4 are $o_p(1)$ by (3.4) and (3.5). Hence, T_{42} converges to 0 in probability. Thus the proof is complete. Q.E.D.

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