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A Chaos Control Method by DFC Using State Prediction

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Abstract

The Delayed Feedback Control method (DFC) proposed by Pyragas applies an input based on the difference between the current state of the system, which is generating chaos orbits, and the τ-time delayed state, and stabilizes the chaos orbit into a target. In DFC, the information about a position in the state space is unnecessary if the period of the unstable periodic orbit to stabilize is known. There exists the fault that DFC cannot stabilize the unstable periodic orbit when a linearlized system around the periodic point has an odd number property. There is the chaos control method using the prediction of the t-time future state (PDFC) proposed by Ushio et al. as the method to compensate this fault.
Then, we propose a method such as improving the fault of the DFC. Namely, we combine DFC and PDFC with parameter W, which indicates the balance of both methods, not to lose each advantage. Therefore, we stabilize the state into the τ periodic orbit, and ask for the ranges of W and gain K using Jury' method, and determine the quasi-optimum pair of (W, K) using a genetic algorithm. Finally, we apply the proposed method to a discrete-time chaotic system, and show the efficiency through some examples of numerical experiments.

Key words : Chaos Control, Delayed Feedback Control, Prediction-based Feedback Control

1. Introduction

In recent years, the chaos control as one of the prospering application of chaos research is to stabilize a state into the peculiar Unstable Periodic Orbits (UPO) embedded in the chaos attractor. The typical methods of the chaos control are the *OGY method* [1, 2, 6] and *Delayed Feedback* Control [3, 4, 5]. *The OGY* method adds a perturbation to the parameter of the system, and *Delayed Feedback Control* (DFC) proposed by Pyragas stabilizes the orbit by the external input. The DFC method stabilizes the chaos orbit to the target UPO by the input based on the difference between the τ-time delayed state and the current state, where τ denotes a period of the stabilized orbits. If the period of UPO stabilized is known, this method has the advantage applied easily without the information about the position in the state space. Moreover, DFC has the very robust characteristic to external noise. There exists the fault that *DFC cannot stabilize UPO when a linearlized system around the each periodic point has an odd number of real eigenvalues greater than one* (odd number property) and has not got the theoretical guarantee related with stability [6, 7]. In various techniques proposed until now as the method compensating this fault, there is Prediction-based Feedback Control (PDFC) which stabilizes UPO using the prediction value of the τ-time future state proposed by Ushio et al. [8. 9]. However, when not receiving restrictions of odd number property, there is no guarantee that PDFC can obtain gain K which stabilizes the system more as compared with DFC. Moreover, PDFC needs to calculate the prediction value of a state analytically against DFC determining the input using the past state obtained already. Therefore, a large error may arise to estimate the future state by PDFC. In order to improve these faults of DFC and PDFC, the method combined DFC and PDFC is described in reference [8].

Then, in this paper, we propose a chaos control method combined DFC and PDFC. PDFC input and DFC input in the proposed method have a common gain and are combined with the parameter which indicates the valance of both inputs. We consider a discrete-time system and perform the numerical experiments to stabilize the fixed point and the 2-periodic orbit with the application of the proposal technique. Furthermore, we show the validity of the proposed method by comparing result of proposal technique with the each result of the same experiments using PDFC and DFC.

2. DFC Method and PDFC Method

We consider the following n-th nonlinear discrete-time system.

$$x(k+1)=f(x(k),u(k)) \tag{1}$$

where $x(k)\in R^n$ is the state, $u(k)\in R^n$ is the input. We assumed that f is differentiable and chaos generates if $u(k)=0$. The linearlized system near the periodic orbits which has periodic coefficients is considered for stabilizing UPO locally. We consider the following linearlized system near the each periodic point $x$pi, i = 1, ⋯, τ.

$$\bar{x}(k+1) = Ai\bar{x}(k) + Biui(k) \quad (2)$$

where $\bar{x}(k+1) = x(k) - xpi, Ai = \frac{\partial}{\partial x} f(xpi, 0) \in R^{n*n}$ and $Bi = \frac{\partial}{\partial x} f(xpi, 0) \in R^{n \times m}$.

2.1 DFC Method

The input of DFC expressed with Eq. (3) is based on the difference between the τ-step delayed state and the current state.

$$u(k) = K_D\{x(k-\tau) - x(k)\} \quad (3)$$

where $K_D \in R^{m \times n}$ is a gain matrix. The trajectory of a state of Eq. (1) surely approaches near the τ periodic orbit as target by ergodicity of the chaotic system when $u(k) = 0$. Therefore, only when the trajectory approaches target UPO so that a state may not become unstable by the input, the input of at Eq. (3) is impressed [3]. Although it is not necessary to search for a target orbit correctly, restriction such as the odd number property exists in the determination of gain K_{Di} of the each periodic point. Therefore, the necessary condition for Eq. (2) to be stable is given by Eq. (4) [7].

$$det(In - Ai) > 0, \quad i = 1, \cdots, \tau. \quad (4)$$

2.2 PDFC Method

The input of PDFC expressed with Eq. (5) which Ushio proposed as the method of compensating the odd number property of DFC is based on the difference between the τ-step future state and the current state.

$$u(k) = Kp\{x(k+\tau) - x(k)\} , \quad (5)$$

where $Kp \in R^{m \times n}$ is a gain matrix an $x(k+\tau)$ is the predicted value of the state with $u(k) = 0$ as

$$x(k+\tau) = f(\cdots 0 f(x), 0) = f^{\tau}(x(k), 0) , \quad (6)$$

Like DFC, only when the trajectory of the state approaches the target, the input is impressed to the system. Eq. (2) using PDFC is expressed with Eq. (7).

$$\bar{x}(k+1) = Ai\bar{x}(k) + BiKpi(Qi - In)\bar{x}(k) , \quad (7)$$

where $Qi = \frac{\partial}{\partial x} f^{\tau}(xpi, 0)$. At this time, if we set $Fpi = Kpi(Qi\text{-}In)$ in Eq. (7), gain Kpi is obtained by Eq. (8). We set Fpi which makes $Ai + BiFpi$ stable matrix.

$$Kpi = Fpi(Qi - In)^{-1} , \quad (8)$$

The necessary and sufficient condition to obtain gain Kpi which makes Eq. (7) stable is that
(Ai, Bi) is a controllable pair and
Qi-In is det (Qi-In)≠0 i.e.., nonsingular [8, 9].

3. Proposed Method

We define the input of the proposed technique combining DFC and PDFC by Eq. (9).

$$u(k) = up(k) + u_D(k) \quad (9)$$

where *up(k)* is corresponding to PDFC input and *uD(k)* is corresponding to DFC input. We determine *up(k)* is Eq. (10) and *uD(k)* is Eq. (11).

$$up(k) = K(1 - W)x(k+\tau) - x(k) , \quad (10)$$
$$u_D(k) = KWx(k-\tau) - x(k) , \quad (11)$$

We set that gain of *up(k)* and gain of *uD(k)* are common K $\in R^{m \times n}$. W is the weight taking the balance of *up(k)* and *uD(k)*, and takes the real value of $0 < W < 1$. That is, we use Eq. (12) which is defined as follows:

$$u(k) = \begin{cases} up(k) + u_D & \text{if } r(k) < \varepsilon \\ 0 & otherwise \end{cases} , \quad (12)$$

where *r(k)* is norm of the vector based on the difference between the τ-step delayed state and the current state, and is described by Eq. (13). ε is the positive real value to be small enough.

$$r(k) = ||x(k-r) - x(k)|| , \quad (13)$$

3.1 Gain K and Weight W

In order to ask for suitable *K* and *W* which guarantee the stability of the linearlized system Eq. (2) around the τ periodic orbit, it is necessary to search for the stability of Eq. (14) which is the extended system of Eq. (2). Since system Eq. (2) is *n*th dimensions, the extended system becomes *N*th dimensions, where $N = n \times (\tau + 1)$. From here, we omit subscript $i = 1, \cdots, \tau$ except for the case where periodic pints are discriminated.

$$\bar{X}(k+1) = \begin{bmatrix} A + BF - WBK \cdots 0 \cdots WBK \\ & \vdots \\ I_{nxt} & 0 \\ & \vdots \end{bmatrix} \bar{X}(k) , \quad (14)$$

where

$$F(W, K) = (1 - W)K(Q - In) \quad (15)$$

and

$$\bar{x}(k) = x(k) - \left[x_{pk}^{T} \cdot \cdot \cdot \cdot \cdot x_{p(k-T)}^{T}\right]^{T}$$

with a periodic point $x_{p(k+T)} = x_{pk}$ and

$$x(k) = x(k) - [x^{T}(k) \cdots x^{T}(k-r)]^{T} .$$

Characteristic polynomial $Z(z, W, K)$ of Eq. (14) becomes Eq. (16).

$$Z(z, W, K) = det[z^{r+1}I_n - z^{t}(A + F - WBK) - WBK] \quad (16)$$

A expansion of Eq. (16) is set with Eq. (17)

$$Z(z, W, K) = a_N z^N + a_{N-1} z^{N-1} + \cdots + a_1 z + a_0 \ , \tag{17}$$

where coefficients a_r , $r=0$,⋯, N are dependent on W and K.

$\lim_{z \to \infty} Z(z, W, K) = \infty > 0$ as $a_N = 1$ When $z=1$ Eq. (16) becomes

$$Z(1, W, K) = det[I_n - (A+F)] \ , \tag{18}$$

Therefore, since the sign of $Z(1, W, K)$ is able to be decided by $F(W, K)$ of Eq. (15), Eq. (2) does not receive restrictions of the odd number property. Then, W and K can be obtained using characteristic equation $Z(z, W, K) = 0$. However, since the dimension of an extended system Eq. (14) becomes large if period r of the target orbit to be stabilized is large, determining the periodic solution of the characteristic equation becomes very hard calculation.

Then, we will judge stability by Jury' method without determining the pole of the system and obtain the ranges of effective gain K and weight W, which make for the linearlized system to be stable, and determine the quasi-optimal pair of (W, K) by the genetic algorithm (GA) from the region $W-K$.

3.2 Determination of Pair (W, K)

Using pairs of (W, K) belonging to region $W-K$ obtained by Jury' method, gene sequence θ of Eq. (19) is set. Using the evaluation function of Eq. (20) the gene manipulation to decide the degree of adaptation is repeated only to individuals belonging to region $W-K$.

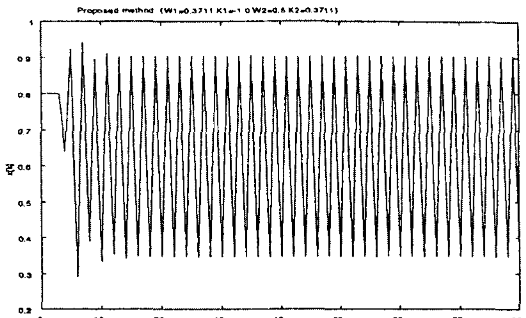

Fig. 1. Controlled behavior $x(k)$ by proposed method.

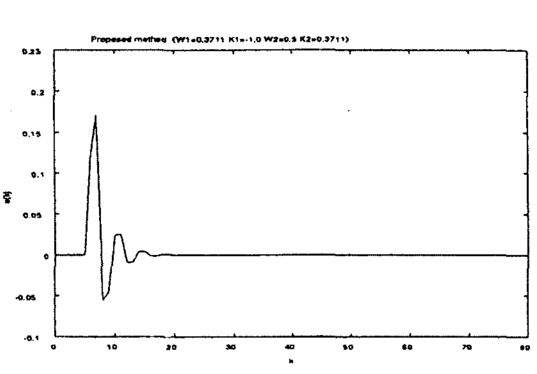

Fig. 2. Input $u(k)$ of proposed method.

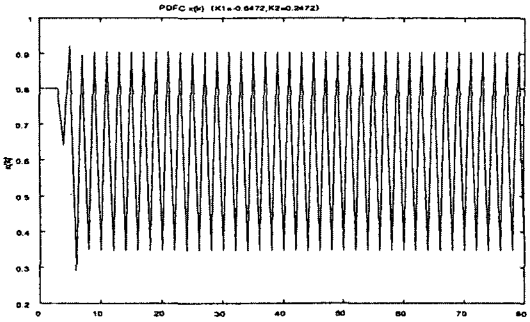

Fig. 3. Controlled behavior $x(k)$ by PDFC.

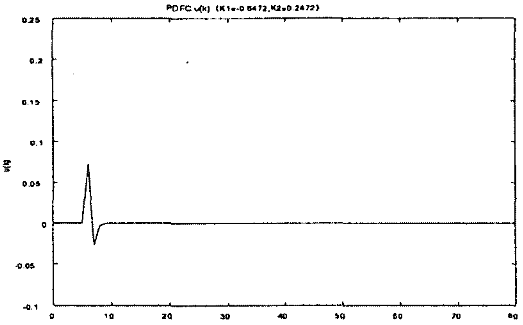

Fig. 4. Input $u(k)$ of PDFC.

$$\theta = [\, W K] \ , \tag{19}$$

$$Fit = \frac{1}{\sum_k (w \| \overline{x}(k) \| + v \| u(k) \|) + j_0} , \tag{20}$$

where and are weight, and j_0 is the positive real value be small enough.

4. Simulations

We examine the performance of our proposed method in this section, and consider 1-dimensional discrete-time nonlinear system with the dynamical equation in the following:

$$x(k+1) = 4x(k)(1 - x(k) + u(k) \ , \tag{21}$$

where $x(k) \in \mathcal{R}$ and $u(k) \in \mathcal{R}$ denote the system state and input. Logistic Map of Eq. (21) has two periodic or-bits, whose two periodic points $x_{p1,2}$ is $5 \pm \sqrt{5}/8$. This orbit will be stabilized by our method. Inequality of W and K of the input are solved by jury's method. Region W_1 - K_1 of Fig. 5 is the command portion of the domain-s which fills all of four condition formulas of Eq. (22). Where parameter W_1 and K_1 denote weight and gain of the input at periodic point x_{p1} .

$$\begin{cases} -\dfrac{2+\sqrt{5}}{-5+5W_1} < K_1, \\ \dfrac{1}{\sqrt{5}(-1+W_1)} > K_1, \\ -\dfrac{1}{W_1} < K_1 < \dfrac{1}{W_1}, \\ 1-(1+\sqrt{5})W_1K_1+W_1)-5+3W_1)K_1^2<0 \end{cases} \quad (22)$$

$x(k)$ of Fig. 1 is the state behavior controlled by pro-posed method. At that time, parameters of the input of Fig. 2 are as $W_1=0.3711$, $K_1=-1.0$, $W_2=0.5$, $K_2=0.3711$. We determined initial value $x(0)=0.8$ and $\varepsilon=0.1$. The result of experiment using PDFC is shown in Fig. 3 for comparison with our method. The input of PDFC at that time is shown in Fig. 4. Two periodic orbit cannot be stabilized by the proposal technique. Region *W-K* of the input parameters by proposed method is Fig. 6. $x(k)$ of Fig. 7 is the state behavior controlled by proposed method. At that time, parameters of the input of Fig. 8 are as $W=0.4$, $K=-0.7$. We determined initial value $x(0)=0.4$, $\varepsilon=0.1$. For comparison, the result of experiment using PDFC is shown in Fig. 9, and the input at that time is shown in Fig. 10. Similarly, the result used DFC is shown in Fig. 11, and the input at that time is shown in Fig. 12. From the results of the numerical experiments used the proposed method in Logistic Map of Eq. (21), When the starting point of control and the converging point of the input and the state were compared, the proposed method can stabilize a state early moreover in the input smaller than PDFC and DFC about the fixed point. About the periodic orbit, the proposed method can control almost like PDFC into two periodic orbit, which is not stabilized by DFC. Although the control result by the proposed method of DFC and PDFC, the proposed method was able to obtain the control result better than which of PDFC and DFC in many cases. By applying two partial inputs adapting PDFC and DFC simultaneously, a state did not become still unstable from the control system which was inferior between the two original control systems. When weight W and gain K could be pertinently chosen in the effective range, we show by numerical experiments that a chaotic system could be stabilized by the proposed method.

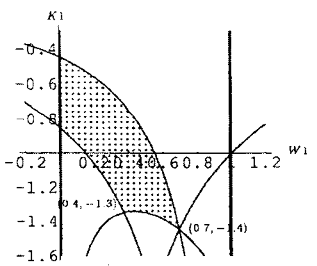

Fig. 5. Region of weight W_1 and gain K_1.

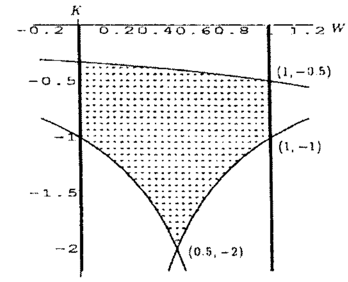

Fig. 6. Region of Weight W and gain K.

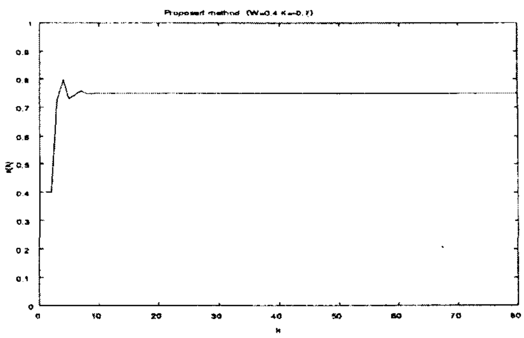

Fig. 7. Controlled behavior $x(k)$ by proposed method.

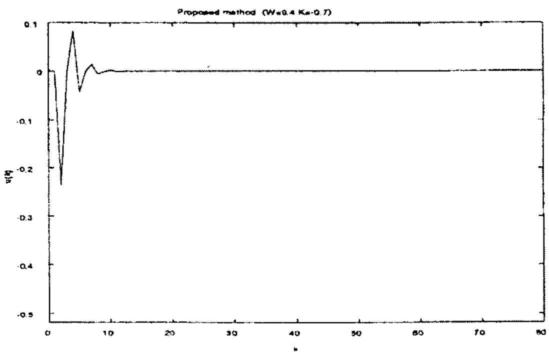

Fig. 8. Input $u(k)$ of proposed method.

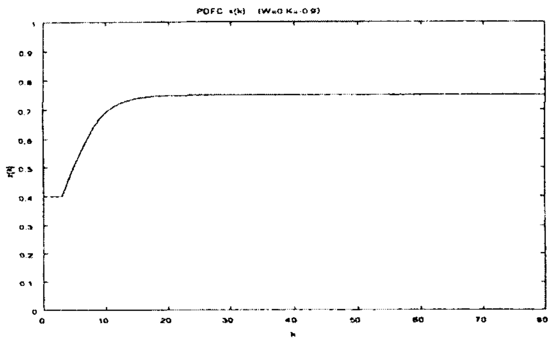

Fig. 9. Controlled behavior $x(k)$ PDFC.

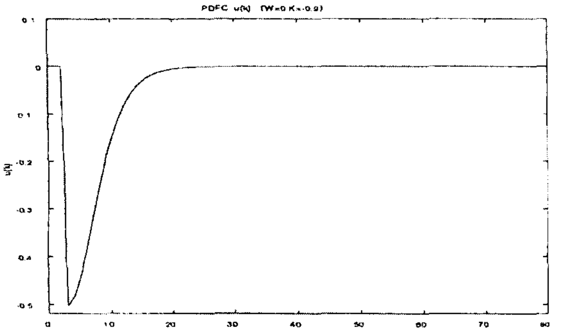

Fig. 10. Input $u(k)$ of PDFC.

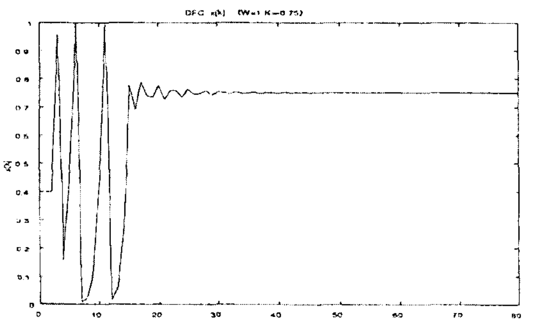

Fig.11. Controlled behavior $x(k)$ by DFC

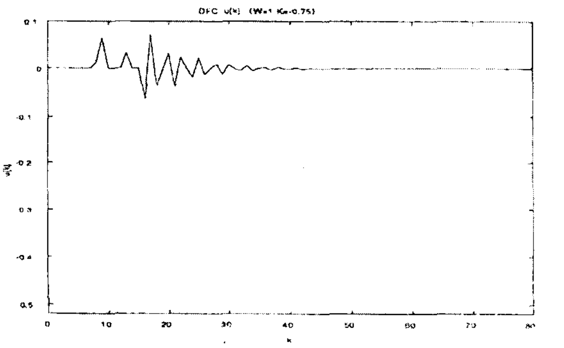

Fig. 12. Input $u(k)$ of DFC

5. Conclusion

As the method of stabilizing the state into the peculiar UPO in chaotic system, We proposed the control technique combined PDFC input and DFC input with the parameter which indicates the balance of both inputs. By the numerical experiments applied the proposed technique to the discrete-time system comparing with the same experiment using DFC and PDFC individually, the validity of the proposal technique was shown. Now, we are considering the application of proposed method to unknown chaotic systems as a future subject, in which the prediction value of a state is not acquired.

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