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Fuzzy least squares polynomial regression analysis using shape preserving operations

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Abstract

In this paper, we describe a method for fuzzy polynomial regression analysis for fuzzy input--output data using shape preserving operations for least-squares fitting. Shape preserving operations simplifies the computation of fuzzy arithmetic operations. We derive the solution using mixed nonlinear program.

Key Words : Polynomial fuzzy regression, shape-preserving operations, membership function, least-square fitting

1. 서 론

For many years statistical linear regression has been used in almost all field of science. The purpose of regression analysis is to explain the variation of a dependent variable Y in terms of the variation of explanatory variables X as $Y=f(X)$ where $f(X)$ is a linear function. The use of statistical linear regression is bounded by some strict assumptions about the given data, that is, the unobserved error term are mutually independent and identically distributed. As a result, the statistical regression model can be applied only if the given data are distributed according to a statistical model, and the relation between x and y is crisp.

Since Tanaka et al. in 1982 [15] proposed a study in linear regression analysis with fuzzy model, the fuzzy regression analysis has been widely studied and applied in a variety of substantive areas. A collection of recent papers dealing with several approaches to fuzzy regression analysis can be found in [11].

Recently, Hong et al.[9] presented a new method to evaluate fuzzy linear regression models for least-square fitting where both input data and output data are fuzzy numbers based on Diamond's[4] fuzzy linear regression model, using shape preserving fuzzy arithmetic operations.

In statistical regression, polynomial are widely used in situations where the response is curvilinear, because even complex nonlinear relationships can be adequately modeled by polynomials over reasonably small range of the dependent variables. In contrast to fuzzy linear regression, there have been only a few articles on fuzzy nonlinear regression[see[2, 3]).

Our approach to fuzzy nonlinear regression is different in that we use the Diamond's metric between fuzzy numbers and we use shape preserving (T_W-based) fuzzy arithmetic operations.

Since T_W-based fuzzy arithmetic operations preserves the shape of fuzzy numbers under addition and multiplication, it simplifies the computation of fuzzy arithmetic operations.

In this paper, using this operations, we consider fuzzy quadratic polynomial regression for least-square fitting. This problem is mixed nonlinear programming problem. We derive the solution using general nonlinear programming problem.

2. Preliminaries

A fuzzy number is a convex subset of the real line R with a normalized membership function.

A triangular fuzzy number $\tilde{a}$ denoted by (a, α, β) is defined as

$$\tilde{a}(t)=\begin{cases}1-\frac{|a-t|}{\alpha} & \text{if } a-\alpha\le t\le a,\\ 1-\frac{|a-t|}{\beta} & \text{if } a\le t\le a+\beta,\\ 0 & \text{otherwise},\end{cases}$$

where $a\in R$ is the center and $\alpha>0$ is the left spread, $\beta>0$ is the right spread of $\tilde{a}$.

If $\alpha=\beta$, then the triangular fuzzy number is called a symmetric triangular fuzzy number and denoted by (a, α).

A $L-R$ fuzzy number $\tilde{a}=(a, \alpha, \beta)_{LR}$ is a function from the reals into the interval $[0, 1]$ satisfying

$$\tilde{a}(t)=\begin{cases}R\left(\frac{t-a}{\beta}\right) & \text{for } a\le t\le a+\beta,\\ L\left(\frac{a-t}{\alpha}\right) & \text{for } a-\alpha\le t\le a,\\ 0 & \text{else},\end{cases}$$

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where L and R are non-decreasing and continuous functions from $[0, 1]$ to $[0, 1]$ satisfying $L(0)=R(0)=1$ and $L(1)=R(1)=0$.

If $L=R$ and $\alpha=\beta$, then the symmetric $L-L$ fuzzy number is denoted $(a, \alpha)_L$.

A binary operation T on the unit interval is said to be triangular norm(t-norm for short) iff T is associative, commutative, non-decreasing and $T(x, 1)=x$ for each $x\in[0, 1]$. Moreover, every t-norm satisfies the following inequality,

$$T_W(a, b)\le T(a, b)\le \min(a, b)=T_M$$

where,

$$T_W(a, b)=\begin{cases} a & \text{if } b=1, \\ b & \text{if } a=1, \\ 0 & \text{otherwise}. \end{cases}$$

The crucial importance of $\min(a, b), a\cdot b, \max(0, a+b-1)$ and $T_W(a, b)$ is emphasized from a mathematical point of view in Ling[13] among others.

The usual arithmetical operations of real can be extended to the arithmetical operations on fuzzy numbers by means of Zadeh's extension principle [16] based on a triangular norm T. Let $\mathcal{A}$, $\mathcal{B}$ be fuzzy numbers of reals line R. The fuzzy number arithmetic operations are summarized as follows:

Fuzzy number addition $\oplus$:

$$(\mathcal{A}\oplus\mathcal{B})(z)=\sup_{x+y=z}T(\mathcal{A}(x), \mathcal{B}(y)), \tag{1}$$

Fuzzy number multiplication $\otimes$:

$$(\mathcal{A}\otimes\mathcal{B})(z)=\sup_{x\cdot y=z}T(\mathcal{A}(x), \mathcal{B}(y)).$$

The addition(subtraction) rule for $L-R$ fuzzy numbers is well known in the case of T_M-based addition and then the resulting sum is again on $L-R$ fuzzy numbers, i.e., the shape is preserved. Diamond [4] used T_M-based addition in his paper. It is also known that T_W-based addition preserves the shape of $L-R$ fuzzy numbers [12, 14]. In practical computation, it is natural to require the preserving the shape of fuzzy numbers during the multiplication. Of course, we know that T_M-based multiplication does not preserve the shape of $L-R$ fuzzy numbers. But it is known by Hong and Do [7] that T_W induces shape preserving multiplication of $L-R$ fuzzy numbers. Recently, Hong [6] showed that T_W is the unique t-norm which induces shape preserving in multiplication of $L-R$ fuzzy numbers.

In [9], Hong et al. used T_W-based fuzzy arithmetic operations.

Let $\mathcal{A}_i=(a_i, \alpha_i)_L$ and $\mathcal{X}_{ij}=(x_{ij}, \gamma_{ij})_L$, $i=1, 2, \cdots, n$, $j=1, 2, \cdots, p$.

Then the membership function of $\mathcal{Y}_i=(\mathcal{A}_1\otimes\mathcal{X}_{i1})\oplus(\mathcal{A}_2\otimes\mathcal{X}_{i2})\oplus\cdots\oplus(\mathcal{A}_p\otimes\mathcal{X}_{ip})$ is given by

$$\mathcal{Y}_i=\left(\sum_{j=1}^{p}a_i x_{ij},\ \max_{1\le j\le p}(|a_i|\gamma_{ij},\ |x_{ij}|\alpha_i)\right)_L. \tag{2}$$

Let $\mathcal{B}_i$, $i=1, 2, \cdots, n$ be fuzzy number. Define $\sum_{i=1}^{n}\mathcal{B}_i=\mathcal{B}_1\oplus\cdots\oplus\mathcal{B}_n$. A possibilistic quadratic polynomial systems whose parameter is defined as

$$\mathcal{Y}=\sum_{j=1}^{p}(\mathcal{A}_j\otimes\mathcal{X}_j)\oplus\sum_{1\le l\le k\le p}(\mathcal{A}_{l,k}\otimes\mathcal{X}_l\otimes\mathcal{X}_k) \tag{3}$$

where $\mathcal{A}=\{\mathcal{A}_j, \mathcal{A}_{l,k}|1\le j\le p, 1\le l\le k\le p\}$ is a fuzzy parameters and $\mathcal{X}=(\mathcal{X}_1,\cdots,\mathcal{X}_p)$ is a fuzzy vector. Using T_W-based arithmetic operations, we have the following lemma by (2).

Proposition 2.1 Let $\mathcal{A}_j=(a_j, \alpha_j)_L$, $\mathcal{A}_{l,k}=(a_{l,k}, \alpha_{l,k})_L$ and $\mathcal{X}_j=(x_j, \gamma_j)_L$. Then the possibilistic quadratic polynomial function with fuzzy parameter $\mathcal{A}_j$, $\mathcal{A}_{l,k}$ and fuzzy variables $\mathcal{X}_j$, $j=1,2, \cdots,p$, $1\le l\le k\le p$ is given by

$$\begin{aligned}\mathcal{Y}=\Big(&\sum_{j=1}^{p}a_j x_j+\sum_{1\le l\le k\le p}a_{l,k}x_l x_k,\\ &\max\{\max_{1\le j\le p}(|a_j|\gamma_j, \alpha_j|x_j|),\\ &\max_{1\le l\le k\le p}(\alpha_{l,k}|x_l||x_k|, |a_{l,k}|\gamma_l|x_k|, |a_{l,k}||x_l|\gamma_k)\}\Big)_L.\end{aligned} \tag{4}$$

3. Fuzzy polynomial regression

In this section, we consider fuzzy quadratic polynomial regression model for least-square fitting.

Let $F_{LR}(R)$ be the set of all $L-R$ fuzzy numbers.

In order to solve fuzzy least squares optimization problem in $F_{LR}(R)$, we use the metric D_{LR} which is defined as distance on triangular fuzzy numbers by Diamond [4] as follows:

$$\begin{aligned}D_{LR}(\mathcal{A}_1, \mathcal{A}_2)^2=&(a_1-a_2)^2+\\ &((a_1-\alpha_1)-(a_2-\alpha_2))^2+((a_1+\beta_1)-(a_2+\beta_2))^2\end{aligned} \tag{5}$$

where $\mathcal{A}_1=(a_1, \alpha_1, \beta_1)_{LR}$, $\mathcal{A}_2=(a_2, \alpha_2, \beta_2)_{LR}$.

Hong et al.[8] considered the following model :

$$(\text{H}):\ \mathcal{Y}=\mathcal{A}\oplus(\mathcal{B}\otimes\mathcal{X})$$

where $\mathcal{A}, \mathcal{B}, \mathcal{X}, \mathcal{Y}\in F_{LR}(R)$.

In this section, we consider the following model:

$$(P):\ \mathcal{Y}=\sum_{j=1}^{p}(\mathcal{A}_j\otimes\mathcal{X}_j)\oplus\sum_{1\le l\le k\le p}(\mathcal{A}_{l,k}\otimes\mathcal{X}_l\otimes\mathcal{X}_k) \tag{6}$$

where $\mathcal{A}_j$, $\mathcal{A}_{l,k}$, $\mathcal{X}$, $Y\in F_{LR}(R), 1\le j\le p$, $1\le l\le k\le p$.

We assume, throughout this section, that $\mathcal{A}_j$, $\mathcal{A}_{ij}$,

$\mathcal{X}$, $\mathcal{Y} \in F_{LR}(R)$ are symmetric $L-R$ fuzzy numbers for computational simplicity.
Suppose that observations consist of data pairs $(\mathcal{X}_i, \mathcal{Y}_i)$, $i=1, 2, \cdots, n$, where $\mathcal{X}_i = (\mathcal{X}_{i1}, \cdots, \mathcal{X}_{ip})$,

$\mathcal{X}_{ij} = (x_{ij}, \gamma_{ij})_L$, $j=1,\cdots,p$, $\mathcal{Y}_i = (y_i, \eta_i)_L$.

Each is to be fitted to the data in the sense of best fit with respect to the D_{LR}-metric. In association with the model (P), consider the least-squares optimization problem

(D): Minimize

$$r(a,\alpha) = \sum_{i=1}^{n} D_{LR}\Big(\sum_{j=1}^{p}(\mathcal{A}_j \otimes \mathcal{X}_{ij}) \oplus \sum_{1\le l\le k\le p}(\mathcal{A}_{l,k} \otimes \mathcal{X}_{il} \otimes \mathcal{X}_{ik}),\ \mathcal{Y}_i\Big). \quad (7)$$

Let $\mathcal{A}_j = (a_j, \alpha_j)_L$ and $\mathcal{A}_{l,k} = (a_{l,k}, \alpha_{l,k})_L$, then by (4)

$$D_{LR}\Big(\sum_{j=1}^{p} a_j x_{ij} + \sum_{1\le l\le k\le p} a_{l,k} x_{il} x_{ik}, \max\{\max_{1\le j\le p}(|a_j|\gamma_{ij}, \alpha_j|x_{ij}|), \max_{1\le l\le k\le p}(\alpha_{l,k}|x_{il}||x_{ik}|, |a_{l,k}|\gamma_{il}|x_{ik}|, |a_{l,k}||x_{il}|\gamma_{ik})\}\}_L, \mathcal{Y}_i\Big)$$
$$= \Big[\sum_{j=1}^{p} a_j x_{ij} + \sum_{1\le l\le k\le p} a_{l,k} x_{il} x_{ik} - \max\{\max_{1\le j\le p}(|a_j|\gamma_{ij}, \alpha_j|x_{ij}|), \max_{1\le l\le k\le p}(\alpha_{l,k}|x_{il}||x_{ik}|, |a_{l,k}|\gamma_{il}|x_{ik}|, |a_{l,k}||x_{il}|\gamma_{ik})\} - (y_i - \eta_i)\Big]^2$$
$$+ \Big[\sum_{j=1}^{p} a_j x_{ij} + \sum_{1\le l\le k\le p} a_{l,k} x_{il} x_{ik} + \max\{\max_{1\le j\le p}(|a_j|\gamma_{ij}, \alpha_j|x_{ij}|), \max_{1\le l\le k\le p}(\alpha_{l,k}|x_{il}||x_{ik}|, |a_{l,k}|\gamma_{il}|x_{ik}|, |a_{l,k}||x_{il}|\gamma_{ik})\} - (y_i + \eta_i)\Big]^2$$
$$+ \Big[\sum_{j=1}^{p} a_j x_{ij} + \sum_{1\le l\le k\le p} a_{l,k} x_{il} x_{ik} - y_i\Big]^2.$$

This problem can be proved by QP problem as follows;

Let $M=\{(j,l,k) | 1\le j\le p, 1\le l\le k\le p\}$, and define

$$A(i,(j,l,k),H_r) = \{((a_1,\alpha_1),\cdots,(a_p,\alpha_p),(a_{1,1},\alpha_{1,1}),\cdots,(a_{p,p},\alpha_{p,p})) \in (R^2)^{\frac{p(p+3)}{2}} \mid \max(|a_j|\gamma_{ij}, |x_{ij}|\alpha_j, \alpha_{l,k}|x_{il}||x_{ik}|, |a_{l,k}|\gamma_{il}|x_{ik}|, |a_{l,k}||x_{il}|\gamma_{ik}) = H_r\}$$

where $H_1 = a_j\gamma_{ij}$, $a_j \ge 0$, $H_2 = -a_j\gamma_{ij}$, $a_j < 0$, $H_3 = |x_{ij}|\alpha_j$, $H_4 = \alpha_{l,k}|x_{il}||x_{ik}|$, $H_5 = a_{l,k}\gamma_{il}|x_{ik}|$, $a_{l,k}\ge 0$, $H_6 = -a_{l,k}\gamma_{il}|x_{ik}|$, $a_{l,k} < 0$, $H_7 = a_{l,k}|x_{il}|\gamma_{ik}$, $a_{l,k}\ge 0$, $H_8 = -a_{l,k}|x_{il}|\gamma_{ik}$, $a_{l,k} < 0$.

Let f and g be functions such that

$$f: \{1,2,\cdots,n\} \to M,$$
$$g: M \to \{H_1, H_2, \cdots, H_8\}.$$

On $\bigcap_{i=1}^{n} A(i, f(i), g(f(i)))$, (7) is a QP problem and

$$Min\ r(a,\alpha) = Min_{f,g} Min_{(a,\alpha)\in \bigcap_{i=1}^{n} A(i,f(i),g(f(i)))} r(a,\alpha).$$

For example, let $n=2$, $p=2$ in (7). Then the model can be written as

$$\mathcal{Y}_i^* = (\mathcal{A}_1^* \otimes \mathcal{X}_{i1}) \oplus (\mathcal{A}_2^* \otimes \mathcal{X}_{i1}) \oplus (\mathcal{A}_{1,1}^* \otimes \mathcal{X}_{i1} \otimes \mathcal{X}_{i2}) \oplus (\mathcal{A}_{1,2}^* \otimes \mathcal{X}_{i1} \otimes \mathcal{X}_{i2}) \oplus (\mathcal{A}_{2,2}^* \otimes \mathcal{X}_{i2} \otimes \mathcal{X}_{i2})$$

and $M=\{(1,1,1),(1,1,2),(1,2,2),(2,1,1),(2,1,2),(2,2,2)\}$. Let $f:\{1,2\}\to M$ be such that $f(1)=(1,1,2)$, $f(2)=(2,1,1)$ and let $g: M \to \{H_1, H_1, \cdots, H_8\}$ be such that $g(f(1)) = g((1,1,2)) = H_5$, $g(f(2)) = g((2,1,1)) = H_3$. Then, on $A(1,f(1),g(f(1))) \bigcap A(2,f(2),g(f(2)))$, (7) is written as

Minimize

$$r(a,\alpha) = \Big[\Big(\sum_{j=1}^{2} a_j x_{1j} + \sum_{1\le l\le k\le 2} a_{l,k} x_{1l} x_{1k}\Big) - a_{1,2}\gamma_{11}|x_{12}| - (y_1 - \eta_1)\Big]^2$$
$$+ \Big[\Big(\sum_{j=1}^{2} a_j x_{1j} + \sum_{1\le l\le k\le 2} a_{l,k} x_{1l} x_{1k}\Big) + a_{1,2}\gamma_{11}|x_{12}| - (y_1 + \eta_1)\Big]^2$$
$$+ \Big[\Big(\sum_{j=1}^{2} a_j x_{1j} + \sum_{1\le l\le k\le 2} a_{l,k} x_{1l} x_{1k}\Big) - y_1\Big]^2$$
$$+ \Big[\Big(\sum_{j=1}^{2} a_j x_{2j} + \sum_{1\le l\le k\le 2} a_{l,k} x_{2l} x_{2k}\Big) - |x_{22}|\alpha_2 - (y_2 - \eta_2)\Big]^2 + \Big[\Big(\sum_{j=1}^{2} a_j x_{2j} + \sum_{1\le l\le k\le 2} a_{l,k} x_{2l} x_{2k}\Big) + |x_{22}|\alpha_2 - (y_2 + \eta_2)\Big]^2 + \Big[\Big(\sum_{j=1}^{2} a_j x_{2j} + \sum_{1\le l\le k\le 2} a_{l,k} x_{2l} x_{2k}\Big) - y_2\Big]^2,$$

$|a_1|\gamma_{11} \le a_{1,2}\gamma_{11}|x_{12}|$, $|x_{11}|\alpha_1 \le a_{1,2}\gamma_{11}|x_{12}|$,
$\alpha_{1,2}|x_{11}||x_{12}| \le a_{1,2}\gamma_{11}|x_{12}|$, $a_{1,2}\ge 0$,
$a_{1,2}|x_{11}|\gamma_{12} \le a_{1,2}\gamma_{11}|x_{12}|$,
$|a_2|\gamma_{22} \le |x_{22}|\alpha_2$, $\alpha_{1,1}|x_{21}||x_{21}| \le |x_{22}|\alpha_2$,
$|a_{1,1}||x_{21}|\gamma_{21} \le |x_{22}|\alpha_2$,

which is a QP problem with respect to a_j, α_j, $j=1,2$, $a_{l,k}$, $\alpha_{l,k}$, $1\le l\le k\le 2$.

Now we consider all such functions f and g and take minimum with regard to them. Then we get the desired solution.

Example. We consider the same artificial data shown in Table 1 in [8].

Table 1. Fuzzy Input-Output Data of Nonlinear Type

Sample number i	$\mathcal{X}_i = (x_i, \gamma_i)$	$\mathcal{Y}_i = (y_i, e_i)$
1	(1.0, 0.5)	(6.3, 2.0)
2	(1.5, 0.5)	(11.5, 1.5)
3	(2.0, 1.0)	(20.0, 2.0)
4	(3.0, 1.0)	(24.0, 1.5)
5	(4.0, 1.0)	(26.1, 1.0)
6	(4.5, 0.5)	(30.0, 3.0)
7	(5.0, 1.5)	(33.8, 2.5)
8	(5.5, 1.0)	(34.0, 3.0)
9	(6.0, 2.0)	(38.1, 2.5)
10	(15.0, 1.0)	(21.9, 1.5)
11	(6.5, 1.5)	(39.9, 3.0)

Sample number i	$X_i=(x_i, \gamma_i)$	$Y_i=(y_i, e_i)$
12	(7.0, 2.5)	(42.0, 1.5)
13	(8.0, 2.0)	(46.1, 2.0)
14	(9.0, 3.0)	(53.1, 4.0)
15	(10.0, 2.0)	(52.0, 5.0)
16	(11.0, 2.0)	(52.5, 3.5)
17	(12.0, 1.0)	(48.0, 3.0)
18	(13.0, 1.0)	(42.8, 2.5)
19	(14.0, 1.0)	(27.8, 2.0)

Noting that

$$A_0 \oplus (A_1 \otimes X_i) \oplus (A_2 \otimes X_i)$$
$$= (a_0 + a_1x_i + a_2x_i^2,$$
$$\max(\alpha_0, |a_1|\,\gamma_i,\ \alpha_1 x_i, |a_2| x_i\gamma_i, \alpha_2 x_i^2))$$

we minimize

$$r(a,\alpha) = \sum_{i=1}^{19}([(a_0+a_1x_i+a_2x_i^2) - \max(\alpha_0, |a_1|\gamma_i, \alpha_1x_i, |a_2|x_i\gamma_i, \alpha_2x_i^2) - (y_i-\eta_i)]^2 + [(a_0+a_1x_i+a_2x_i^2) + \max(\alpha_0, |a_1|\gamma_i, \alpha_1x_i, |a_2|x_i\gamma_i, \alpha_2x_i^2) - (y_i+\eta_i)]^2 + [(a_0+a_1x_i+a_2x_i^2) - y_i]^2)$$

$$f(a_0, a_1, a_2) = \sum_{i=1}^{19}[2(a_0+a_1x_i+a_2x_i^2)^2 + (y_i-\eta_i)^2 + (y_i+\eta_i)^2 - 2(a_0+a_1x_i+a_2x_i^2)(y_i-\eta_i) - 2(a_0+a_1x_i+a_2x_i^2)(y_i+\eta_i) + [(a_0+a_1x_i+a_2x_i^2) - y_i]^2]$$

$$r(a,\alpha) = f(a_0, a_1, a_2) + \sum_{i=1}^{19}\{2[\max(\alpha_0, |a_1|\gamma_i, \alpha_1x_i, |a_2|x_i\gamma_i, \alpha_2x_i^2)]^2 - 4\eta_i\max(\alpha_0, |a_1|\gamma_i, \alpha_1x_i, |a_2|x_i\gamma_i, \alpha_2x_i^2)\}$$

$$\frac{\partial r}{\partial a_0} = \frac{\partial f}{\partial a_0} = \sum_{i=1}^{19}[4(a_0+a_1x_i+a_2x_i^2) - 2(y_i-\eta_i) - 2(y_i+\eta_i) + 2(a_0+a_1x_i+a_2x_i^2 - y_i)]$$
$$= 6\sum_{i=1}^{19}(a_0+a_1x_i+a_2x_i^2 - y_i)$$
$$= 6na_0 + 6a_1\sum_{i=1}^{19}x_i + 6a_2\sum_{i=1}^{19}x_i^2 - 6\sum_{i=1}^{19}y_i,$$
$$\frac{\partial r^2}{\partial^2 a_0} = \frac{\partial f^2}{\partial^2 a_0} = 6n.$$

So a solution for a_0 is given by the solution a_0^* to the equation

$$a_0^* = \hat{y} - a_1\hat{x} - a_2\widehat{x^2},\ \text{where}\ \hat{y} = \frac{\sum_{i=1}^{19} y_i}{19},$$
$$\hat{x} = \frac{\sum_{i=1}^{19} x_i}{19}\ \text{and}\ \widehat{x^2} = \frac{\sum_{i=1}^{19} x_i^2}{19}.$$

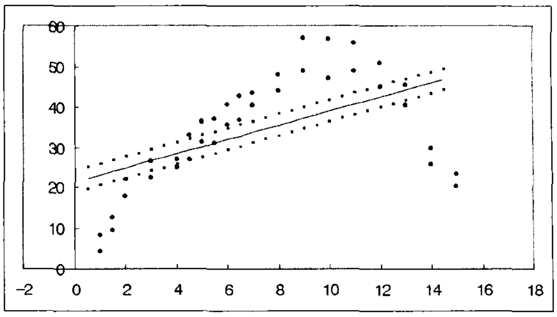

Figure 1. The Fuzzy Linear Regression Model

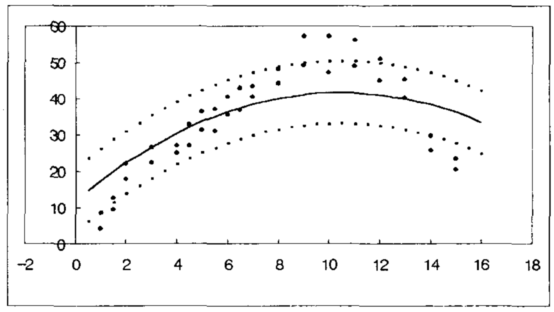

Figure 2. The Fuzzy Quadratic polynomial Regression Model

Then the solution for fuzzy linear regression model is $A_0^* = (21.474, 2)$, $A_1^* = (1.75, 0.13)$ with $r(a^*, \alpha^*) = (7161.78)$ and the solution for fuzzy polynomial regression model is $A_0^* = (12.035, 0 \sim 0.47)$, $A_1^* = (5.66, 0 \sim 0.35)$, $A_2^* = (-0.27, 0.02)$ with $r(a^*, \alpha^*) = (4649.995)$.

In Fig. 1 and Fig. 2, the 19 pairs of dots are $(x_i, y_i \pm e_i)$, $i = 1, 2, \cdots, 19$. The curve and two broken curves are loci of the membership function of the fuzzy linear regression model and fuzzy polynomial regression model, where $\bar{\xi}$ is $\sum_{i=1}^{19}\gamma_i/19$.

As you can see Fig.1 and Fig. 2, both fuzzy linear regression model and fuzzy polynomial regression model don't fit well. But fuzzy polynomial regression model fits better than fuzzy linear regression model. We need to study about detection of outliers or develop other types of non-linear fuzzy regression model with fuzzy input-output data.

4. Conclusion

We suggested fuzzy quadratic polynomial regression for least-square fitting using shape preserving operations. We use general nonlinear programming problem to derive the optimal solutions. An artificial example is given.

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