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Strong fuzzy hyperK-subalgebra

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Abstract

In this paper, we define a strong fuzzy hyperK-subalgebra and investigate between a strong fuzzy hyperK-subalgebra and a fuzzy hyperK-subalgebra. And then we give some properties of a weak homomorphism and a strong fuzzy hyperK-subalgebra.

Key Words : strong fuzzy hyperK-subalgebra, fuzzy hyperK-subalgebra, weak homomorphism.

1. INTRODUCTION

The hyper algebraic structure theory was introduced in 1934 by F. Marty [8] at the 8th Congress of Scandinavian Mathematicians. Since then many researchers have worked on this area. Recently, Y. B. Jun et al. [6] applied the hyperstructures to BCK-algebras and introduced the concept of a hyperBCK-algebras which is a generalization of a BCK-algebra, and investigated some properties. They also introduced the notion of a hyperBCK-ideal and a weak hyperBCK-ideal, and gave relations between hyperBCK-ideals and weak hyperBCK-ideals. R. A. Brozoei et al. [1] defined the notion of a hyperK-algebra. Hyperstructures have many applications to several sectors of both pure and applied sciences. In this paper, we define a strong fuzzy HyperK-subalgebra and investigate between a strong fuzzy hyperK-subalgebra and a fuzzy hyperK-subalgebra. And then we give some properties of a weak homomorphism and a strong fuzzy hyperK-subalgebra.

2. PRELIMINARIES

R. A. Borzoei, A. Hasankhani and M. M. Zahedi established the notion of hyperI/hyperK-algebras as follows: By *a hyperI-algebra* we mean a nonempty set H endowed with a hyperoperation "o" and a constant 0 satisfying the following axioms:

(HI1) $(x \circ z) \circ (y \circ z) < x \circ y$,
(HI2) $(x \circ y) \circ z = (x \circ z) \circ y$,
(HI3) $x < x$,
(HI4) $x < y$ and $y < x$ imply $x = y$

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for all $x, y, z \in H$, where $x < y$ is defined by $0 \in x \circ y$ and for every $A, B \subseteq H$, $A < B$ is defined by $\exists a \in A$ and $\exists b \in B$ such that $a < b$. If a hyperI- algebra $(H; \circ, 0)$ satisfies

(HI5) $0 < x$ for all $x \in H$,

then $(H; \circ, 0)$ is called a *hyperK-algebra*, Let $(H; \circ, 0)$ be *a hyperK-algebra* and let S be a subset of H containing 0. If S is a hyperK-algebra with respect to the hyperoperation "$\circ$" on H, we say that S is a *hyperK-subalgebra* of H.

We now review some fuzzy logic concepts. A fuzzy set in a set X is a function $\mu : X \to [0, 1]$. For a fuzzy set μ in X and $\alpha \in [0,1]$, define $\bigcup(\mu; \alpha)$ to be the set $\bigcup(\mu; \alpha) := \{x \in X \mid \mu(x) \geq \alpha\}$, which is called a *level set of* μ.

In what follows, H denotes a hyperK-algebra unless otherwise specified.

Definition 2.1 ([3]). A fuzzy set μ In H is said to be a *fuzzy hyperK-subalgebra* of H it is satisfies the inequality:

$$\inf_{z \in x \circ y} \mu(z) \geq \min\{\mu(x), \mu(y)\}$$

for all $x, y \in H$

Proposition 2.2 ([3]). *Let μ be a fuzzy hyperK-subalgebra of H. Then $\mu(0) \geq \mu(x)$ for all $x \in H$.*

Lemma 2.3 ([3]). *Let S be a non-empty subset of H. Then S is a hyperK-subalgebra of H if and only if $x \circ y \subseteq S$ for all $x, y \in S$.*

Theorem 2.4 ([3]). *Let μ be a fuzzy set in H. Then μ is a fuzzy hyperK-subalgebra of H if and only if for every $\alpha \in [0, 1]$ the non-empty level set $U(\mu; \alpha)$ of μ is a hyperK-subalgebra of H.*

We then call $U(\mu; \alpha)$ a *level hyperK-subalgebra of* μ.

3. Strong fuzzy hyperK-subalgebra

Definition 3.1. A fuzzy set μ in H is said to be a *strong fuzzy hyperK-subalgebra* of H if it satisfies the inequality:

$$\inf_{z\in x\circ y}\mu(z)\geq\mu(x),$$

for all $x,\ y\in H$.

Proposition 3.2. *Let μ be a strong fuzzy hyperK-subalgebra of H. Then $\mu(0)\geq\mu(x)$ for all $x\in H$.*

Proof. Using (HI3), we see that $0\in x\circ x$ for all $x\in H$. Hence $\mu(0)\geq\inf_{z\in x\circ x}\mu(z)\geq\mu(x)$, for all $x\in H$. □

Example 3.3. Let $H=\{0,\ 1,\ 2\}$. Consider the following table:

∘	0	1	2
0	{0}	{0}	{0}
1	{1}	{0, 1}	{0, 1}
2	{2}	{1, 2}	{0, 1, 2}

Then $(H;\circ,0)$ is a hyperK-algebra. Define a fuzzy set $\mu: H\rightarrow[0,\ 1]$ by $\mu(0)=\mu(1)=\alpha_1>\alpha_2=\mu(2)$. Then μ is a fuzzy hyperK-subalgebra

Theorem 3.4. Let $(H;\circ,0)$ *be a hyperK-algebra Then every strong fuzzy hyperK-subalgebra of H is a fuzzy hyperK-subalgebra of H.*

Proof. Since μ is a strong fuzzy hyperK-algebra, $\inf_{z\in x\circ y}\mu(z)\geq\mu(x)$, for all $x,\ y\in H$, Then $\inf_{z\in x\circ y}\mu(z)\geq\mu(x)\geq\min\{\mu(x),\ \mu(y)\}$ for all $x,\ y\in H$. Hence μ is a fuzzy hyperK-subalgebra of H.

Example 3.5. Let $H=\{0,1,2\}$. Consider the following table:

∘	0	1	2
0	{0}	{0, 1, 2}	{0, 1, 2}
1	{1}	{0, 1, 2}	{0, 1, 2}
2	{2}	{1, 2}	{0, 1, 2}

Then $(H;\circ,0)$ is a hyperK-algebra. Define a fuzzy set $\mu: H\rightarrow[0,1]$ by $\mu(0)=1$ and $\mu(1)=\mu(2)=0$. Then μ is a fuzzy hyperK- subalgebra of H, but not a strong fuzzy hyperK-subalgebra of H, since $\inf_{z\in 0\circ 1}\mu(z)=0\not\geq 1=\mu(0)$

Theorem 3.6. *Let μ be a strong fuzzy hyperK-subalgebra of H. Then for every $\alpha\in[0,1]$ the non-empty level set $U(\mu;\alpha)$ of μ is a hyperK-subalgebra of H.*

Proof. Suppose that μ is a strong fuzzy hyperK-subalgebra of H and let $x,y\in U(\mu:\alpha)$ for $\alpha\in[0,1]$. Let $z\in x\circ y$. Then

$$\mu(z)\geq\inf_{w\in x\circ y}\mu(w)\geq\mu(x)\geq\alpha,$$

and so $z\in U(\mu;\alpha)$. This shows that $x\circ y\subseteq U(\mu;\alpha)$ Hence $U(\mu;\alpha)$ is a hyperK-subalgebra of H. □

Theorem 3.7. *Let S be a non-empty subset of H and let μ_s be a fuzzy set in H defined by*

$$\mu_s(x):=\begin{cases}\alpha_1 & if x\in S\\ \alpha_2 & otherwise,\end{cases}$$

for all $x\in H$ and $\alpha_1>\alpha_2$ in [0,1]. Then μ_s is a strong fuzzy hyperK-subalgebra of H if and only if S is a hyperK-subalgebra of H.

Proof. Assume that $\mu_s\mu_s$ is a strong fuzzy hyperK-subalgebra of H and let $x,y\in S$. Then $\mu_s(x)=\alpha_1=\mu_s(y)$. For any $z\in x\circ y$ we have

$$\mu_s(z)\geq\inf_{w\in x\circ y}\mu_s(w)\geq\mu_s(x)=\alpha_1$$

and so $\mu_s(z)=\alpha_1$. Hence $z\in S$, which shows that $x\circ y\subseteq S$. Therefore S is a hyperK-subalgebra of H by Lemma 2.3. Conversely, suppose that S is a hyperK-subalgebra of H and let $x,y\in H$. If $x\notin S$ or $y\notin S$, then clearly

$$\inf_{w\in x\circ y}\mu_s(w)\geq\alpha_2=\mu_s(x).$$

Assume that $x\in S$ and $y\in S$. Then $x\circ y\subseteq S$, and μ thus

$$\inf_{z\in x\circ y}\mu_s(z)=\alpha_1=\mu_s(x).$$

Consequently, μ_s is a strong fuzzy hyperK-subalgebra of H. □

Definition 3.8 ([3]). Let H_1 and H_2 be hyperK-algebras. A mapping $f: H_1\rightarrow H_2$ is called a *weak homomorphism if*

(i) $f(0)=0$,
(ii) $f(x\circ y)\subseteq f(x)\circ f(y)$ for all $x,y\in H_1$

Theorem 3.9. *Let $f: H_1\rightarrow H_2$ be a weak homomorphism of hyperK-algebras. If μ is a fuzzy hyperK-subalgebra of H_2, then μ_f is a strong fuzzy hyperK-subalgebra of H_1 where μ_f is defined by $\mu_f(x):=\mu(f(x))$, for all $x\in H_1$.*

Proof. For any $x,y\in H_1$, we have

$$\inf_{z\in x\circ y}\mu_f(z)=\inf_{z\in x\circ y}\mu(f(z))$$
$$\geq\inf_{f(z)\in f(x\circ y)}\mu(f(z))$$

$$\geq \inf_{f(z)\in f(x)\circ f(y)} \mu(f(z))$$
$$\geq \mu(f(x))$$
$$= \mu_f(x),$$

which shows that μ_f is a strong fuzzy hyperK-subalgebra of H_1. □

Let $t \geq 0$ be a real number. If $\alpha \in [0, 1]$, α^t shall mean the positive root in case $t < 1$. We define $\mu^t: H \to [0, 1]$ by $\mu^t(x) := (\mu(x))^t$ for all $x \in H$.

Theorem 3.10. *If μ is a strong fuzzy hyperK-subalgebra of H, then so is μ^t for all $t \geq 0$.*

Proof. For any $x, y \in H$ and $t \geq 0$, we have

$$\inf_{z\in x\circ y} \mu^t(z) = \inf_{z\in x\circ y} (\mu(z))^t$$
$$= (\inf_{z\in x\circ y} \mu(z))^t$$
$$\geq (\mu(x))^t$$
$$= \mu^t(x)$$

Hence μ^t is a strong fuzzy hyperK-subalgebra of H.□

Theorem 3.11. *Let μ be a strong fuzzy hyperK-subalgebra of H and $\theta: [0, \mu(0)] \to [0, 1]$ be an increasing function. Let μ_θ be a fuzzy set in H defined by $\mu_\theta(x) := \theta(\mu(x))$ for all $x \in H$. Then μ_θ is a strong fuzzy hyperK-subalgebra of H.*

Proof. Let $x, y \in H$. Then

$$\inf_{z\in x\circ y} \mu_\theta(z) = \inf_{z\in x\circ y} \theta(\mu(z))$$
$$\geq \theta(\inf_{z\in x\circ y} \mu(z))$$
$$\geq \theta(\mu(x))$$
$$= \mu_\theta(x)$$

Hence μ_θ is a strong fuzzy hyperK-subalgebra of H.□

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