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Fuzzy semi-topogenous orders and fuzzy supra topologies

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Abstract

We investigate the properties of fuzzy (semi-)topogenous orders in the framework of fuzzy (supra) topologies and fuzzy (supra) interior operators. We study the relationship between fuzzy (semi-)topogenous structures, fuzzy (supra)topologies and fuzzy (supra)interior operators.

Key Words : Fuzzy (semi-)topogenous order, Fuzzy (supra) interior operator, Fuzzy (supra) topology.

1. Introduction and Preliminaries

Csaszar [3] introduced the concept of a syntopogenous structure to develop to the three main structures of topologies, proximities and uniformities.

Katsaras and Petalas [11] extended them to the theory of fuzzy sets. Katsaras [4-11] has developed in many directions. El-Monsef and Ramadan [1] defined and studied the concept of fuzzy supra topological spaces.

In this paper, we investigate the properties of fuzzy (semi-) topogenous orders, fuzzy (supra)topological spaces and fuzzy (supra) interior operators. We study the relationship between them.

Thought this paper, let X be a nonempty set, $I=[0,1]$ and I^X the family of all fuzzy subsets of X. For $\alpha\in I$, $\overline{\alpha}(x)=\alpha$ for all $x\in X$. For a subset A of X, χ_A is a characteristic function of A.

Definition 1.1([1,2]) A subset τ of I^X is called a fuzzy supra topology on X if it satisfies the following conditions:

(O1) $\overline{0},\overline{1}\in\tau$,

(O2) $\bigvee_{i\in\Gamma}\mu_i\in\tau$ for any $\mu_i\in\tau$.

A fuzzy supra topology τ is called a fuzzy topology if it satisfies

(O3) $\mu_1\wedge\mu_2\in\tau$ for any $\mu_1,\mu_2\in\tau$.

The pair (X,τ) is called a fuzzy (resp. supra) topological space.

Definition 1.2([1,2]) A function $\text{int}:I^X\to I^X$ is called a fuzzy supra interior operator on X if it satisfies the following conditions:

(I1) $\text{int}(\overline{1})=\overline{1}$.

(I2) $\text{int}(\lambda)\le\lambda$.

(I3) If $\lambda_1\le\lambda_2$, then $\text{int}(\lambda_1)\le\text{int}(\lambda_2)$.

A fuzzy supra interior operator int is called a fuzzy interior operator if it satisfies

(I) $\text{int}(\lambda_1\wedge\lambda_2)=\text{int}(\lambda_1)\wedge\text{int}(\lambda_2)$.

A fuzzy supra interior operator int is called topological if it satisfies

(T) $\text{int}(\text{int}(\lambda))=\text{int}(\lambda)$.

Theorem 1.3([1,2]) Let (X,τ) be a fuzzy (resp. supra) topological space. We define a function $\text{int}_\tau:I^X\to I^X$ as follows:

$$\text{int}_\tau(\lambda)=\bigvee\{\mu\in I^X \mid \mu\le\lambda,\mu\in\tau\}$$

Then int_τ is a topological fuzzy (resp. supra) interior operator on X.

2. Fuzzy (semi-)topogenous orders and fuzzy (supra) topologies

Let $\ll$ be a binary relation on X;i.e. $\ll\subset I^X\times I^X$. The facts that $(\lambda,\mu)\in\ll$ and $(\lambda,\mu)\in\ll$ are denoted by $\lambda\ll\mu$ and $\lambda\overline{\ll}\mu$, respectively.

Definition 2.1 ([11]) A binary relation $\ll$ on $I^X$is called a fuzzy semi-topogenous order on X if it satisfies:

(T1) $\overline{1}\ll\overline{1}$ and $\overline{0}\ll\overline{0}$,

(T2) if $\lambda\ll\mu$, then $\lambda\le\mu$,

(T3) if $\lambda\le\lambda_1\ll\mu_1\le\mu$, then $\lambda\ll\mu$.

Remark 2.2 Let $\ll$ be a fuzzy semi-topogenous order on X. Define by

$$\lambda\ll^s\mu \text{ iff } (\overline{1}-\mu)\ll(\overline{1}-\lambda).$$

Then $\ll^s$ is a fuzzy semi-topogenous order on X.

Definition 2.3 ([11]) A fuzzy semi-topogenous order $\ll$ is called:

(1) symmetric if $\ll=\ll^s$, that is,

(T4) $\lambda\ll\mu$ iff $(\overline{1}-\mu)\ll(\overline{1}-\lambda)$

(2) fuzzy topogenous if for any $\lambda,\lambda_1,\lambda_2,\mu,\mu_1,\mu_2\in I^X$,

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(T5) $\lambda_1 \vee \lambda_2 \ll \mu$ iff $\lambda_1 \ll \mu$, $\lambda_2 \ll \mu$

(T6) $\lambda \ll \mu_1 \wedge \mu_2$ iff $\lambda \ll \mu_1$, $\lambda \ll \mu_2$

(3) perfect if, for any $\{\mu, \lambda_i \mid i \in \Gamma\} \subset I^X$,

(T7) $\bigvee_{i \in \Gamma} \lambda_i \ll \mu$ iff $\lambda_i \ll \mu$, for all $i \in \Gamma$.

(4) biperfect if it is perfect and for any $\{\lambda, \mu_i \mid i \in \Gamma\} \subset I^X$,

(T8) $\lambda \ll \bigwedge_{i \in \Gamma} \mu_i$ iff $\lambda \ll \mu_i$, for all $i \in \Gamma$.

Definition 2.4 ([11]) Let $\langle\langle_1$ and $\langle\langle_2$ be fuzzy semi-topogenous orders on X. $\langle\langle_1$ is finer than $\langle\langle_2$ ($\langle\langle_2$ is coarser than $\langle\langle_1$) if $\lambda \langle\langle_1 \mu$ for all $\lambda \langle\langle_2 \mu$.

Definition 2.5 ([11]) A fuzzy topogenous order $\ll$ on X is called a fuzzy topogenous structure satisfying the following condition:

(T) $\ll \circ \ll$ is finer than $\ll$ where $\ll \circ \ll$ is defined by, for any $\lambda, \mu \in I^X$, $\lambda (\ll \circ \ll) \mu$ iff there exists $\rho \in I^X$ such that $\lambda \ll \rho$ and $\rho \ll \mu$.

The pair $(X, \ll)$ is called a fuzzy topogenous space.

A fuzzy topogenous structure $\ll$ is called perfect (resp. biperfect, symmetric, etc.) if $\ll$ is perfect (resp. biperfect, symmetric, etc.).

Theorem 2.6 Let $\ll$ be a fuzzy semi-topogenous order on X. A mapping $\text{int}_{\langle\langle} : I^X \to I^X$ is defined by

$$\text{int}_{\langle\langle}(\lambda) = \bigvee \{\mu \in I^X \mid \mu \ll \lambda\}.$$

Then we have the following properties:

(1) $\text{int}_{\langle\langle}$ is a fuzzy supra interior operator.

(2) If $\ll$ satisfies (T6), then $\text{int}_{\langle\langle}$ is a fuzzy interior operator.

(3) If $\ll$ satisfies (T5), then $\text{int}_{\langle\langle'}$ is a fuzzy interior operator.

(4) If $\ll \circ \ll$ is finer than $\ll$, then, for each $\lambda \in I^X$, $\text{int}_{\langle\langle}(\text{int}_{\langle\langle}(\lambda)) = \text{int}_{\langle\langle}(\lambda)$.

(5) If $\ll$ is a fuzzy topogenous structure, then $\text{int}_{\langle\langle}$ is a topological fuzzy interior operator.

Proof (1) (I1) Since $\overline{1} \ll \overline{1}$, $\text{int}_{\langle\langle}(\overline{1}) = \overline{1}$.

(I2) Let $\mu \ll \lambda$. By (T2), $\mu \le \lambda$. It implies $\text{int}_{\langle\langle}(\lambda) \le \lambda$.

(I3) If $\lambda_1 \le \lambda_2$ and $\mu \ll \lambda_1$, by (T3), $\mu \ll \lambda_2$. Thus, $\text{int}_{\langle\langle}(\lambda_1) \le \text{int}_{\langle\langle}(\lambda_2)$

(2) From (I3), we have

$$\text{int}_{\langle\langle}(\lambda_1 \wedge \lambda_2) \le \text{int}_{\langle\langle}(\lambda_1) \wedge \text{int}_{\langle\langle}(\lambda_2)$$

Conversely, suppose there exist $\lambda_1, \lambda_2 \in I^X$ such that

$$\text{int}_{\langle\langle}(\lambda_1 \wedge \lambda_2) \ge \text{int}_{\langle\langle}(\lambda_1) \wedge \text{int}_{\langle\langle}(\lambda_2)$$

There exist $x \in X$ and $t \in I_1$ such that

$$\text{int}_{\langle\langle}(\lambda_1 \wedge \lambda_2)(x) < t < \text{int}_{\langle\langle}(\lambda_1)(x) \wedge \text{int}_{\langle\langle}(\lambda_2)(x)$$

Since $\text{int}_{\langle\langle}(\lambda_i)(x) > t$, for each $i \in \{1,2\}$, there exists $\mu_i \in I^X$ with $\mu_i \ll \lambda_i$ such that

$$\text{int}_{\langle\langle}(\lambda_i)(x) \ge \mu_i(x) > t$$

On the other hand, since

$\mu_1 \ll \lambda_1$, $\mu_2 \ll \lambda_2 \Rightarrow (\mu_1 \wedge \mu_2) \ll \lambda_i$, $i \in \{1,2\}$ ((T3))

$\Rightarrow (\mu_1 \wedge \mu_2) \ll (\lambda_1 \wedge \lambda_2)$ ((T6))

we have $\text{int}_{\langle\langle}(\lambda_1 \wedge \lambda_2)(x) \ge (\mu_1 \wedge \mu_2)(x) > t$.

It is a contradiction. Thus,

$$\text{int}_{\langle\langle}(\lambda_1 \wedge \lambda_2) \ge \text{int}_{\langle\langle}(\lambda_1) \wedge \text{int}_{\langle\langle}(\lambda_2)$$

(3) It is similarly proved as (2).

(4) Since $\text{int}_{\langle\langle}(\lambda) \le \lambda$,by (T3),

$$\text{int}_{\langle\langle}(\text{int}_{\langle\langle}(\lambda)) \le \text{int}_{\langle\langle}(\lambda)$$

Suppose there exists $\lambda \in I^X$ such that

$$\text{int}_{\langle\langle}(\text{int}_{\langle\langle}(\lambda)) \ge \text{int}_{\langle\langle}(\lambda)$$

There exist $x \in X$ and $t \in I_1$ such that

$\text{int}_{\langle\langle}(\text{int}_{\langle\langle}(\lambda))(x) < t < \text{int}_{\langle\langle}(\lambda)(x)$.

Since $\text{int}_{\langle\langle}(\lambda)(x) > t$, there exists $\mu \in I^X$ with $\mu \ll \lambda$ such that $\text{int}_{\langle\langle}(\lambda)(x) \ge \mu(x) > t$.

Since $\ll \circ \ll$ is finer than $\ll$, then $\mu \ll \lambda$ implies $\mu (\ll \circ \ll) \lambda$ .Then there exists $\rho \in I^X$ such that $\mu \ll \rho$ and $\rho \ll \lambda$. Hence $\mu \ll \rho \le \text{int}_{\langle\langle}(\lambda)$ implies $\mu \ll \text{int}_{\langle\langle}(\lambda)$. Thus

$$\text{int}_{\langle\langle}(\text{int}_{\langle\langle}(\lambda))(x) \ge \mu(x) > t.$$

It is a contradiction. Thus,

$$\text{int}_{\langle\langle}(\text{int}_{\langle\langle}(\lambda)) \ge \text{int}_{\langle\langle}(\lambda).$$

(5) It is trivial from (2) and (4).

Theorem 2.7 Let $\ll$ be a fuzzy semi-topogenous order on X. Define a fuzzy topology on X by

$$\tau_{\langle\langle} = \{\lambda \in I^X \mid \text{int}_{\langle\langle}(\lambda) = \lambda\}.$$

Then:

(1) $\tau_{\ll}$ is a fuzzy supra topology on X induced by $\ll$.

(2) If $\ll$ satisfies (T6), then $\tau_{\ll}$ is a fuzzy topology on X.

(3) If $\ll$ is perfect, then $\lambda \in \tau_{\langle\langle}$ iff $\lambda \ll \lambda$, for each $\lambda \in I^X$.

Proof (1)(O1) Since $\text{int}_{\langle\langle}(\overline{0}) = \overline{0}$ and $\text{int}_{\langle\langle}(\overline{1}) = \overline{1}$, then $\overline{0}, \overline{1} \in \tau_{\langle\langle}$.

(O2) Let $\lambda_j \in \tau_{\langle\langle}$, for each $j \in \Gamma$. Then $\lambda_j = \text{int}_{\langle\langle}(\lambda_j)$. By Theorem 2.6 (1), we have

$$\text{int}_{\langle\langle}(\bigvee_{j \in \Gamma} \lambda_j) \ge \bigvee_{j \in \Gamma} \text{int}_{\langle\langle}(\lambda_j) = \bigvee_{j \in \Gamma} \lambda_j.$$

So, $\text{int}_{\langle\langle}(\bigvee_{j \in \Gamma} \lambda_j) = \bigvee_{j \in \Gamma} \lambda_j$. Hence $\bigvee_{j \in \Gamma} \lambda_j \in \tau_{\langle\langle}$.

Thus, $\tau_{\langle\langle}$ is a fuzzy supra topology on X.

(2) Let $\lambda_1, \lambda_2 \in \tau_{\ll}$. Then

$$\lambda_i = \text{int}_{\ll}(\lambda_i), i=1,2.$$

From Theorem 2.6 (2), we have
$\text{int}_{\ll}(\lambda_1 \wedge \lambda_2) = \text{int}_{\ll}(\lambda_1) \wedge \text{int}_{\ll}(\lambda_2) = \lambda_1 \wedge \lambda_2$. Consequently, $\lambda_1 \wedge \lambda_2 \in \tau_{\ll}$.
(3) Let $\mu \in \tau_{\ll}$. Then

$$\mu = \text{int}_{\ll}(\mu) = \bigvee\{\rho \in I^X \mid \rho \ll \mu\}.$$

Since $\ll$ is perfect, $\mu \ll \mu$.
Conversely, let $\lambda \ll \lambda$. Then $\text{int}_{\ll}(\lambda) \geq \lambda$.
So, $\text{int}_{\ll}(\lambda) = \lambda$. Thus, $\lambda \in \tau_{\ll}$.

Example 2.8 We define binary relations $\ll_1, \ll_2$ on I^X as follows

$$\lambda \ll_1 \mu \text{ iff } \lambda \leq \mu,$$
$$\lambda \ll_2 \mu \text{ iff } \lambda = \overline{0} \text{ or } \mu = \overline{0}.$$

We easily show that $(X, \ll_i)$ are fuzzy biperfect and symmetric topogenous spaces for $i=1,2$.
From Theorem 2.6, we can obtain fuzzy interior operators $\text{int}_{\ll_i}: I^X \to I^X$ as follows:

$$\text{int}_{\ll_1}(\lambda) = \lambda,$$
$$\text{int}_{\ll_2}(\lambda) = \begin{cases} \overline{1}, \text{ if } \lambda = \overline{1}, \\ \overline{0}, \text{ otherwis.} \end{cases}$$

From Theorem 2.7, $\tau_{\ll_1} = I^X$ and $\tau_{\ll_2} = \{\overline{0}, \overline{1}\}$.

Theorem 2.9 Let int be a fuzzy supra interior operator on X. Define a binary relation $\ll_{\text{int}}$ as

$$\lambda \ll_{\text{int}} \mu \text{ iff } \lambda \leq \text{int}(\mu).$$

Then :
(1) $\ll_{\text{int}}$ is a perfect fuzzy semi-topogenous order on X such that $\text{int}_{\ll_{\text{int}}}(\lambda) = \text{int}(\lambda)$ for each $\lambda \in I^X$.
(2) If int is a fuzzy interior operator on X, then $\ll_{\text{int}}$ is a fuzzy topogenous order on X.
(3) If $\text{int}(\text{int}(\lambda)) = \text{int}(\lambda)$ for each $\lambda \in I^X$, then $(\ll_{\text{int}} \circ \ll_{\text{int}})$ is finer than $\ll_{\text{int}}$.
(4) If int be a topological fuzzy interior operator on X, then $\ll_{\text{int}}$ is a fuzzy topogenous structure on X.
(5) If $\ll$ is a fuzzy semi-topogenous order, then $\ll_{\text{int}_{\ll}}$ is finer than $\ll$.
(6) If $\ll$ is a perfect fuzzy semi-topogenous order, then $\ll = \ll_{\text{int}_{\ll}}$.

Proof (1) (T1) Since $\text{int}(\overline{0}) = \overline{0}$ and $\text{int}(\overline{1}) = \overline{1}$, then $\overline{0} \ll_{\text{int}} \overline{0}$, $\overline{1} \ll_{\text{int}} \overline{1}$.
(T2) If $\lambda \ll_{\text{int}} \mu$, then $\lambda \leq \text{int}(\mu) \leq \mu$
(T3) Let $\lambda \leq \lambda_1 \ll_{\text{int}} \mu_1 \leq \mu$. Since $\lambda_1 \leq \text{int}(\mu_1)$, then $\lambda_1 \leq \text{int}(\mu)$. Hence $\lambda \ll_{\text{int}} \mu$.

(T7) Let $\bigvee_{i\in\Gamma} \lambda_i \ll_{\text{int}} \mu$ .Since $\lambda_i \leq \bigvee_{i\in\Gamma} \lambda_i \ll_{\text{int}} \mu$, by (T3), $\lambda_i \ll_{\text{int}} \mu$ for all $i \in \Gamma$.
Let $\lambda_i \ll_{\text{int}} \mu$ for all $i \in \Gamma$. Then $\lambda_i \leq \text{int}(\mu)$ for all $i \in \Gamma$. Thus, $\bigvee_{i\in\Gamma} \lambda_i \leq \text{int}(\mu)$.
It implies $\bigvee_{i\in\Gamma} \lambda_i \ll_{\text{int}} \mu$ Thus, $\ll_{\text{int}}$ is a perfect fuzzy semi-topogenous order on X.
Let $\mu \ll_{\text{int}} \lambda$. Then $\mu \leq \text{int}(\lambda)$. By the definition of $\text{int}_{\ll_{\text{int}}}$, we have $\text{int}_{\ll_{\text{int}}}(\lambda) \leq \text{int}(\lambda)$
Since $\text{int}(\lambda) \leq \text{int}(\lambda)$, then $\text{int}(\lambda) \ll_{\text{int}} \lambda$
Thus, $\text{int}_{\ll_{\text{int}}}(\lambda) \geq \text{int}(\lambda)$.
(2) From (1), we only show $\ll_{\text{int}}$ satisfies (T6).
Let $\lambda \ll_{\text{int}} (\mu_1 \wedge \mu_2)$. Since $\lambda \ll_{\text{int}} (\mu_1 \wedge \mu_2) \leq \mu_i$ for $i=1,2$, then , by (T3), $\lambda \ll_{\text{int}} \mu_i$.
Let $\lambda \ll_{\text{int}} \mu_i$ for $i=1,2$. Then $\lambda \leq \text{int}(\mu_i)$.
Since int is a fuzzy interior operator,
$\lambda \leq \text{int}(\mu_1) \wedge \text{int}(\mu_2) = \text{int}(\mu_1 \wedge \mu_2)$
Thus, $\lambda \ll_{\text{int}} (\mu_1 \wedge \mu_2)$.
(3) Let $\lambda \ll_{\text{int}} \mu$. Then $\lambda \leq \text{int}(\mu)$. Since $\lambda \leq \text{int}(\mu) = \text{int}(\text{int}(\mu))$ and $\text{int}(\mu) \leq \text{int}(\mu)$ then $\lambda \ll_{\text{int}} \text{int}(\mu)$ and $\text{int}(\mu) \ll_{\text{int}} \mu$. Thus, $\lambda (\ll_{\text{int}} \circ \ll_{\text{int}}) \mu$.
(4) It is trivial from (2) and (3).
(5) Let $\lambda \ll \mu$. Then $\lambda \leq \text{int}_{\ll}(\mu)$.
Thus $\lambda \ll_{\text{int}_{\ll}} \mu$ Thus, $\ll_{\text{int}_{\ll}}$ is finer than $\ll$.
(6) Let $\lambda \ll_{\text{int}_{\ll}} \mu$. Then

$$\lambda \leq \text{int}_{\ll}(\mu) = \bigvee\{\rho \mid \rho \ll \mu\}.$$

Since $\ll$ is perfect, then $\text{int}_{\ll}(\mu) \ll \mu$
From (T3), $\lambda \leq \text{int}_{\ll}(\mu) \ll \mu$ implies $\lambda \ll \mu$.

Theorem 2.10 Let τ be a fuzzy supra topology on X. Then:
(1) $\lambda \ll_{\text{int}_\tau} \mu$ iff there exists $\rho \in \tau$ such that $\lambda \leq \rho \leq \mu$
(2) $\tau_{\ll_{\text{int}_\tau}} = \tau$.

Proof (1) Let $\lambda \ll_{\text{int}_\tau} \mu$.
Then $\lambda \leq \text{int}_\tau(\mu) \leq \mu$ By Definition 1.1 (O2), we have $\text{int}_\tau(\mu) \in \tau$.
Let $\rho \in \tau$ such that $\lambda \leq \rho \leq \mu$.
Then $\lambda \leq \text{int}_\tau(\rho) = \rho \leq \mu$.
Hence $\lambda \ll_{\text{int}_\tau} \rho \leq \mu$. By (T3), $\lambda \ll_{\text{int}_\tau} \mu$.
(2) Since $\ll_{\text{int}_\tau}$ is perfect semi-topogenous order on X, by Theorem 2.7(3),
$\lambda \in \tau_{\ll_{\text{int}_\tau}} \Rightarrow \lambda \ll_{\text{int}_\tau} \lambda$
Then there exists $\rho \in \tau$ such that
$\lambda \leq \rho \leq \lambda$. So $\lambda \in \tau$.
Conversely, let $\lambda \in \tau$. Then $\lambda = \text{int}_\tau(\lambda)$. It implies $\lambda \ll_{\text{int}_\tau} \lambda$. Thus, $\lambda \in \tau_{\ll_{\text{int}_\tau}}$.

Example 2.11 Let $X = \{x, y, z\}$ be a set.

Define binary relations $\langle\langle_i$ on I^X as follows:

$$\lambda\langle\langle_1\mu \text{ iff } \begin{cases} \lambda=\overline{0} \text{ or } \mu=\overline{1}, \\ \overline{0}\neq\lambda\leq\chi_{\{x\}}\ \overline{1}\neq\mu\geq\chi_{\{x,y\}} \\ \overline{0}\neq\lambda\leq\chi_{\{y\}}\ \overline{1}\neq\mu\geq\chi_{\{x,y\}} \end{cases}$$

$$\lambda\langle\langle_2\mu \text{ iff } \begin{cases} \lambda=\overline{0} \text{ or } \mu=\overline{1}, \\ \overline{0}\neq\lambda\leq\chi_{\{y\}}\ \overline{1}\neq\mu\geq\chi_{\{x,y\}} \\ \overline{0}\neq\lambda\leq\chi_{\{y\}}\ \overline{1}\neq\mu\geq\chi_{\{y,z\}} \end{cases}$$

$$\lambda\langle\langle_3\mu \text{ iff } \begin{cases} \lambda=\overline{0} \text{ or } \mu=\overline{1}, \\ \overline{0}\neq\lambda\leq\chi_{\{x\}}\ \overline{1}\neq\mu\geq\chi_{\{x,y\}} \end{cases}$$

(1) $\langle\langle_1$ is a fuzzy semi-topogenous order on X. but not topogenous because

$\chi_{\{x\}}\langle\langle_1\chi_{\{x,y\}}$, $\chi_{\{y\}}\langle\langle_1\chi_{\{x,y\}}$ but $(\chi_{\{x\}}\vee\chi_{\{y\}})\overline{\langle\langle_1}\chi_{\{x,y\}}$

From Theorem 2.6, we can obtain fuzzy supra interior operator $\text{int}_{\langle\langle_1}: I^X\rightarrow I^X$ as follows:

$$\text{int}_{\lambda\langle\langle_1}(\lambda)=\begin{cases} \overline{1} \text{ if } \lambda=\overline{1}, \\ \chi_{\{x,y\}} \text{ if } \chi_{\{x,y\}}\leq\lambda\neq\overline{1} \\ \overline{0} \quad \text{otherwise} \end{cases}$$

From Theorem 2.7, we can obtain fuzzy supra topology $\tau_{\langle\langle_1}$ as follows:

$$\tau_{\langle\langle_1}=\{\overline{0},\overline{1},\chi_{\{x,y\}}\}.$$

Since $\chi_{\{x,y\}}\in\tau_{\langle\langle_1}$, but $\chi_{\{x,y\}}\overline{\langle\langle_1}\chi_{\{x,y\}}$,
by Theorem 2.7 (3), $\langle\langle_1$ is not perfect
From Theorem 2.9(5), $\langle\langle_{\text{int}_{\langle\langle_1}}$ is finer than $\langle\langle_1$,
but $\langle\langle_1\neq\langle\langle_{\text{int}_{\langle\langle_1}}$ as follows:

$$\lambda\langle\langle_{\text{int}_{\langle\langle_1}}\mu \text{ iff } \begin{cases} \lambda=\overline{0} \text{ or } \mu=\overline{1}, \\ \overline{0}\neq\lambda\leq\chi_{\{x,y\}}\ \overline{1}\neq\mu\geq\chi_{\{x,y\}} \end{cases}$$

(2) $\langle\langle_2$ is a perfect fuzzy semi-topogenous order on X but not topogenous because:
$\chi_{\{y\}}\langle\langle_2\chi_{\{x,y\}}$, $\chi_{\{y\}}\langle\langle_2\chi_{\{y,z\}}$
but $\chi_y\overline{\langle\langle_2}(\chi_{\{x,y\}}\wedge\chi_{\{y,z\}})$.
From Theorem 2.6, we can obtain fuzzy supra interior operator $\text{int}_{\langle\langle_2}: I^X\rightarrow I^X$ as follows:

$$\text{int}_{\lambda\langle\langle_2}(\lambda)=\begin{cases} \overline{1} \text{ if } \lambda=\overline{1}, \\ \chi_{\{y\}} \text{ if } \chi_{\{x,y\}}\leq\lambda\neq\overline{1} \\ \chi_{\{y\}} \text{ if } \chi_{\{y,z\}}\leq\lambda\neq\overline{1} \\ \overline{0} \quad \text{otherwise} \end{cases}$$

But it is not a fuzzy interior operator because

$\overline{0}=\text{int}_{\langle\langle_2}(\chi_{\{x,y\}}\wedge\chi_{\{y,z\}})$
$\neq\text{int}_{\langle\langle_2}(\chi_{\{x,y\}})\wedge\text{int}_{\langle\langle_2}(\chi_{\{y,z\}})=\chi_{\{y\}}$.
From Theorem 2.9(6), since $\langle\langle_2$ is perfect, we have $\langle\langle_2=\langle\langle_{\text{int}_{\langle\langle_2}}$ as follows:

$$\lambda\langle\langle_{\text{int}_{\langle\langle_2}}\mu \text{ iff } \begin{cases} \lambda=\overline{0} \text{ or } \mu=\overline{1}, \\ \overline{0}\neq\lambda\leq\chi_{\{y\}}\ \overline{1}\neq\mu\geq\chi_{\{x,y\}} \\ \overline{0}\neq\lambda\leq\chi_{\{y\}}\ \overline{1}\neq\mu\geq\chi_{\{y,z\}} \end{cases}$$

Furthermore,

$\tau_{\langle\langle_2}=\{\overline{0},\overline{1}\}$ iff $\lambda\langle\langle_2\lambda$, for all $\lambda\in\{\overline{0},\overline{1}\}$.

(3) $\langle\langle_3$ is a fuzzy topogenous order on X but not topogenous structure from the following statements:
For any $\rho\in I^X$ with $\chi_{\{x\}}\leq\rho\leq\chi_{\{x,y\}}$,we have

$$\chi_{\{x\}}\overline{\langle\langle_3}\rho,\quad \rho\overline{\langle\langle_3}\chi_{\{x,y\}}.$$

Thus, $\chi_{\{x\}}\overline{(\langle\langle_3\circ\langle\langle_3)}\chi_{\{x,y\}}$ but $\chi_{\{x\}}\langle\langle_3\chi_{\{x,y\}}$. From Theorem 2.6(2), $\text{int}_{\langle\langle_3}$ is a fuzzy interior operator from:

$$\text{int}_{\lambda\langle\langle_3}(\lambda)=\begin{cases} \overline{1} \text{ if } \lambda=\overline{1}, \\ \chi_{\{x\}} \text{ if } \chi_{\{x,y\}}\leq\lambda\neq\overline{1} \\ \overline{0} \quad \text{otherwise} \end{cases}$$

From Theorem 2.6(3), since $\langle\langle_3\circ\langle\langle_3$ is not finer than $\langle\langle_3$, in general, we have

$$\chi_{\{x\}}=\text{int}_{\langle\langle_3}(\chi_{\{x,y\}})\neq\text{int}_{\langle\langle_3}(\text{int}_{\langle\langle_3}(\chi_{\{x,y\}}))=\overline{0}.$$

(4) We easily show that $\langle\langle_1, \langle\langle_2$ and $\langle\langle_3$ are not symmetric.

(5) We define a fuzzy interior operator $\text{int}: I^X\rightarrow I^X$ as follows:

$$\text{int}(\lambda)=\begin{cases} \overline{1} \text{ if } \lambda=\overline{1}, \\ \chi_{\{x\}} \text{ if } \chi_{\{x,y\}}\leq\lambda\neq\overline{1} \\ \overline{0} \quad \text{otherwise} \end{cases}$$

From Theorems 2.6 and 2.9, we obtain the followings:

$$\lambda\langle\langle_{\text{int}}\mu \text{ iff } \begin{cases} \lambda=\overline{0} \text{ or } \mu=\overline{1}, \\ \overline{0}\neq\lambda\leq\chi_{\{x\}}\ \overline{1}\neq\mu\geq\chi_{\{x,y\}} \end{cases}$$

$$\text{int}_{\langle\langle_{\text{int}}}(\lambda)=\begin{cases} \overline{1} \text{ if } \lambda=\overline{1}, \\ \chi_{\{x\}} \text{ if } \chi_{\{x,y\}}\leq\lambda\neq\overline{1} \\ \overline{0} \quad \text{otherwise} \end{cases}$$

We have $\text{int}_{\langle\langle_{\text{int}}}(\lambda)=\text{int}(\lambda)$ for all $\lambda\in I^X$.

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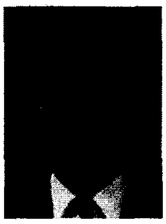

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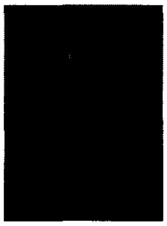

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