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NONPARAMETRIC ONE-SIDED TESTS FOR MULTIVARIATE AND RIGHT CENSORED DATA†

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ABSTRACT

In this paper, we formulate multivariate one-sided alternatives and propose a class of nonparametric tests for possibly right censored data. We obtain the asymptotic tail probability (or p-value) by showing that our proposed test statistics have asymptotically multivariate normal distributions. Also, we illustrate our procedure with an example and compare it with other procedures in terms of empirical powers for the bivariate case. Finally, we discuss some properties of our test.

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1. INTRODUCTION

Suppose that a laboratory has developed a medicine or a treatment which may have some effects on several (≥ 2) symptoms simultaneously and conducted a drug-placebo experiment. The data consist of d dimensional vectors of which some component may be right censored. In this situation, one may perform a nonparametric test for checking the equality between two multivariate survival functions under the general alternatives. However, it is often of interest to test if the medicine or treatment has significant effects on some of the d symptoms. This corresponds to the so-called one-sided testing problem for multivariate data. For example, consider the data from the National Cooperative Gallstone Study

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(NCGS) (*cf.* Schoenfield *et al.*, 1981). To identify the efficacy of *chenodiol* used in a treatment for cholesterol gallstones, Wei and Lachin (1984) raised an issue whether there exists any difference in progression of gallbladder disease between the control and the high dose groups. Here progression of the disease is indicated by occurrence of gallbladder pain or presence of pain accompanied by other symptoms that require surgical removal of the gallbladder (cholecystectomy). Each observation in either group contains the time lengths of experiencing gallbladder pain and cholecystectomy. Therefore it is more appropriate for the NCGS data to consider one-sided alternatives rather than general ones. Wei and Lachin (1984) proposed a testing procedure in this direction based on sum of d univariate test statistics.

For the complete bivariate data case, Bhattacharyya and Johnson (1970) proposed a nonparametric test for the ordered alternatives based on the layer ranks (*cf.* Barndorff-Nielsen and Sobel, 1966). Also, Johnson and Mehrotra (1972) developed another nonparametric test based on linear rank statistic (*cf.* Puri and Sen, 1971). The limiting distributions of these test statistics are univariate normal. Boyett and Shuster (1977) proposed a nonparametric test based on maximal T-statistic but did not provide the limiting distribution. More recently, Park *et al.* (2001) suggested a nonparametric procedure based on the univariate nonparametric test statistics and showed its asymptotic normality. All the above mentioned works, adopt the permutation principle to obtain the exact null distribution for the small sample case. For censored data, Wei and Knuiman (1987) proposed a nonparametric procedure modifying the idea of the layer ranks for censored data. An extension of Gehan test for bivariate data can be also considered. However, since the censoring distribution is involved even under the null hypothesis, an exact null distribution based on the permutation principle may not be available. Hence, the asymptotic univariate normality was derived. We note that extensions to the cases of three or higher dimension for the procedure of Wei and Knuiman (1987) are not easy since the method of the layer ranks may not be applicable to the data of three or higher dimension.

In this paper, we propose an asymptotically nonparametric test procedure for censored data using the method adopted by Boyett and Shuster (1977). Our procedure can be applied to the data with three or more dimensions. We show that the limiting distributions of our proposed test statistics are related to multivariate normal distribution. Therefore calculation of the limiting tail probability (or p-value) for any given data depends completely on the software such as S-PLUS (*cf.* S-PLUS 6 Programmer's Guide, 2001) or M_X (*cf.* Neale *et al.*, 1998) program.

We illustrate our procedure using the NCGS data. We compare the performance of our procedure with that of Wei and Knuiman (1987) in terms of empirical powers through computer simulations. Also we discuss briefly some properties of our procedure. Finally we derive the limiting covariance and the limiting power of the test under the Pitman translation alternatives in the appendices.

2. Nonparametric One-sided Tests for Right Censored Data

Let $\{\mathbf{X}_{ij} = (X_{i1j}, \ldots, X_{idj})', j = 1, \ldots, n_i\}$ be independently and identically distributed d-variate life time random vectors with non-negative components and a continuous distribution function $F_i(x_1, \ldots, x_d)$, $i = 1, 2$. Also, let $\{\mathbf{U}_{ij} = (U_{i1j}, \ldots, U_{idj})', j = 1, \ldots, n_i\}$ be independently distributed d-variate censoring random vectors with an arbitrary distribution function $G_i(x_1, \ldots, x_d)$, $i = 1, 2$. In order to avoid the identifiability problem, we assume that the life time random vectors are independent of the censoring random vectors. Because of the random censoring scheme, we may only observe for each k, $k = 1, \ldots, d$, that

$$T_{ikj} = \min(X_{ikj}, U_{ikj}) \quad \text{and} \quad \delta_{ikj} = I(X_{ikj} \leq U_{ikj})$$

for $i = 1, 2$ and $j = 1, \ldots, n_i$, where $I(\cdot)$ is an indicator function. Since we are concerned about the location translation model, we assume that there exists $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_d)' \in \mathbb{R}^d$ with $\theta_k \geq 0$ such that

$$F_2(\mathbf{t}) = F_1(\mathbf{t} - \boldsymbol{\theta})$$

for all $\mathbf{t} \in \mathbb{R}^d$. Based on this model, we formulate a hypothesis as follows:

$$H_0 : \theta_1 \leq 0, \theta_2 \leq 0, \ldots, \theta_d \leq 0$$
$$vs. \quad H_1 : \text{at least one of } \theta_k\text{'s is strictly larger than } 0.$$

Before constructing the test statistics for this problem, we introduce some notations. For each k^{th} component, let $N_{ik}(t) = \sum_{j=1}^{n_i} I(T_{ikj} \leq t,\ \delta_{ikj} = 1)$ be the number of deaths that occur no later than time t in group i and $Y_{ik}(t) = \sum_{j=1}^{n_i} I(T_{ikj} \geq t)$, the corresponding number at risk by time t. Also, let Q_k be a bounded non-negative predictable process which is a function of observations and satisfies $Q_k(t) = 0$ whenever $\min\{Y_{1k}(t), Y_{2k}(t)\} = 0$. Furthermore, let $n = n_1 + n_2$ and

$$T_{kn} = \sqrt{n} \int_0^\infty Q_k(t) \left\{ \frac{dN_{1k}(t)}{Y_{1k}(t)} - \frac{dN_{2k}(t)}{Y_{2k}(t)} \right\}. \tag{2.1}$$

We note that T_{kn} forms a class of linear rank statistics by varying Q_k. For example, the Gehan and log-rank statistics correspond to $Q_k = Y_{1k}Y_{2k}/n^2$ and $Q_k = Y_{1k}Y_{2k}/\{n(Y_{1k}+Y_{2k})\}$, respectively. Also, we note that T_{kn}'s are used for testing $H_0^k : \theta_k = 0$ against $H_1^k : \theta_k \neq 0$ and the testing rule is to reject H_0^k in favor of H_1^k for large values of $|T_{kn}|$. In order to obtain the critical value for any given significance level or p-value, we need the null distribution of T_{kn}. Since the censoring distribution is involved in the distribution of T_{kn} even under H_0^k, we shall obtain the limiting distribution using the large sample approximation theory. For this, we introduce some additional notations. For each $i = 1, 2$ and for each $k = 1, \ldots, d$, let Λ_{ik} denote the k^{th} marginal cumulative hazard function. Also, let F_{ik} and G_{ik} be the k^{th} marginal distribution functions for F_i and G_i, respectively.

Now, we collect the assumptions below:

(A1) $n/n_i \longrightarrow \rho_i \in (1, \infty)$, as $n \longrightarrow \infty$, $i = 1, 2$;

(A2) For each k, the random weight function Q_k converges in probability to a function q_k uniformly on each closed subinterval of $[0, \infty)$ as $n \longrightarrow \infty$.

Under (A1) and (A2), we have the following well-known result, which is due to Gill (1980).

LEMMA 1. *Suppose (A1) and (A2) hold. Then, under $H_0^k : \theta_k = 0$ the test statistic T_{kn} converges in distribution to a normal random variable with mean 0 and variance σ_k^2, where*

$$\sigma_k^2 = \sum_{i=1}^{2} \rho_i \int \frac{q_k^2}{(1-F_{ik})(1-G_{ik})}(1-\Delta\Lambda_{ik})d\Lambda_{ik}, \tag{2.2}$$

and $\Delta\Lambda_{ik}(t) = \Lambda_{ik}(t) - \Lambda_{ik}(t-)$ *for each* $i = 1, 2$ *and* $k = 1, \ldots, d$.

We need a consistent estimate $\widehat{\sigma}_k^2$ of σ_k^2 for implementing T_{kn} to real testing problems. We use the following consistent estimate of σ_k^2:

$$\widehat{\sigma}_k^2 = n \sum_{i=1}^{2} \sum_{j=1}^{n_i} \frac{Q_k^2(T_{ikj})}{Y_{ik}^2(T_{ikj})} \left\{1 - \frac{\Delta N_{ik}(T_{ikj}) - 1}{Y_{ik}(T_{ikj}) - 1}\right\}\delta_{ikj}, \tag{2.3}$$

where $\Delta N_{ik}(t) = N_{ik}(t) - N_{ik}(t-)$ for each i and k.

Without loss of generality, we assume that one rejects $H_0^k : \theta_k \leq 0$ in favor of the alternatives $H_1^k : \theta_k > 0$ for large positive values of T_{kn}. We propose a nonparametric one-sided test statistic for censored and multivariate data following Boyett and Shuster (1977) as follows:

$$T_n = \max\left(\frac{T_{1n}}{\widehat{\sigma}_1}, \dots, \frac{T_{dn}}{\widehat{\sigma}_d}\right), \tag{2.4}$$

where $\widehat{\sigma}_k = (\widehat{\sigma}_k^2)^{1/2}$ for each k. We take the maximum among all the d studentized univariate test statistics. We note that the weight function Q_k is allowed to vary with component. When H_1 is true, at least one of $(T_{kn}/\widehat{\sigma}_k)$'s would tend to have large positive value. Thus we may reject H_0 for large values of T_n in favor of H_1.

In order to determine the critical value $C_n(\alpha)$ for any given significance level α or obtain the p-value, we need to derive the null distribution of T_n. For this end, we note that for any $t > 0$,

$$\begin{aligned} P(T_n > t) &= P\left\{\max\left(\frac{T_{1n}}{\widehat{\sigma}_1}, \dots, \frac{T_{dn}}{\widehat{\sigma}_d}\right) > t\right\} \\ &= 1 - P\left\{\max\left(\frac{T_{1n}}{\widehat{\sigma}_1}, \dots, \frac{T_{dn}}{\widehat{\sigma}_d}\right) \leq t\right\} \\ &= 1 - P\left(\frac{T_{1n}}{\widehat{\sigma}_1} \leq t, \dots, \frac{T_{dn}}{\widehat{\sigma}_d} \leq t\right). \end{aligned} \tag{2.5}$$

We have to consider the limiting distribution. To begin with, we obtain the following asymptotic result for $(T_{1n}, \dots, T_{dn})'$ from Lemma 1.

THEOREM 1. *Under H_0 with (A1) and (A2), $(T_{1n}, \dots, T_{dn})'$ converges in distribution to a d-variate normal random vector with mean zero vector and covariance matrix $(\sigma_{kl})_{k,l=1,\dots,d}$.*

PROOF. It follows from Lemma 1 that any linear combination of $T_{1n}, \dots, T_{dn}$ is asymptotically normally distributed if the limiting covariance matrix exists. Therefore from Cramér-Wold device (*cf.* Shorack and Wellner, 1986), Theorem 1 follows if we show that σ_{kl} is the limit of $\mathrm{Cov}(T_{kn}, T_{ln})$ for $k \neq l$. The expression and derivation of σ_{kl} will be given briefly in Appendix A. □

In order to implement our procedure, we have to obtain a consistent estimate of (σ_{kl}). It is enough to obtain a consistent estimate of σ_{kl} for $k \neq l$. For each i and k, let

$$\widehat{V}_{ik}(s) = \sum_{j=1}^{n_i} \frac{Q_k(T_{ikj})}{Y_{ik}(T_{ikj})} \cdot \frac{\delta_{ikj} I(T_{ikj} \leq s)}{Y_{ik}(T_{ikj})}. \tag{2.6}$$

A consistent estimate $\widehat{\sigma}_{kl}$ of σ_{kl} is defined by

$$\begin{aligned}\widehat{\sigma}_{kl} = n\sum_{i=1}^{2}\sum_{j=1}^{n_i}\frac{Q_k(T_{ikj})}{Y_{ik}(T_{ikj})}\frac{Q_l(T_{ilj})}{Y_{il}(T_{ilj})}\delta_{ikj}\delta_{ilj} - n\sum_{i=1}^{2}\sum_{j=1}^{n_i}\frac{Q_k(T_{ikj})}{Y_{ik}(T_{ikj})}\widehat{V}_{il}(T_{ilj})\delta_{ikj} \\ -n\sum_{i=1}^{2}\sum_{j=1}^{n_i}\frac{Q_l(T_{ilj})}{Y_{il}(T_{ilj})}\widehat{V}_{ik}(T_{ikj})\delta_{ilj} + n\sum_{i=1}^{2}\sum_{j=1}^{n_i}\widehat{V}_{ik}(T_{ikj})\widehat{V}_{il}(T_{ilj}).\end{aligned} \tag{2.7}$$

The proof for the consistency of $\widehat{\sigma}_{kl}$ can be found in Wei and Lachin (1984).

By Theorem 1 and Slutsky's theorem, one can easily show that the random vector $(T_{1n}/\widehat{\sigma}_1, \ldots, T_{dn}/\widehat{\sigma}_d)'$ converges in distribution to a d-variate normal random vector with mean zero vector and covariance matrix $\mathbf{R} = (r_{kl})$ whose all diagonal elements equal one. The off-diagonal elements can be consistently estimated by $\widehat{r}_{kl} = \widehat{\sigma}_{kl}/(\widehat{\sigma}_k\widehat{\sigma}_l)$ for $k \neq l$. Thus for $t > 0$, we may obtain the tail probability, $P(T_n > t)$, from the d-variate normal distribution with the estimated covariance matrix $\widehat{\mathbf{R}} = (\widehat{r}_{kl})$. In the case of bivariate normal distributions with mean zero vector and unit variances, Owen (1962) tabulated cumulative probabilities when both coordinates have the same values. However, the tables are not enough since they do not contain the cumulative probabilities for all the values of correlation coefficient. We may approximate $P(T_n > t)$ by using the *pmvnorm* function provided by S-PLUS for bivariate data. For the $d(\geq 3)$-variate case, we may use the M_X program (*cf.* Neale *et al.*, 1998). The program and documentation can be downloaded from the website *http://www.vipbg.vcu.edu/mxgui.*

3. Example and Simulation Results

For illustration of our procedure, we consider a part of the NCGS data, which are given in Table 1 of Wei and Lachin (1984). In the table, each observation in the placebo ($n_1 = 48$) and high dose ($n_2 = 65$) groups consist of the time lengths of experiencing gallbladder pain (X_{i1j}) and cholecystectomy (X_{i2j}). From the data, it would be of interest to see if there is any effect of the drug chenodiol on the gallbladder disease. For the general alternatives, Wei and Lachin (1984) obtained 0.042 and 0.145 as the p-values for the Gehan and the log-rank score tests, respectively. For the one-sided alternatives, the procedure by Wei and Knuiman (1987) gives 0.037 as its p-value. For our testing procedure using the Gehan score for both components, the values of T_n and $\widehat{\rho}_{12}$ are 2.4579 and 0.637, respectively. The corresponding approximate p-value is 0.013, which shows a strong evidence of delays of disease progression of gallbladder pain or cholecystectomy for the high dose group. If we adopt the log-rank score for both components, we obtain $T_n = 1.9735$ and $\widehat{\rho}_{12} = 0.605$. The corresponding approximate p-value is 0.042. We note that our procedure compared to Wei and Lachin (1984)'s achieves much smaller p-values and that the test based on the Gehan score is more powerful than the one based on the log-rank score in this case. This result was also noted by Wei and Lachin (1984). By some suitable choice of scores, we may improve the power of test. In this example, we used the S-PLUS for the numerical results.

Next, we compare the performance of our test with that of Wei and Knuiman (1987)'s (U_n) in terms of empirical powers through simulations with S-PLUS. This is done for bivariate distributions. In the following tables, we present the empirical powers for various censoring distributions and a fixed the life time distribution. The results are based on 1,000 simulations with sample sizes $n_1 = 30$ and $n_2 = 40$. The simulations were carried out under the nominal significance levels(α) 0.01 and 0.05. In the tables, T_n(Gehan) and T_n(log-rank) denote the test statistics which use the Gehan and log-rank scores, respectively, for their components. For the life time distribution, we used the Marshall-Olkin type of exponential distribution, whose joint survival function is as follows: for each i, $i = 1, 2$,

$$\begin{aligned} S(x_{i1}, x_{i2}) &= P(X_{i1} > x_{i1}, X_{i2} > x_{i2}) \\ &= \exp\{-\lambda_{i1}x_{i1} - \lambda_{i2}x_{i2} - \lambda_{i3}\min(x_{i1}, x_{i2})\}. \end{aligned} \tag{3.1}$$

We generated the life time random vectors, (X_{i1}, X_{i2}), with $\lambda_{i1} = \lambda_{i2} = \lambda_{i3} = 1$ for each i and took $\theta_1 = 0.0, 0.1, \ldots, 0.7$ with θ_2 being fixed at zero. For the censoring distributions, we considered the bivariate exponential distributions whose joint survival functions are of the form,

$$C(u_{i1}, u_{i2}) = P(U_{i1} > u_{i1}, U_{i2} > u_{i2}) = \exp\ (-\lambda_{i1}u_{i1} - \lambda_{i2}u_{i2}). \tag{3.2}$$

We considered four different cases: $\lambda_{11} = \lambda_{12} = \lambda_{21} = \lambda_{22} = 1$; $\lambda_{11} = \lambda_{21} = 1$ and $\lambda_{12} = \lambda_{22} = 2$; $\lambda_{11} = \lambda_{12} = 1$ and $\lambda_{21} = \lambda_{22} = 2$; $\lambda_{11} = \lambda_{22} = 1$ and $\lambda_{12} = \lambda_{21} = 2$. The results are given in Tables 1 through 4. The tables show the same trends regardless of the nominal significance level. In all tables, we note that our procedure achieves better empirical powers than U_n regardless of the censoring patterns for both scores. This suggests that our procedure with a suitable score may be a reasonable alternative for the one-sided test of the multivariate and possibly censored data. Finally, we note that the T_n(Gehan) test gives better empirical powers than T_n(log-rank) and achieves well the nominal significance level.

TABLE 1 *Comparison of empirical powers when* $\lambda_{11} = \lambda_{12} = \lambda_{21} = \lambda_{22} = 1$

Test Statistic	α	(θ_1,θ_2) (0,0)	(0.1,0)	(0.2,0)	(0.3,0)	(0.4,0)	(0.5,0)	(0.6,0)	(0.7,0)
U_n	0.01	0.001	0.020	0.114	0.271	0.424	0.589	0.752	0.797
	0.05	0.046	0.209	0.464	0.694	0.848	0.936	0.972	0.982
T_n(Gehan)	0.01	0.007	0.107	0.470	0.827	0.956	0.992	0.997	1.000
	0.05	0.046	0.362	0.780	0.958	0.993	1.000	1.000	1.000
T_n(log − rank)	0.01	0.008	0.069	0.241	0.477	0.718	0.836	0.942	0.971
	0.05	0.059	0.203	0.490	0.748	0.905	0.949	0.985	0.996

TABLE 2 *Comparison of empirical powers when* $\lambda_{11} = \lambda_{21} = 1$ *and* $\lambda_{12} = \lambda_{22} = 2$

Test Statistic	α	(θ_1,θ_2) (0,0)	(0.1,0)	(0.2,0)	(0.3,0)	(0.4,0)	(0.5,0)	(0.6,0)	(0.7,0)
U_n	0.01	0.004	0.016	0.064	0.156	0.253	0.364	0.454	0.536
	0.05	0.029	0.166	0.360	0.554	0.768	0.849	0.917	0.945
T_n(Gehan)	0.01	0.006	0.109	0.455	0.806	0.958	0.993	0.998	1.000
	0.05	0.049	0.331	0.777	0.956	0.996	1.000	1.000	1.000
T_n(log − rank)	0.01	0.010	0.061	0.246	0.488	0.715	0.867	0.946	0.969
	0.05	0.061	0.201	0.407	0.736	0.902	0.969	0.985	0.994

TABLE 3 *Comparison of empirical powers when* $\lambda_{11} = \lambda_{12} = 1$ *and* $\lambda_{21} = \lambda_{22} = 2$

Test Statistic	α	(θ_1,θ_2) (0,0)	(0.1,0)	(0.2,0)	(0.3,0)	(0.4,0)	(0.5,0)	(0.6,0)	(0.7,0)
U_n	0.01	0.003	0.029	0.116	0.232	0.400	0.537	0.627	0.681
	0.05	0.035	0.203	0.461	0.703	0.874	0.939	0.971	0.984
T_n(Gehan)	0.01	0.006	0.113	0.515	0.855	0.964	0.966	1.000	1.000
	0.05	0.049	0.396	0.823	0.968	0.999	1.000	1.000	1.000
T_n(log − rank)	0.01	0.010	0.059	0.259	0.520	0.736	0.896	0.961	0.983
	0.05	0.055	0.226	0.531	0.775	0.921	0.976	0.993	0.999

TABLE 4 *Comparison of empirical powers when* $\lambda_{11} = \lambda_{22} = 1$ *and* $\lambda_{12} = \lambda_{21} = 2$

Test Statistic	α	(θ_1,θ_2) (0,0)	(0.1,0)	(0.2,0)	(0.3,0)	(0.4,0)	(0.5,0)	(0.6,0)	(0.7,0)
U_n	0.01	0.005	0.025	0.101	0.227	0.341	0.444	0.534	0.601
	0.05	0.032	0.201	0.449	0.688	0.855	0.938	0.962	0.981
T_n(Gehan)	0.01	0.008	0.118	0.518	0.846	0.973	0.996	0.999	1.000
	0.05	0.044	0.367	0.815	0.977	0.999	1.000	1.000	1.000
T_n(log − rank)	0.01	0.008	0.060	0.252	0.538	0.763	0.886	0.953	0.989
	0.05	0.058	0.223	0.517	0.811	0.920	0.966	0.991	0.997

4. CONCLUDING REMARKS

As noted before T_{kn} can produce a class of linear rank statistics by varying the weight function Q_k for the univariate case. Prentice (1978) derived another type of linear rank statistics for right censored data from the marginal likelihood, which can produce locally most powerful tests. Mehrotra *et al.* (1982) and Park (2000) showed that those two types of linear rank statistics are equivalent. One may improve the power by choosing suitable Q_k for each component. This is the reason why we allow Q_k to vary with component. Also, we note that the covariance matrices are not required to be nonsingular since we do not consider the quadratic form of test statistics which requires the inverse matrix of the covariance matrix. This may be an advantage of our approach. We note that for each k, the sequence (T_{kn}) is consistent for testing $H_0^k : \theta_k \leq 0$ against the one-sided alternatives $H_1^k : \theta_k > 0$. This implies that the sequence (T_n) is consistent for the one-sided test of the multivariate data.

For the limiting power of our test, we consider the following Pitman translation alternatives: For each n and for each k, $k = 1, \ldots, d$, let

$$H_{1n} : \theta_{kn} = \frac{c_k}{\sqrt{n}}, \tag{4.1}$$

where c_k is a fixed positive real number. Also let $\boldsymbol{\theta}_n = (\theta_{1n}, \ldots, \theta_{dn})'$. Then we may obtain the limiting power of the test as follows:

$$\lim_{n\to\infty} P_{\theta_n}\{T_n \geq C_n(\alpha)\} = 1 - \Phi_{\mathbf{R}}\big(C(\alpha) - c_1 m_1, \ldots, C(\alpha) - c_d m_d\big), \tag{4.2}$$

where $\Phi_{\mathbf{R}}$ is the d-variate normal cumulative distribution function with zero mean vector and covariance matrix $\mathbf{R}$ with unit value for the diagonal elements. Here, $C(\alpha)$ satisfies $\lim_{n\to\infty} P\{T_n \geq C(\alpha)\} = \alpha$ and $m_k = \lim_{n\to\infty} \mu'_{kn}(0)/\{\sqrt{n}\sigma_{kn}(0)\}$ for each k. The derivation of the limiting power appears in Appendix B.

Finally, we note that the limiting distribution of U_n is univariate normal. Therefore the efficacy for U_n would be the non-centrality parameter value of the chi-square distribution with one degree of freedom. However, as we already have seen, the limiting distribution of $(T_{1n}, \ldots T_{dn})$ is multivariate normal. Thus, the efficacy of the tests based on T_n would be the non-centrality parameter value of the chi-square distribution with two degrees of freedom in the case of bivariate data. The comparison of the performance between the two tests in terms of efficacy may not be easily done. This is the reason why we compare the performance by computer simulations only.

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APPENDIX

A. Proof of Theorem 1

In this appendix, we derive the limiting covariance σ_{kl} for $l \neq k$. First of all, we note that under H_0,

$$\begin{aligned} T_{kn} &= \sqrt{n}\int_0^\infty Q_k(t)\left\{\frac{dN_{1k}(t)}{Y_{1k}(t)} - \frac{dN_{2k}(t)}{Y_{2k}(t)}\right\} \\ &= \sqrt{nn_1}\int_0^\infty \frac{Q_k(t)}{Y_{1k}(t)}dM_{1k}(t) - \sqrt{nn_2}\int \frac{Q_k(t)}{Y_{2k}(t)}dM_{2k}(t), \end{aligned}$$

where $M_{ik}(t) = (1/\sqrt{n_i})\{N_{ik}(t) - \int_0^t Y_{ik}(u)d\Lambda_{ik}(u)\}$ is a martingale for each $i = 1, 2$. Thus, under H_0, we have

$$\mathrm{Cov}(T_{kn}, T_{ln}) = nn_1 E\left(\int \frac{Q_k}{Y_{1k}}dM_{1k}\int \frac{Q_l}{Y_{1l}}dM_{1l}\right) + nn_2 E\left(\int \frac{Q_k}{Y_{2k}}dM_{2k}\int \frac{Q_l}{Y_{2l}}dM_{2l}\right),$$

since M_{1k} and M_{2l} (also M_{1l} and M_{2k}) are perpendicular. One may obtain the following expression for σ_{kl} by applying the Fubini's theorem and Slutsky's theorem under (A1) and (A2):

$$\begin{aligned} \sigma_{kl} = \sum_{i=1}^{2}\rho_i\Bigg\{&\iint \frac{q_k(s)q_l(t)}{\pi_{ik}(s)\pi_{il}(t)}S_{ikl}^C(s,t)d^2F_{ikl}(s,t) \\ &- \iint \frac{q_k(s)q_l(t)}{\pi_{ik}(s)\pi_{il}(t)}S_{ikl}^C(s,t)S_{ilk}(t|s)dF_{ik}(s)d\Lambda_{il}(t) \\ &- \iint \frac{q_k(s)q_l(t)}{\pi_{ik}(s)\pi_{il}(t)}S_{ikl}^C(s,t)S_{ikl}(s|t)dF_{il}(t)d\Lambda_{ik}(s) \\ &+ \iint \frac{q_k(s)q_l(t)}{\pi_{ik}(s)\pi_{il}(t)}S_{ikl}^C(s,t)S_{ikl}(s,t)d\Lambda_{ik}(s)d\Lambda_{il}(t)\Bigg\}, \end{aligned}$$

where

$$\begin{aligned} S_{ikl}(s,t) &= P(X_{ik1} > s, X_{il1} > t), \qquad S_{ikl}^C(s,t) = P(U_{ik1} > s, U_{il1} > t), \\ S_{ilk}(t|s) &= P(X_{il1} > t | X_{ik1} = s), \qquad S_{ikl}(s|t) = P(X_{ik1} > s | X_{il1} = t), \\ \pi_{ik}(t) &= \{1 - F_{ik}(t)\}\{1 - G_{ik}(t)\}. \end{aligned}$$

B. Proof of (4.2)

In this appendix, we derive the limiting power of our proposed test under the Pitman translation alternatives. First of all, we note that

$$\begin{aligned}\mu_{kn}(H_{1n}) &= E(T_{kn}|H_{1n}) = \frac{1}{\sqrt{n}}E\left[\int_0^\infty Q_k(t)\left\{\frac{dN_{1k}(t)}{Y_{1k}(t)} - \frac{dN_{2k}(t)}{Y_{2k}(t)}\right\}\middle|H_{1n}\right] \\ &= \sqrt{n}\int_0^\infty Q_k(t)d\{\Lambda_{1k}(t) - \Lambda_{2k}(t)|H_{1n}\} \\ &= \sqrt{n}\int_0^\infty Q_k(t)\left\{\lambda_{1k}(t)dt - \lambda_{1k}\left(t - \frac{c_k}{\sqrt{n}}\right)dt\middle|H_{1n}\right\}.\end{aligned}$$

We see that

$$\left.\frac{d\mu_{kn}(H_{1n})}{dc_k}\right|_{c_k=0} = \int_0^\infty Q_k(t)\lambda'_{1k}(t)dt \neq 0,$$

where $\lambda'_{1k}$is the derivative of λ_{1k}. Also, we note that

$$\sigma_k^2(H_{1n}) \longrightarrow \sigma_k^2 \text{ and } \sigma_{kl}(H_{1n}) \longrightarrow \sigma_{kl} \text{ for } k \neq l.$$

Thus, we obtain the limiting power for (T_n) as follows from the fact that $\widehat{\sigma}_k$ and $\widehat{\sigma}_{kl}$ are the consistent estimates of σ_k and σ_{kl}, respectively. In the following, $\mu_{kn}(\theta_{kn})$, $\sigma_{kn}^2(\theta_{kn})$ and $\mathbf{R}(\boldsymbol{\theta}_n)$ denote the mean, variance and correlation coefficient matrix, respectively, under the Pitman translation alternatives.

$$\begin{aligned}&\lim_{n\to\infty} P_{\theta_n}\{T_n \geq C(\alpha)\} \\ &= 1 - \lim_{n\to\infty} P_{\theta_n}\left\{\frac{T_{1n}}{\widehat{\sigma}_{1n}} < C(\alpha), \dots, \frac{T_{dn}}{\widehat{\sigma}_{dn}} < C(\alpha)\right\} \\ &= 1 - \lim_{n\to\infty} P_{\theta_n}\left\{\frac{T_{1n} - \mu_{1n}(\theta_{1n})}{\sigma_1(\theta_{1n})} < C(\alpha)\frac{\widehat{\sigma}_{1n}}{\sigma_1(\theta_{1n})} - \frac{\mu_{1n}(\theta_{1n})}{\sigma_1(\theta_{1n})}, \dots, \right. \\ &\qquad\qquad \left. \frac{T_{dn} - \mu_{dn}(\theta_{dn})}{\sigma_d(\theta_{dn})} < C(\alpha)\frac{\widehat{\sigma}_{dn}}{\sigma_d(\theta_{dn})} - \frac{\mu_{dn}(\theta_{dn})}{\sigma_d(\theta_{dn})}\right\} \\ &= 1 - \Phi_{\mathbf{R}}(C(\alpha) - c_1 m_1, \dots, C(\alpha) - c_d m_d).\end{aligned}$$

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