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Nonlinear Phenomena in Resonant Excitation of Flexural-Gravity Waves

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Abstract

The influence of nonlinear phenomena on the behavior of stationary forced flexural-gravity waves on the surface of deep water is investigated, when the perturbation of external pressure moves with near-resonant velocity. It is shown that there are three branches of bounded stationary solutions turning into asymptotic solutions of the linear problem with zero initial conditions. For the first time ice sheet destruction by turbulent fluctuations of atmosphere pressure in ice adjacent layer in wind conditions is studied.

Keywords: nonlinear wave, resonance, asimptotic expansions, ice destruction, fracture toughness

1 Introduction

The impact of nonlinear effects is one of the physical factors restricting the vibration amplitude of particles of a continuum under an external driving force (load). In the linear approximation neglecting dissipation, the vibration amplitude is infinite in case resonant conditions are held. In continuum with wave propagation of perturbations, the resonance conditions at a load localized in a single spatial direction are reduced to fulfillment of the two relations: $V = \omega'(k)$, $\omega_f = \omega(k) - Vk$, where V is the load velocity, ω_f is the load vibration frequency, $\omega(k)$ and $\omega'(k)$ are the frequency and group velocity of a wave with the wave number k. In the linear approximation, the continuum properties are defined by the dispersion equation $\omega = \omega(k)$. The two resonance conditions represent an algebraic relation between V, k, and ω_f. As the conditions are held, the amplitude of a wave with the wave number k infinitely grows under a load. The resonance conditions mean that the load is to move with the group velocity of a wave, whose frequency coincides with the load vibration frequency in the accompanying frame of reference.

A typical example of a continuum, allowing fulfillment of the resonance conditions, is a layer of free-surface ideal fluid. The external load is pressure applied to the free surface. The resonance conditions can be held neglecting the surface phenomena only at $\omega_f \neq 0$ (Debnath and Rosenblat 1969). The resonance conditions can be held at $\omega_f = 0$ if capillary-gravitational forces are taken into account (Rayleigh 1883) or when an elastic plate floats on the fluid surface (Kheysin 1963). In 1883, Rayleigh studied stationary solutions to the problem of excitation of capillary–gravity waves by a pressure source localized near a horizontal straight line moving over the surface of infinitely deep fluid. In the linear approximation, he showed that the amplitude of the fluid surface perturbation infinitely grows as the load velocity tends to its critical value. The latter was found from the condition of coinciding wavenumbers of the waves carrying energy away from the pressure loading point to infinity. The resonance of the pressure loading with flexural-gravity (Kheysin 1963) or gravity (Debnath and Rosenblat 1969) waves is related to the same behavior of the waves carrying energy to infinity. Schulkes and Sneyd (1988) constructed time-dependent solution describing the response of floating elastic plate on the pressure loading. Bates and Shapiro (1981) and Hosking et al (1988) investigated the influence of viscous-elastic damping on the bounding of flexural-gravity wave amplitude in the vicinity of the resonance. The wave resistance in the Rayleigh problem was calculated by Richard and Raphael (1999) taking into account viscous properties of the fluid. It was shown to remain limited as the load velocity tends to the critical value. Wave resistance emerging due to emission of capillary-gravity waves in the problem considered by Rayleigh is zero if the load velocity is lower than the critical one and nonzero otherwise.

The problem of resonant excitation of surface gravity waves in the ideal fluid, its surface under a running oscillating load, was studied in (Akylas 1984), with a nonlinear Schrödinger equation (NSE) derived for the resonant wave envelope. The external load was simulated by the NSE force term proportional to the Dirac δ-function. A stationary solution to the problem was shown to be limited and have a few branches. The dynamics of capillary-gravity and flexural-gravity waves excited by an external load and moving at a near-resonance velocity was studied in (Marchenko and Dias 1999). Similar (Akylas 1984) the problem was formally reduced to the NSE with a force term proportional to the Dirac δ-function. The nonlinearity-accounting method proposed in (Akylas 1984) is based on asymptotic expansions presupposing slow changes of the wave and load amplitudes. A formal substitution of the δ-function into the right-hand side of the NSE does not meet this condition.

The studies of resonant excitation of flexural-gravity waves by moving vehicles should take into account the restriction of the vehicle sizes. The results of theoretical and experimental studies are contained in the definitive book (Squire et al 1996). The other field of the application of the problem is related to the study of wind influence on ice covered water layer. The interaction of the wind with ice ridges excites air vorticity and air pressure perturbations propagating with wind velocity. The extension of ice ridges in the same space direction causes air perturbations extended in the same direction. The simplest model of such air perturbation is based on the using of the Dirac δ-function for the force term in dynamic boundary condition. Present study is devoted to the studies of forced nonlinear flexural-gravity waves excited by a load modeled by the external pressure δ-function. We study forced waves of sufficiently small steepness, which don't cause ice sheet breakup. In the case of deep water the steepness parameter can be used for the construction of asymptotical series describing the solution of the problem (Fenton 1979). For the studies of flexural-gravity waves of large steepness more comprehensive technique should be used (Forbes 1986 1988).

2 Basic equations

We consider potential motions of a layer of infinitely deep fluid against the background of a flow with a constant velocity V (Figure 1a). The fluid is covered by a thin elastic plate. The motion

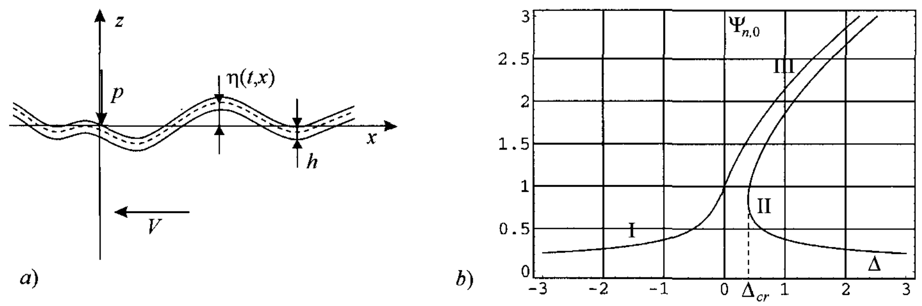


Figure 1: The flow of infinitely deep fluid beneath an elastic ice plate (a). The dependence of the amplitude of stationary forced wave in the loading point from the velocity of external pressure in the resonant vicinity (b)

equations in the dimensionless form are written as

$$\nabla^2\varphi = 0, \qquad z < \varepsilon\eta \tag{1}$$

$$\frac{D\varphi}{Dt} + \frac{\varepsilon}{2}(\nabla\varphi)^2 + \eta + \frac{\partial^4\eta}{\partial x^4} + p = 0, \qquad z = \varepsilon\eta \tag{2}$$

$$\frac{D\eta}{Dt} + \varepsilon\frac{\partial\eta}{\partial x}\frac{\partial\varphi}{\partial x} = \frac{\partial\varphi}{\partial z}, \qquad z = \varepsilon\eta \tag{3}$$

$$\varphi \to 0, \qquad z \to -\infty \tag{4}$$

where $\nabla = (\partial/\partial x, \partial/\partial z)$, $D/Dt = \partial/\partial t - V\partial/\partial x$, $\varphi(t,x,z)$ is the velocity potential, $z = \eta(t,x)$ is the fluid surface equation, $p = p(x)$ is the external pressure applied to the fluid surface, t is the time, x and z are the horizontal and vertical coordinates. In the frame reference moving with water particles with velocity V the problem under the consideration is equal to the problem about the influence of the moving pressure field on the rest fluid.

When writing the equations in the dimensionless form, the characteristic length scale l is put to $\left(Eh^3/(12\rho g(1-\nu^2))\right)^{1/4}$, where E and ν are Young's modulus and Poisson ratio of the elastic plate, ρ is the fluid density, and g is the gravity acceleration. The typical time scale is $(l/g)^{1/2}$. The dimensional velocity potential and flow velocity are $a\,(lg)^{1/2}\,\varphi$ and $V\,(lg)^{1/2}$, respectively, a is the typical value of the function $\eta(t,x)$. The dimensionless parameter is $\varepsilon = a/l$. Assuming $E = 3\cdot 10^9 Nm^{-2}$, $\nu = 0.34$, $\rho = 1020kgm^{-3}$ and $h = 1m$ one finds $l \approx 13m$, and typical time is about 1 sec. The estimation of the typical time shows that the influence of creep deformations can be disregarded, and the elasticity model for the ice is reasonable in the problem (Ashton 1986).

We suppose the external pressure in dimensionless variables is written as

$$p(x) = p_0\delta(x), \qquad \delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx}dk \tag{5}$$

Averaging over the interval $(-L_p/2, L_p/2)$, where L_p defines typical size of the perturbation of external pressure filed, dimensional pressure $\langle p\rangle$ is related to p_0 as

$$p_0 = \langle p\rangle\, L_p(\rho g a l)^{-1} \tag{6}$$

3 Solution to the linearized problem

The solution to the set of (1)–(4), describing excitation of flexural-gravity waves by pressure source (5) and meeting the initial conditions

$$\eta = 0, \qquad \partial\eta/\partial t = 0, \qquad t = 0 \tag{7}$$

is written in the linear approximation ($\varepsilon = 0$) as (Schulkes and Sneyd 1988)

$$\varphi = -\frac{ip_0\Psi_\varphi}{4\pi} + c.c., \qquad \eta = -\frac{p_0\Psi_\eta}{4\pi} + c.c.,$$
$$\Psi_\varphi = \int_{-\infty}^{\infty} \frac{-kV + \omega e^{-i(\omega-kV)t}}{\omega(\omega - kV)} e^{ikx+|k|z} dk, \qquad \Psi_\eta = \int_{-\infty}^{\infty} \frac{|k|(1 - e^{-i(\omega-kV)t})}{\omega(\omega - kV)} e^{ikx} dk \tag{8}$$

where the dispersion function $\omega = \omega(k)$ is given by

$$\omega = \sqrt{|k|(1 + k^4)} \tag{9}$$

We represent the functions Ψ_φ and Ψ_η as the sum of the stationary and nonstationary terms

$$\Psi_\varphi = -VI_\varphi + \Psi_\varphi^{ns}, \qquad \Psi_\eta = I_\eta + \Psi_\eta^{ns} \tag{10}$$

$$I_\varphi = \int_C \frac{ke^{ikx+|k|z}}{\omega(\omega - Vk)} dk, \qquad I_\eta = \int_C \frac{|k|e^{ikx}}{\omega(\omega - Vk)} dk$$
$$\Psi_\varphi^{ns} = \int_C \frac{e^{-i(\omega-Vk)t}}{\omega - Vk} e^{ikx+|k|z} dk, \qquad \Psi_\eta^{ns} = -\int_C \frac{|k|e^{-i(\omega-Vk)t}}{\omega(\omega - Vk)} e^{ikx} dk \tag{11}$$

The integrals I_φ, I_η, Ψ_φ^{ns} and Ψ_η^{ns} depend on the contour C position with respect to the roots of the equation

$$\omega - Vk = 0 \tag{12}$$

located at the complex plane k. Using (9), one finds that (12) has six roots. There are two real positive roots k_+ and k_-, when $V > V_r$, $V_r = (256/27)^{1/8} \approx 1.32$. At $V = V_r$, the roots merge at the point $k_+ = k_- = k_r$, where $k_r = 3^{-1/4} \approx 0.76$. At $V \in (0, V_r)$, the roots k_+ and k_- are became complex conjugate. The roots k_+ and k_- are supposed to lie in the upper and lower semiplanes of the complex variable k, respectively. We suppose that the contour C traces the roots $k = k_+$ and $k = k_-$ from below and above, respectively. At $V \in (0, V_r)$, the contour C coincides with axis $\mathrm{Im}k = 0$. Such a choice of the contour maintains fulfillment of the radiation conditions in the asymptotic stationary solution described by functions I_φ and I_η.

Using the stationary phase method, one can readily show that the functions Ψ_φ^{ns} and Ψ_η^{ns} are exponentially damped at $V \in (0, V_r)$ when $t \to \infty$; at $V > V_r$, the damping is proportional to $t^{-1/2}$ (Schulkes and Sneyd 1988). At $V = V_r$, solution (8) infinitely grows in time proportionally to $t^{1/2}$. Therefore, the load velocity V_r is referred to as resonant. The functions I_φ and I_η are continuous at any x, since the integrands in (8) are damped proportionally to k^{-4} at $|k| \to \infty$. Using the Jordan lemma, we arrive at

$$I_\eta = A_+e^{ik_+x} + I_\eta^+, \quad x > 0; \qquad I_\eta = A_-e^{ik_-x} + I_\eta^-, \quad x < 0 \tag{13}$$

Functions $I_\eta^+(x)$ and $I_\eta^-(x)$ are reduced to the integrals along the cuts located in the upper and lower half-planes of the Riemann surface. The amplitudes $A_\pm$ are given by $A_\pm = \pm 2\pi i k_\pm(\omega_\pm(\omega'_\pm - V))^{-1}$. It follows from (9) that

$$\frac{\partial^3 I_\eta}{\partial x^3} = -i \int\limits_C \frac{\omega V k - |k|}{\omega k(\omega - Vk)} e^{ikx} dk + 2\pi\theta(x), \qquad \theta(x) = -\frac{i}{2\pi} \int\limits_{-\infty}^{\infty} \frac{e^{ikx}}{k} dk \tag{14}$$

The integral in formula (14) is interpreted in the sense of a principal value in the vicinity of points $k = k_\pm$. The integrand is damped proportionally to $|k|^{-5/2}$ at $|k| \to \infty$. Therefore, the integral converges and is continuous at any x. The Heaviside function $\theta(x)$ is leapwise changed from zero to unity at $x = 0$. Hence, $[\partial^3 I_\eta/\partial x^3] = 2\pi$, where $[f]$ means the difference of values of an arbitrary function $f(x)$ at $x \to \pm 0$.

Thus, using (8) and (10), one finds that $[\partial^3\eta/\partial x^3] = p_0$ in the linearized problem, and derivations of more lower orders are continuous in the point of the loading. We assume that this property remains in the general formulation of the problem as well, taking into account the nonlinear effects. Let us show that the boundary condition (2) including the external pressure defined by (5) is equivalent to the boundary condition (2), in which the external pressure $p = 0$, and to a certain contact-boundary condition at $x = 0$. Let us integrate (2) over coordinate x from $-A$ to A and then tend A to zero. The velocity potential at the fluid free surface and the fluid free surface itself are continuous together with their first derivations with respect to t, x and z. Therefore, the integrals of $\partial\varphi/\partial t$, η and $(\nabla\varphi)^2$ tend to zero at $A \to 0$. Finally, we arrive at the contact-boundary condition

$$[\partial^3\eta/\partial x^3] = -p_0 \tag{15}$$

Further we suppose that the load velocity is little different from resonant V_r,

$$V = V_r + \varepsilon^2\Delta \tag{16}$$

where $\varepsilon << 1$ and $|\Delta| \le O(1)$. The parameter Δ is real and can have any sign. It follows from (12) and (16) that $k_\pm = k_r \pm \varepsilon\lambda\sqrt{\Delta} + O(\varepsilon^2\Delta)$, $\lambda = \sqrt{2k_r/\omega_r''}$. The amplitudes $A_\pm$ near the resonance approximately are equal to each other $A_\pm = A_r(\varepsilon\sqrt{\Delta})^{-1} + O(\varepsilon\sqrt{\Delta})$, where $A_r = 2\pi i/(V_r\omega_r''\lambda)$ and $\omega_r'' = \partial^2\omega/\partial k^2|_{k=k_r}$. Hence, near the resonance we have

$$I_\varphi = \frac{A_r}{\varepsilon\sqrt{\Delta}} e^{(k_r \pm \varepsilon\lambda\sqrt{\Delta})(ix+z)} + O(1), \qquad I_\eta = \frac{A_r}{\varepsilon\sqrt{\Delta}} e^{i(k_r \pm \varepsilon\lambda\sqrt{\Delta})x} + O(1) \tag{17}$$

where signs "+" and "–" correspond to regions $x > 0$ and $x < 0$, respectively. One can see from (10) and (17) that the solution in the resonance vicinity is close to the plane wave with the wavenumber $k = k_r$, propagating at the load velocity. The wave amplitude $p_0 A_r/(2\pi\varepsilon|\Delta|^{1/2})$ tends to infinity when $\Delta \to 0$. However, it follows from (15) that it is important to take into account the derivatives of perturbation (described by functions $I_\eta^\pm$) exponentially weakening as receding from its coordinate origin in order to meet this contact-boundary condition.

4 Nonlinear theory of a forced perturbation

From (17) it follows that the nonlinear terms in (2) and (3) are the same order as the linear terms near the resonance. Therefore, the nonlinearity should necessarily be taken into account. A prime objective of this section is to derive an asymptotic nonlinear equation describing the stationary shape of the fluid surface in the resonance vicinity. As the initial equations, we use set (1)–(4) at $p = 0$ and contact-boundary condition (15).

Assuming the velocity potential and fluid surface amplitudes to be bounded, we search for the solution to the considered set in the form of series with respect to the powers of the series expansion parameter ε,

$$\varphi = \varphi_0 + \varepsilon\varphi_1 + \varepsilon^2\varphi_2 + \ldots, \qquad \eta = \eta_0 + \varepsilon\eta_1 + \varepsilon^2\eta_2 + \ldots \tag{18}$$

with the first expansion terms are given by

$$\begin{aligned} &\varphi_0(x,z,X) = -i\beta V\Phi(X)I_\varphi(x,z) + \text{c.c.}, \\ &\eta_0(x,X) = \beta\Phi(X)I_\eta(x) + \text{c.c.}, \qquad \beta = i\varepsilon\sqrt{\Delta}(A_rV_r)^{-1} \end{aligned} \tag{19}$$

where $X = \varepsilon x$ is the slow spatial coordinate along the motion. It follows from (17) that the function $\Phi(X)$ describes slow amplitude variations of the fluid velocity potential, caused by nonlinear and dispersion effects. Substituting expansions (18) and (19) and using Fourier transform with respect to the x-coordinate one find "short" asymptotic equation as (Marchenko 2001)

$$\frac{\omega_r''}{2}\frac{\partial^2\Psi}{\partial X^2} + \Delta k_r\Psi + \kappa|\Psi|^2\Psi = 0, \qquad \kappa = -\frac{7\cdot 3^{5/8}}{66} \tag{20}$$

where $\Psi = \Phi\exp(i\lambda\sqrt{\Delta}X)$ when $X > 0$, and $\Psi = \Phi\exp(-i\lambda\sqrt{\Delta}X)$ when $X < 0$. From (18)- (20) it follows that contact-boundary condition (15) is written as

$$i\,[\partial\Psi/\partial X] + \text{c.c.} = p_0V_r/(3\varepsilon k_r^2) \tag{21}$$

5 Nonlinear stationary driven waves

It follows from (10), (19) and (21) that the function Ψ corresponding to a solution to the linearized problem is written as

$$\Psi_l = \Psi_{l,0}\begin{cases} e^{i\lambda\sqrt{\Delta}X}, & X > 0 \\ e^{-i\lambda\sqrt{\Delta}X}, & X < 0 \end{cases}, \qquad \Psi_{l,0} = -\frac{p_0}{2\varepsilon\omega_r''\lambda\sqrt{\Delta}} \tag{22}$$

We study the properties of solutions to nonlinear problem (20), (21), transforming into solutions to linear problem (22), when the parameter κ formally tending to zero.
Three branches of the solution to (20) is given by

$$I: \Psi_n = i\tau\lambda\sqrt{\frac{|\Delta|\omega_r''}{\kappa}}\begin{cases} \sinh^{-1}\left(\lambda\sqrt{|\Delta|}(X+X_0)\right), & X > 0 \\ -\sinh^{-1}\left(\lambda\sqrt{|\Delta|}(X-X_0)\right), & X < 0 \end{cases}, \qquad \Delta < 0 \tag{23}$$

$$II: \Psi_n = -\tau\Psi_{n,0}\begin{cases} e^{iRX}, & X > 0 \\ e^{-iRX}, & X < 0 \end{cases}, \qquad R = \sqrt{\lambda^2\Delta + \frac{2\kappa}{\omega_r''}\Psi_{n,0}^2}, \qquad \Delta > 0 \tag{24}$$

$$III: \Psi_n = i\tau\lambda\sqrt{-\frac{\Delta\omega_r''}{2\kappa}}\begin{cases}\tanh\left(\lambda\sqrt{\frac{\Delta}{2}}(X+X_0)\right), & X>0\\ -\tanh\left(\lambda\sqrt{\frac{\Delta}{2}}(X-X_0)\right), & X<0\end{cases}, \quad \Delta>0 \tag{25}$$

where $\tau = |p_0|/p_0$. It follows from condition (21) that $\Psi_{n,0}$ ($\Psi_{n,0} = |\Psi_n|$ at $X=0$) relating to the nonlinear solutions (23)-(25) respectively satisfy to the equations

$$I: 4\Psi_{n,0}\sqrt{\lambda^2|\Delta| - \frac{\kappa}{\omega_r''}\Psi_{n,0}^2} = \frac{V_r|p_0|}{3\varepsilon k_r^2}, \quad \Delta<0 \tag{26}$$

$$II: 4\Psi_{n,0}\sqrt{\lambda^2\Delta + \frac{2\kappa}{\omega_r''}\Psi_{n,0}^2} = \frac{V_r|p_0|}{3\varepsilon k_r^2}, \quad \Delta>0 \tag{27}$$

$$III: 4(\frac{\lambda^2\Delta\omega_r''}{2\kappa} + \Psi_{n,0}^2) = \frac{V_r|p_0|}{3\varepsilon k_r^2}\sqrt{-\frac{\omega_r''}{\kappa}}, \quad \Delta>0 \tag{28}$$

It follows from (26) and (27), that $i\Psi_{n,0} \to \Psi_{l,0}$, and branch III turns into infinity at $\kappa \to 0$. From (26) and (28) one finds that

$$\Psi_{n,0} \to \Psi_0, \quad \Delta \to 0; \qquad \Psi_0 = |p_0|V_r\sqrt{-\omega_r''/\kappa}(12\varepsilon k_r^2)^{-1} \tag{29}$$

Equation (27) has two real roots in the region $\Delta > \Delta_{cr}$, where $\Delta_{cr} \approx 0.4$. Let the typical value of the dimensional amplitude a of a wave be defined by equality $\Psi_0 = 1$. The dependences $\Psi_{n,0}(\Delta)$ defined by formulas (26)-(28) are shown in Figure 1b (curves I, II, III). Curve IV is defined by equations $\Psi_{nl} = \lambda\sqrt{-\Delta\omega_r''/(2k)}$ and describes the dependence of the amplitude of free periodic wave on the parameter Δ. The work $dA = -\int_{-\infty}^{\infty} p(x)D\eta/Dtdxdt$ is equal to the work of the wave resistance force F_R with an opposite sign, $dA = -F_R V dt$ Simple calculations based on the law of energy conservation for (1)-(4) show that averaged wave resistance $\langle F_R\rangle = k_r(2\pi)^{-1}\int_0^{2\pi/k_r} F_R dx$ is equal to zero at the branches I and III of the solution. At the branch II one finds, to the accuracy of O(ε), that wave resistance is bounded

$$\langle F_R\rangle = \varepsilon\left(\frac{5k_r^3}{V_r^2} - \frac{1}{2}\right)\Psi_{n,0} \approx 1.2\varepsilon\Psi_{n,0}, \qquad \Delta \geq \Delta_{cr} \tag{30}$$

Field experiments show that peak deflections of the ice sheet in the vicinity of moving vehicle are observed when its speed is closed to resonant velocity $V_r\sqrt{lg}$ (Squire et al 1996). Therefore we assume that there are physical reasons due to which branches II and III of constructed solution become unstable when $\Psi_{n,0} > 1$ and $\Delta \in (0, \Delta_{cr})$. Therefore further we analyze stresses in forced wave in explicit resonance $\Delta = 0$.

6 Estimations of ice destruction modes

Peak tensile stress σ_{max}, forming on the wave crests, is estimated using formulas

$$\sigma_{\max} = 6h^{-2}\max_x M, \quad M = \frac{Eh^3}{12(1-\nu^2)}\frac{\partial^2\eta}{\partial x^2} \tag{31}$$

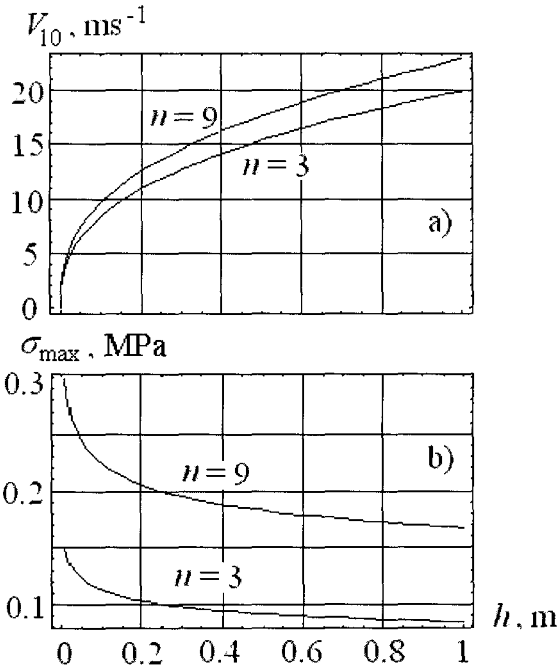


Figure 2: Resonant wind velocity V_{10}(a) and peak stress σ_{max} (b) on the wave crests versus ice thickness h. Young modulus is equal to $E = n \cdot GPa$; values of coefficient n are shown on the figures

It is assumed that typical value of the fluctuations of the atmosphere pressure in ice adjacent atmosphere layer has the order of the drag force (Landau and Lifshitz 1988). The drag force is estimated over the value of wind velocity on $10m$ distance from the ice surface as $\sigma = \rho_a C_a V_{10}^2$, where $C_a = 2 \cdot 10^{-3}$ is drag coefficient and ρ_a is air density (Andreas 1998). Dimensionless amplitude p_0 is assumed to be equal to

$$p_0 = \frac{\sigma L_s}{\rho g a l} \tag{32}$$

where L_s is the horizontal size of the roughness of ice cover surface. Further we assume for the estimations that $L_s = 1m$.

The velocity profile in the adjacent layer is defined as $V_a = u_*/k \ln(z/z_0)$, where $z_0 = 10^{-3}m$ (Andreas 1998). It is assumed that the velocity of the motion of the pressure fluctuation in the adjacent layer is closed to wind velocity V_1 on the distance $z \approx 1m$ from the surface of the ice cover. Since $V_1 \approx 0.75 V_{10}$, then for the resonance between the pressure fluctuation and flexural-gravity waves we have to set

$$V_r \sqrt{lg} = 0.75\, V_{10} \tag{33}$$

where $V_r\sqrt{lg}$ is dimensional value of resonant velocity V_r. The dependencies of wind velocity $V_{10} \approx 1.33\sqrt{lg}V_r$ from ice thickness h following from (33) are shown on Figure 2a for two values of Young modulus $E = 3GPa$ and $9GPa$.

Peak value of the dimensional quantity $\partial^2\eta/\partial x^2$ around the loading point in the case of explicit resonance $\Delta = 0$ is estimated using (26) and (32) as

$$\max_x \frac{\partial^2\eta}{\partial x^2} \approx \frac{2k_r^2}{l^2}A \tag{34}$$

where

$$A^2 = \frac{\alpha\sigma L_s}{\rho g}, \quad \alpha = \frac{V_r}{12k_r^2}\sqrt{-\frac{\omega_r''}{\kappa}} \approx 0.67$$

Substituting (34) into (31) one finds the peak tensile stress as

$$\sigma_{\max} = \frac{Eh}{\sqrt{3}\,l^2(1-\nu^2)}\sqrt{\frac{\alpha\sigma L_s}{\rho g}} \tag{35}$$

The dependencies of $\sigma_{\max}$ from ice thickness h are shown on Figure 2b for the same values of Young modulus as on Figure 2a.

The destruction of the ice sheet on wave crest is occurred when

$$\sigma_{\max} = \sigma_t \tag{36}$$

where σ_t is ice strength under the extension. Numerous field and laboratory experiments set strong dependence of σ_t from brine concentration ν_b inside the ice. When tensile stresses applied in normal direction to optical axis (c-axis) the following formula is used.

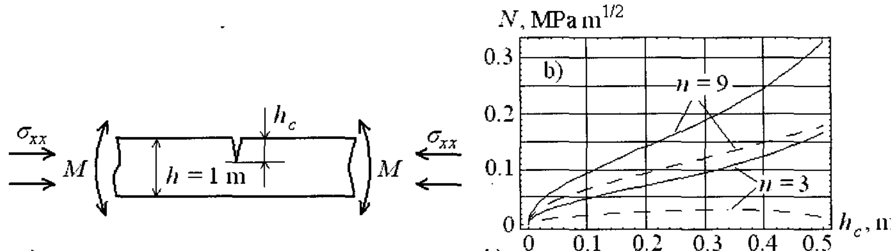
$$\sigma_t = 6.86\cdot 10^5\left(1-\sqrt{\nu_b/0.275}\right)(Nm^{-2}) \tag{37}$$

Typical value of brine concentration ν_b in sea ice is smaller 0.05 (Squire et al 1996). In this case from (37) it follows $\sigma_t \approx 0.39MPa$. From Figure 2b it follows that ice destruction is impossible in this case. On the other hand in summer period brine concentration can reach 0.2. Assuming $\nu_b = 0.1$ and 0.2 one finds $\sigma_t \approx 0.27MPa$ and $\approx 0.1MPa$ respectively. Therefore from Figure 2b it follows that ice destruction by turbulent fluctuations of the atmosphere pressure in wind conditions is possible in the warm time of a year.

Criterion (37) is formulated for continuous ice without defects. In natural conditions ice cover has many cracks (Ashton 1986), which can evaluate under the influence of wave induced bending deformations. For the analysis of possible modes of ice destruction let us consider the following problem. It is assumed that there is initial crack of depth h_c in the ice sheet of thickness $h = 1m$ (Figure 3a), and the direction of crack extension is perpendicular to the direction of wave propagation. A single crack has small influence on wave propagation, but the influence of the wave on the crack can be significant, since wave induced bending stresses can be enough for the starting of crack growth. Taking into account that crack length is much larger $1m$ we consider two-dimensional problem about the evaluation of rectilinear crack in elastic band performing the cross-section of the ice sheet by vertical plane. It is assumed that the stresses applied to the band are reduced to the sum of bending moment M (due to wave influence) and given external compression σ_{xx}.

The condition of crack equilibrium is formulated as

$$N \equiv N_I(M) + N_I(\sigma_{xx}) = K_{IC} \tag{38}$$



a) b)

Figure 3: Initial crack of depth h_c in the ice sheet of thickness $h = 1m$(a). Fracture toughness versus crack depth (b). Continuous lines correspond to uncompressed ice sheet ($\sigma_{xx} = 0$) and dashed lines correspond to compressed ice ($\sigma_{xx} = -30kPa$). Young modulus is equal to $E = n \cdot GPa$; values of coefficient n are shown at Figure 3b

where $K_{IC} = 0.1 \div 0.5MPa \cdot m^{1/2}$ is sea ice fracture toughness and $N_I(M)$ and $N_I(\sigma_{xx})$ are defined by formulas (Goldstein and Marchenko 1989)

$$\begin{aligned} N_I(M) &\approx 4.2Mh^{-3/2}\left((1-h_c/h)^{-3}-(1-h_c/h)^3\right)^{1/2},\ h_c/h > 0.005 \\ N_I(\sigma_{xx}) &\approx \sigma_{xx}\sqrt{2\pi h_c}\frac{1.11+5(h_c/h)^4}{1-h_c/h} \end{aligned} \qquad (39)$$

Values $K_{IC} \approx 0.1 \div 0.2MPa \cdot m^{1/2}$ are found for in-situ experiments with first-year sea ice, while values $K_{IC} \approx 0.3 \div 0.5MPa \cdot m^{1/2}$ are found in laboratory conditions for the fresh ice (Dempsey et al 1999). Most typical value of fracture toughness for first-year sea ice $K_{IC} \approx 0.22MPa \cdot m^{1/2}$ was defined at temperature about -14^{o}C.

The condition for crack growth is formulated as

$$N \geq K_{IC} \qquad (40)$$

The dependencies of N from crack depth h_c in the ice of thickness $h = 1m$ are shown on Figure 3b for uncompressed and compressed ice. One can see that condition (40) is valid when $K_{IC} \approx 0.2MPa \cdot m^{1/2}$ and $h_c > 0.3m$ for $E = 9GPa$. The destruction of compressed ice is not occurred when $\sigma_{xx} = -30kPa$.

7 Conclusions

The dynamic boundary condition at the fluid surface, taking into account an external δ-shaped pressure, has been shown to be equivalent to the boundary condition neglecting a load and an additional contact-boundary condition. Substitution of the asymptotic expansions into the contact-boundary condition leads to the condition of a leap of the first harmonic amplitude at the loading point. An analysis of the solutions to the NSE with the found contact-boundary condition shows that there are three branches of stationary solutions bounded in the resonance vicinity.

It is shown that pressure perturbations running with super-resonant velocity can force flexural-gravity waves with higher amplitudes than at explicit resonance $\Delta = 0$. It has contradiction with experimental data. Therefore we assume that solutions of branch III and highest part of branch

II are unstable. Including of dissipation in the model can cause the turning of the three branches into one branch of the solution, which maximum is displaced from the origin ($\Delta = 0$) to the right (Barnard et al 1977).

Constructed solutions were used to estimate the possibility of ice destruction by turbulent fluctuations of atmosphere pressure in ice adjacent layer in wind conditions. We selected only perturbations moving with resonant velocity assuming that they can cause most strong influence on the ice sheet, and set that their amplitude is about drag force at the ice surface. Estimations show that the destruction of ice sheet of thickness $1m$ is possible due to the growth of initial cracks which depth is larger $0.3m$. the destruction of continuous ice without cracks is possible only in the warm time of a year, when ice tensile strength is minimal due to high concentration (0.1-0.2) of the brine inside the ice.

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References

AKYLAS, T.R. 1984 On the excitation of nonlinear water waves by a moving pressure distribution oscillating at resonant frequency. Phys. Fluids 27(12), pp. 2803-2807

ANDREAS, E.L. 1998 The atmospheric boundary layer over polar marine surfaces. Physics of ice-covered seas, **2**, Lepparanta M. (ed), Helsinki University Printing House, Helsinki, pp. 715-773

ASHTON, G.D. 1986 River and lake ice engineering. Water Resourses Publications, USA

BARNARD, B.J.S., MAHONY, J.J. AND PRITCHARD, W.G. 1977 The excitation of surface waves near a cut-off frequency. Phil. Trans. R. Soc. London, A286, pp. 87-123

BATES, H.F. AND SHAPIRO, L.H. 1981 Plane waves in a viscoelastic floating ice sheet. J. Geoph. Res. **86(5)**, pp. 4269-4273

GOLDSTEIN, R.V. AND MARCHENKO, A.V. 1989 On the long waves in the fluid beneath compressed ice cover. In Electro-physical and Physical-mechanical Properties of Ice (V.V. Bogorodskii and V.P.Gavrilo, eds), pp. 188-205

DEBNATH, L. AND ROSENBLAT, S. 1969 Quar. J. of Mech. Appl. Math, **22**, 221

DEMPSEY, J.P., DEFRANCO, S.J., ADAMSON, R.M. AND MULMULE, S.V. 1999 Scale effects on the in-situ tensile strength and fracture of ice. Part II: First-yaer sea ice at Resolute, N.W.T. International. J. of Fracture, **95**, pp. 347-366

FENTON, J.D. 1979 A high-order cnoidal wave theory. J. of Fluid Mech, **94**, pp. 129-161

FORBES, L.K. 1986 Surface waves of large amplitude beneath an elastic sheet. Part 1. High -order series solution. J. of Fluid Mech, **169**, pp. 409-428

FORBES, L.K. 1988 Surface waves of large amplitude beneath an elastic sheet. Part 2. Galerkin solution. J. of Fluid Mech, **188**, pp. 491-508

FOX, C. 1992 Large Amplitude Sea/Ice Coupling. Advances in Ice Technology, Sackinger, W.M. and Wadhams, P. (ed), Cambridge, Comp. Mech. Pub., pp. 291-304

HOSKING, R.J., SNEYD, A.D. AND WAUGH, D.W. 1988 Viscoelastic response of a floating ice plate to a moving load. J. of Fluid Mech, **196**, pp. 409-430

KHEYSIN, D.YE. 1963 Moving load on an elastic plate which floats on the surface of an ideal fluid (in Russian). Izvestya Akademii Nauk SSSR, Otd. Tekh. Nauk, Mekhanika i Mashinostroenie **1**, pp. 178-180

LANDAU, L.D. AND LIFSHITZ, E.M. 1988 Fluid Mechanics. Nauka, Moscow, (in Russian), pp. 736

MARCHENKO, A. AND DIAS, F. 1999 The influence of nonlinearity on the restriction of wave amplitude by the resonance with external pressure. Wave dynamics at the fluid surface. Voliak, K. (ed.), Moscow: Nauka. Fizmatlit, pp. 141-152

MARCHENKO, A. 2001 Nonlinear effect at resonant excitation of capillary-gravity waves. Physics of vibrations **9(4)**, pp. 224-234

MELLOR, M. 1983 Mechanical behavior of sea ice. USA CRREL Monograph, 83-1, pp. 105

LORD RAYLEIGH. 1883 The form of standing waves on the surface of running water. Proc. London Math. Soc., **15**, pp. 69-78

RICHARD, D. AND RAPHAEL, E. 1999 Capillary-gravity waves: The effect of viscosity on the wave resistance. Europhys. Lett., **48(1)**, pp. 49-52

SCHULKES, R.M.S.M. AND SNEYD, A.D. 1988 Time-dependent response of a floating ice sheet to a steadily moving load. J. of Fluid Mech., **186**, pp. 25-46

SQUIRE, V.A., HOSKING, R.G., KERR, A.D. AND LANGHORNE, P.J. 1996 Moving load on ice plates. Kluwer Academic Publishers, Netherlands, pp. 230