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A PRODUCT FORMULA OF SEIBERG-WITTEN INVARIANTS

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ABSTRACT. Let X be a 4-manifold obtained by gluing two symplectic 4-manifolds X_i, $i = 1, 2$, along embedded surfaces. Using the gradient flow of a functional on 3-dimensional Seiberg-Witten theory along the cylindrical end, we study the Seiberg-Witten equations on X and have a product formula of Seiberg-Witten invariants on X from the ones on X_i, $i = 1, 2$.

Suppose that X_1 and X_2 are a closed, oriented 4-manifolds. Let Σ be a closed, oriented Riemann surface with genus $g(\Sigma) > 1$. And let $\Sigma \to X_i$ for $i = 1, 2$ be smooth embeddings representing homologies of infinite order. Suppose that the self-intersections $\Sigma \cdot \Sigma$ in X_i are zero. Then there is a regular neighborhood of Σ diffeomorphic to $D^2 \times \Sigma$. We have X_1^0 and X_2^0 by removing the interiors of these regular neighborhoods. Denote by Y the boundary $S^1 \times \Sigma$. There is an orientation reversing diffeomorphism $\partial X_1^0 \to \partial X_2^0$ which is identity on Σ and is complex conjugation on S^1 factor. We denote by $Z = X_1 \sharp_\Sigma X_2$ manifold obtained by gluing X_1^0 and X_2^0 along the boundary Y into Z with $Z - Y = X_1^0 \coprod X_1^0$. Fix metrics on X_1^0 and X_2^0 which have cylindrical ends with orientation preserving isometric to $[-1, \infty) \times Y$ and $[-1, \infty) \times \bar{Y}$ respectively. Let Z_s be the compact manifold obtained truncating the ends of X_1^0 and X_2^0 at $s \times Y$ and $s \times \bar{Y}$, respectively and then identifying two truncated manifolds along their common boundary Y. Thus the diffeomorphism type of Z is independent of s.

From now on we fix Spin^c structures $\tilde{P}_1$ and $\tilde{P}_2$ on X_1^0 and X_2^0 whose determinant line bundles restricted to Y are both isomorphic to the

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pullback from Σ of a line bundle of degree $2-2g(\Sigma_1)$ on Σ. We choose small perturbation of monopole equations for X_1^0 and X_2^0 so that the equations on X_1 are

$$\begin{cases} F_A^+ = q(\psi) + \sqrt{-1}\varphi_1(*n + dt \wedge n) + \sqrt{-1}\eta_1^+, \\ D_A\psi = 0, \end{cases}$$

where n is a harmonic 1-form on Σ and η_1^+ is a compactly supported self-dual 2-form and φ_1 is a smooth function which is 1 on $[0,\infty)\times Y$ and vanishes off of $[-1,\infty)\times Y$.

Let $\mathcal{M}_{d_1}(\tilde{P}_1, n, \eta_1^+)$ be the moduli space of finite energy solutions to the perturbed equations with dimension $2d_1$. In a similar way, we define the moduli spaces $\mathcal{M}_{d_2}(\tilde{P}_2, n, \eta_2^+)$ of finite energy solutions on X_2^0 with dimension $2d_2$. Let S_d be the set of isomorphism classes of Spin^c structures $\tilde{P}$ on Z with the property that $\bar{P}\mid_{X_1^0} = \bar{P}_1$, $\bar{P}\mid_{X_2^0} = \bar{P}_2$ and $\frac{1}{4}(c_1(det\bar{P})^2[Z] - (2\chi + 3\sigma)(Z)) = 2d$. If $\bar{P} \in S_d$, for each $s \geq 0$ we have the corresponding Spin^c structure $\bar{P}_s$ over Z_s. For sufficiently large s, let $\eta^+ = \eta_1^+ + \eta_2^+$ and define the moduli space $\mathcal{M}_d(\bar{P}_s, n, \eta^+)$ of solutions to the perturbed Seiberg-Witten equations

$$\begin{cases} F_A^+ = q(\psi) + \sqrt{-1}\varphi_s(*n + dt \wedge n) + \sqrt{-1}\eta_1^+, \\ D_A\psi = 0, \end{cases}$$

where $\varphi_s : Z_s \to [0,1]$ is the function which is φ_1 on $X_1^0(s)$ and φ_2 on $X_2^0(s)$.

For simplicity, we shall write $\mathcal{M}_{d_1}(\bar{P}_1)$ and $\mathcal{M}_{d_2}(\bar{P}_2)$ for $\mathcal{M}_{d_1}(\bar{P}_1, n, \eta_1^+)$ and $\mathcal{M}_{d_2}(\bar{P}_2, n, \eta_2^+)$, respectively.

Now we will define the moduli space of solutions to the monopole equations on non-compact 4-manifolds with ends isometric to $[-1,\infty)\times Y$. We consider only solutions to the equations with finite energy on the cylinderical end. Let $Y = S^1 \times \Sigma$ and let X^0 be a Riemannian 4-manifold whose end is orientation-preserving isometric to $[-1,\infty)\times Y$. Let a Spin^c structure $\tilde{P}$ on X^0 be given and denote the restriction of $\tilde{P}$ to Y by $\tilde{P_Y}$. Then for any solution (A,ψ) to the Siberg-Witten equations with respect to this Spin^c structure, there is a temporal gauge for $\tilde{P}$ restricted to the cylindrical end so that the flow line γ satisfies the gradient flow equation. A finite energy solution is a solution for which an associated flow line $\gamma : [0,\infty) \to C^*(\tilde{P_Y})$ satisfies $\lim_{t\to\infty}(f(\gamma(t)) - f(\gamma(0))) < \infty$. Here $C^*(\tilde{P_Y})$ is the space of pairs of a connection on the determinant line bundle of $\tilde{P_Y}$ and a section of spinor bundle on Y. And f is the function defined by

$$f(A,\psi) = \int_Y (F_A \wedge A + \langle \psi, \partial_A \psi \rangle).$$

And the difference is

$$\begin{aligned}
& f(\gamma(t)) - f(\gamma(0)) \\
&= \int_0^t \frac{\partial (f \circ \gamma)}{dt} dt \\
&= \int_0^t \frac{\partial}{\partial t} \int_Y (F_A \wedge A + \langle \psi, \partial_A \psi \rangle) \\
&= \int_0^t \int_Y \left(\frac{\partial}{\partial t} \langle *F_A, A \rangle + \frac{\partial}{\partial t} \langle \psi, \partial_A \psi \rangle \right) \\
&= \int_0^t \int_Y \left(2\langle *F_A, \frac{\partial A}{\partial t} \rangle + 2\langle \frac{\partial \psi}{\partial t}, \partial \psi_A \psi \rangle - 2\langle \frac{\partial A}{\partial t}, q(\psi) \rangle \right) \\
&= \int_0^t \int_Y \left(2|\frac{\partial A}{\partial t}|^2 + 2|\frac{\partial \psi}{\partial t}|^2 \right) < \infty.
\end{aligned}$$

Intuitively, the finiteness of energy implies that (A, ψ) approaches to the static solution as $t \to \infty$. Taubes obtains the following :

THEOREM 1. [17]. *Let $X^0, Y, \tilde{P}$ be given as above. Let (A, ψ) be a finite energy solution to the Seiberg-Witten equations associated $\tilde{P}$. Then there is a C^∞-product structure for $\tilde{P}|_{[0,\infty)\times Y}$ such that in this product structure (A, ψ) converges exponentially fast to a static solution.*

Let $\mathcal{M}_d(\tilde{P})$ be the moduli space of all finite energy solutions to the Seiberg-Witten equations.

$$\begin{aligned}
\mathcal{M}_d(\tilde{P}) = \{(A,\psi) | & \text{(i) } (A,\psi) \text{ is in a temporal gauge on } [0,\infty) \times Y, \\
& \text{(ii) } \gamma(t) = (A(t), \psi(t)) \text{ satisfies the gradient flow equation}, \\
& \text{(iii) } \lim_{t\to\infty} f(\gamma(t)) - f(\gamma(0)) < \infty \}.
\end{aligned}$$

Then $\mathcal{M}_d(\tilde{P})$ is a smooth compact manifold of dimension 2d except singularity.

Theorem 1 shows the existence of limit of gauge equivalence classes

$$r(A,\psi) = \lim_{t\to\infty} (A_t, \psi_t).$$

The limit defines a continuous map

$$r : \mathcal{M}_d(\tilde{P}) \longrightarrow \mathcal{R}(Y)$$

to the moduli space of solutions of the Seiberg-Witten equations on Y.

Suppose that $(A_i, \psi_i) \in \mathcal{M}_{d_i}(\tilde{P}_i)$ and $r_1(A_1, \psi_1) = r_2(A_2, \psi_2) = \rho \in \mathcal{R}(Y)$. For sufficiently large s, we can make a small modification to (A_1, ψ_1) so that it is equivalent to $\rho \in \mathcal{R}(Y)$ on the end $[s, \infty) \times Y$ of X_1^0. Making similar modification to (A_2, ψ_2) allows us to join two solutions to form $(A_1, \psi_1)\#(A_2, \psi_2)$ on Z_s which may not satisfy the equations on the neck region. But this can be deformed to a solution on Z_s. In fact, we have the following result.

THEOREM 2. *Suppose that (A_1, ψ_1) and (A_2, ψ_2) are regular points of their moduli spaces and suppose that r_1, r_2 are transverse at $((A_1, \psi_1), (A_2, \psi_2))$. Then gluing and deforming determine a diffeomorphism*

$$\coprod_{d_1+d_2-g(\Sigma_2)=d} \mathcal{M}_{d_1}(\tilde{P}_1) \times_r \mathcal{M}_{d_2}(\tilde{P}_2) \to \coprod_{\tilde{P}_s \in S_d} \mathcal{M}_d(\tilde{P}_s).$$

Here d_1 or d_2 are greater than or equal to $g(\Sigma_2)$ and $\mathcal{M}_{d_1} \times_r \mathcal{M}_{d_2}$ is the fiber product

$$\mathcal{M}_{d_1} \times_r \mathcal{M}_{d_2} = \{((A_1, \psi_1), (A_2, \psi_2)) | r_1(A_1, \psi_1) = r_2(A_2, \psi_2)\}.$$

This theorem shows that the formal dimension of the moduli space is as follows:

$$\dim \mathcal{M}_d(\tilde{P}_s) = \dim \mathcal{M}_{d_1} + \dim \mathcal{M}_{d_2} - \dim \mathcal{R}(\Sigma).$$

Let $X = X^0 \cup (D^2 \times \Sigma) = X^0 \cup W^0$ be a compact symplectic 4-manifold. The condition on $\tilde{P}_{X^0} \to X^0$ implies that it has an extension to a Spin^c structure over X. The extended Spin^c structure differs by an even multiple of $PD[\Sigma]$, which is Poincare dual of $[\Sigma] \in H_2(X, Z)$.

Now we will consider only 0-dimensional moduli spaces with respect to an extended Spin^c structure $\tilde{P}$ over X. Applying the above Theorem 2 to $X = X^0 \cup W^0$ we obtain a diffeomorphism

$$\mathcal{M}_{d_1}(X^0) \times_r \mathcal{M}_{d_2}(W^0) \to \mathcal{M}_0(\tilde{P}).$$

Here (d_1, d_2) is one of $(0, g(\Sigma_2))$ or $(g(\Sigma_2), 0)$. It follows from the fact that d_1 or d_2 is greater than or equal to $g(\Sigma_2)$ and $d_1 + d_2 = g(\Sigma_2)$. Otherwise, the fiber product $\mathcal{M}_{d_1} \times_r \mathcal{M}_{d_2}$ is empty for a Spin^c structure $\tilde{P}$ with nonzero Seiberg-Witten invariant.

COROLLARY 3. *Let X_1 and X_2 be symplectic manifolds and $Z = X_1 \sharp_\Sigma X_2$. Let $\tilde{P}_{X_1^0}$ and $\tilde{P}_{X_2^0}$ be Spin^c structures on X_1^0 and X_2^0 such that $c_1(det\tilde{P}_{X_i^0})(\Sigma) = 2-2g(\Sigma_1)$ for $i=1,2$. Then there is a diffeomorphism*

$$\mathcal{M}_{d_1}(X_1^0, \tilde{P}_{X_1^0}) \times_r \mathcal{M}_{d_2}(X_2^0, \tilde{P}_{X_2^0}) \to \mathcal{M}_{d_1+d_2-g(\Sigma_2)}(Z, \tilde{P}),$$

where $(d_1, d_2) = (0, g(\Sigma_2)), (g(\Sigma_2), 0)$ or $(g(\Sigma_2), g(\Sigma_2))$.

THEOREM 4. *For d=0, there is a diffeomorphism*

$$\mathcal{M}_0(X_1^0, \tilde{P}_{X_1^0}) \times_r \mathcal{M}_{g(\Sigma_2)}(X_2^0, \tilde{P}_{X_2^0}) \to \mathcal{M}_0(Z, \tilde{P}),$$

and we have a relation

$$SW_Z(\tilde{P}) = SW_{X_1^0}(\tilde{P}_{X_2^0}) \cdot p,$$

where p is the degree of $r : \mathcal{M}_{g(\Sigma_2)}(X_2^0, \tilde{P}_{X_2^0}) \to \mathcal{R}(\Sigma)$.

Proof. Let $\{s_i\}$ be a sequence approaching to infinity and let (A_i, ψ_i) be a sequence in $\mathcal{M}_0(Z_{s_i}, \tilde{P})$ with respect to the metric g_{s_i}. After passing to a subsequence, (A_i, ψ_i) converges to $((A_0, \psi_0), (B_0, \phi_0))$ in compact topology. If $f(A_0, \psi_0) = f((A_i, \psi_i|_{X_1(s)})$, then (A_0, ψ_0) is a solution to the SW-equations and is in $\mathcal{M}_0(X_1^0, \tilde{P}_{X_1^0})$. But if $f(A_0, \psi_0) < f((A_i, \psi_i|_{X_1(s_i)})$, then there are a sequence of $t_i \to \infty$ and $\epsilon > 0$ such that

$$\left| \int_{t_i}^{\infty} F_{A_i} \wedge F_{A_i} \right| \geq \epsilon.$$

Since the solution $(A_i,\ \psi_i)$ decays exponentially to a static solution, we can assume that

$$\left| \int_{t_i}^{t_i+1} F_{A_i} \wedge F_{A_i} \right| \geq \epsilon.$$

Take a sequence $r_i < t_i$ with $\lim_{i\to\infty} r_i = \infty$ and an embedding $I_i : (-r_i, r_i) \times Y \to X_1^0$ by $(t, x) \to (t + t_i, x)$. After taking a subsequence $I_i^*(A_i, \psi_i)$ converges to $(\bar{A}_1, \bar{\psi}_1)$ on $\mathbb{R} \times Y$ in compact topology, which satisfies a gradient flow equation and

$$\left| \int_0^1 F_{\bar{A}_1} \wedge F_{\bar{A}_1} \right| \geq \epsilon.$$

Repeating the same process we obtain finite numbers $(\bar{A}_i, \bar{\psi}_i), i = 1, 2, \ldots, k$, such that $\lim_{t\to\infty}(A_0, \psi_0) = \lim_{t\to-\infty}(\bar{A}_1, \bar{\psi}_1)$ and $\lim_{t\to-\infty}$

$(\bar{A}_{i-1}, \bar{\psi}_{i-1}) = \lim_{t\to-\infty}(\bar{A}_i, \bar{\psi}_i)$ in $\mathcal{R}(Y)$ for all $i = 2, \dots, k$. Then $f(A_i, \psi_i)[X_1^0] = f(A_0, \psi_0) + \Sigma_{i=1}^k f(\bar{A}_i, \bar{\psi}_i) = (2\chi + 3\sigma)(X_1^0)$.

Since $(A_0, \psi_0) - (2\chi + 3\sigma)(X_1^0) \geq 0$ and $f(\bar{A}_i, \bar{\psi}_i) \geq 0$ for all i, we have k=0 and $f(A_0, \psi_0) = (2\chi + 3\sigma)(X_1^0)$.

Similarly, on X_2^0 we have a limit of subsequence (A_i, ψ_i) of solutions restricted to $X_2(s_i)$ with respect to the metric g_{s_i}, of the form $(B_0, \phi_0)\sharp(\bar{B}_1, \bar{\phi}_1)\sharp \cdots \sharp(\bar{B}_l, \bar{\phi}_l)$, where $\lim_{t\to\infty}(B_0, \phi_0)=\lim_{t\to-\infty}(\bar{B}_l, \bar{\phi}_l)$, $\lim_{t\to\infty}(\bar{B}_{i-1}, \bar{\phi}_{i-1}) = \lim_{t\to-\infty}(\bar{B}_i, \bar{\phi}_i)$ for all $i = 2, \dots, k$ and $\lim_{t\to\infty} (\bar{B}_l, \bar{\phi}_l) = \lim_{t\to-\infty}(A_0, \phi_0)$.

Therefore we have

$$\begin{aligned} f(B_0, \phi_0) + \Sigma_{i=1}^l f(\bar{B}_i, \bar{\phi}_i) &= f(A_i, \psi_i)[Z] - f(A_0, \psi_0)[X_1^0] \\ &= (2\chi + 3\sigma)(X_2^0), \end{aligned}$$

and so l= 0. Then up to sign $SW_Z(\tilde{P})$ equals to $p \cdot SW_{X_1^0}(\tilde{P}_{x_1^0})$. □

THEOREM 5. *For the canonical Spin^c structure $K_X^{\pm 1}$ on X, dim$\mathcal{M}_{X_0}$ $(K_X^{\pm 1}|_{X_0})$ is zero and the Seiberg-Witten invariant $SW_{X^0}(K_X^{\pm 1}|_{X_0})$ is nonzero.*

Proof. We show that the dimension of moduli space $\mathcal{M}_{W^0}(K_X^{-1}|_{W^0})$ is $2g(\Sigma_2)$. For a general Spin^c structure $L = K_{W^0}^{-1} \otimes F^2$, by the index theorem

$$\dim\mathcal{M}_{W_0}(L) = \frac{1}{4}(c_1(L)^2[W^0] - (2\chi + 3\sigma)(W^0)).$$

We assume that $c_1(L)(\Sigma) = 2 - 2g(\Sigma_1)$. A finite energy solution $(A, \psi) = (A, \alpha, 0) \in \mathcal{M}_{W^0}(L)$ exponentially decays to a static solution in an appropriate gauge. Since $F_B^* \wedge F_B = 0$ for a static solution B,

$$c_1(L)^2[W^0] = -\frac{1}{4\pi^2}\int_{W^0} F_A \wedge F_A$$

is finite. Let $\lim_{t\to\infty}(A_t, \psi_t) = (A_0, \alpha_0, 0) \in \mathcal{R}(Y)$. Then the limit (A_0, α_0) is pull back of a solution $(A_\Sigma, \alpha_\Sigma)$ which is represented by $\alpha_0^{-1}(0) = \cup_{i=1}^{g(\Sigma_2)}\{x_i\} \times S^1$. Let $N(x_i)$ be a neighborhood of x_i in Σ.

We can assume that

$$\int_{N(x_i)} \frac{\sqrt{-1}}{2\pi} F_A = 1,$$

by a choice of metric in Σ. The holomorphic section α converges exponentially fast to a section which is defined by zero at end of $\{x_i\} \times D^2$ and nonzero at end of $\{y\} \times D^2$ for $y \notin \{x_1, \cdots, x_g\}$ and so α can be extended to a holomorphic section over $\Sigma \times S^2$. Also $A_0|_{D^2}$ can be extended to a holomorphic connection on a line bundle over S^2. Then we have

$$\int_{y \times D^2} \frac{\sqrt{-1}}{2\pi} F_{A_0} = \text{the number of zeros of } \alpha \text{ over } D^2 (:= n),$$

and

$$\int_{x_i \times D^2} \frac{\sqrt{-1}}{2\pi} F_{A_0} = n + 1.$$

So we have

$$\begin{aligned}
& c_1(L)^2[W^0] \\
&= 2 \int_\Sigma \frac{\sqrt{-1}}{2\pi} F_A \int_{D^2} \frac{\sqrt{-1}}{2\pi} F_A \\
&= 2 \left(\int_{\Sigma - \cup_{i=1}^{g(\Sigma_2)} N(x_i)} \frac{\sqrt{-1}}{2\pi} F_A + \sum_{i=1}^{g(\Sigma_2)} \int_{N(x_i)} \frac{\sqrt{-1}}{2\pi} F_A \right) \\
&\quad \times \left(1 + 2 \cdot \frac{\sqrt{-1}}{2\pi} \int_{D_2} F_{A_0} \right) \\
&= 2(2 - 2g(\Sigma_1) + 2 \int_{\Sigma - \cup_{i=1}^{g(\Sigma_2)} N(x_i)} \frac{\sqrt{-1}}{2\pi} F_A \int_{D^2} \frac{\sqrt{-1}}{2\pi} F_{A_0} \\
&\quad + 2 \sum_{i=1}^{g(\Sigma_2)} \int_{N(x_i)} \frac{\sqrt{-1}}{2\pi} F_A \int_{D^2} \frac{\sqrt{-1}}{2\pi} F_{A_0} \\
&= 2[2 - 2g(\Sigma_1) + 2(2 - 2g(\Sigma_1) - g(\Sigma_2))n + 2g(\Sigma_2)(n+1)] \\
&= 4 - 4g(\Sigma_1) + 4n(2 - 2g(\Sigma_1)) + 4g(\Sigma_2).
\end{aligned}$$

Thus we have

$$\begin{aligned}
& \dim \mathcal{M}_{W^0}(L) \\
&= \frac{1}{4}(4 - 4g(\Sigma_1) + 4n(2 - 2g(\Sigma_1)) + 4g(\Sigma_2) - (4 - 4g(\Sigma))) \\
&= 2g(\Sigma_2) + n(2 - 2g(\Sigma_1)),
\end{aligned}$$

where

$$n = \int_{y\times D^2} \frac{\sqrt{-1}}{2\pi} F_{A_0} = \sharp \text{ of zeros of } \alpha \text{ over } D^2.$$

For a Spin^c structure $L = K_X^{-1} = K_{W_0}^{-1} \otimes F^2$ with $c_1(F)(\Sigma) = g(\Sigma_2)$, $2c_1(F)[\{y\} \times D^2] = c_1(K_X^{-1})[\{y\} \times D^2] - c_1(K_{W_0}^{-1})[\{y\} \times D^2] = 0$.

Then $n = 0$ and $\dim\mathcal{M}_{W^0}(K_X^{-1}|_{W^0}) = 2g(\Sigma_2)$. From Theorem 4 $\dim\mathcal{M}_X(K_X^{-1}) = 0$ and $\dim\mathcal{M}_{X^0}(L) = 0$, $SW_X(K_X^{-1})$ is equal to $SW_{X^0} \cdot p$ up to sign.

Therefore SW_{X^0} is not trivial. □

Using the results of [3] and [17] we can prove the following.

THEOREM 6. *There is a Spin^c structure L on X^0 with $c_1(L)(\Sigma) = 2 - 2g(\Sigma_1)$ such that the degree of r is nonzero.*

THEOREM 7. *Let X be a symplectic manifold with an embedded submanifold Σ_1 of minimal genus > 1 in its homology class $[\Sigma_1] \in H_2(X, \mathbb{Z})$ and of self-intersection number ≥ 0. Let $Z = X\sharp_\Sigma X$. Then there is a Spin^c structure $\tilde{L}$ on Z with nonzero Seiberg-Witten invariant and degree $2 - 2g(\Sigma_1)$ on Σ.*

Proof. Let $\tilde{L}$ be a Spin^c structure on $Z = X\sharp_\Sigma X$ obtained by gluing restriction $K_X^{-1}|_{X^0}$ of canonical Spin^c structure on X to X^0 as in Theorem 6. Applying Theorem 4 to $Z = X\sharp_\Sigma X$, we have

$$SW_Z(\tilde{L}) = \pm SW_{X^0}(K_X^{-1}|_{X^0}) \cdot p,$$

where p is the degree of map $r : \mathcal{M}_{X^0}(L|_{X^0}) \to \mathcal{R}(\Sigma)$. Then the Seiberg-Witten invariant on Z with respect to a Spin^c structure L is nonzero.□

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