개선지수를 고려한 주기적 예방보전의 최적화에 관한 연구[1)]

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Optimal Periodic Preventive Maintenance with Improvement Factor

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Key Words : periodic preventive maintenance, minimal repair, improvement factor, hazard rate, expected cost rate

Abstract

In this paper, we consider a periodic preventive maintenance(PM) policy in which each PM reduces the hazard rate but remains the pattern of hazard rate unchanged. And the system undergoes only minimal repairs at failures between PM's. The expected cost rate per unit time is obtained. The optimal number N of PM and the optimal period x, which minimize the expected cost rate per unit time are discussed. Explicit solutions for the optimal periodic PM are given for the Weibull distribution case.

1. Introduction

Preventive Maintenance(PM) has played an important role in effective operation and economic management of industrial systems. PM prevents unexpected catastrophic failure of system and ultimately extends the system life.

PM problems have been studied by many authors. Barlow and Hunter (1960) propose two types of PM policies. One policy is that PM is done periodically and minimal repair at any intervening failure between periodic PM's.

In almost earlier PM policy, it is assumed that a system is as good as

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new one after PM. The imperfect PM policy, in which PM is imperfect with probability p, is introduced by Chan and Down(1978) and is discussed in Nakagawa(1979), Murthy and Nguyen(1981), Brown and Proschan(1983) and Fontenot and Proschan(1984). Canfield(1986) considers PM policy which slows system degradation while the level remains unchanged and the hazard function is increased.

Recently, Park, Jung and Yum(2000) consider Canfield's PM model with various cost structures of running the system and find the optimal number of PM's and the optimal period, which minimize the expected cost rate per unit time for an infinite time span.

Lie and Chun(1986) introduce an improvement factor in hazard rate after maintenance. PM policy with improvement factor is studied by several authors. Nakagawa(1980) considers imperfect PM policies for which the system after PM has a reduced age. Nakagawa(1986) considers periodic and sequential PM policies for which the system has a different failure distribution between PM's in such a way that $h_k(t) < h_{k+1}(t)$ for any $t>0$, where $h_k(t)$ is the hazard rate in the kth period of PM. And Nakagawa(1988) considers a sequential imperfect PM policies in which the hazard rate after PM k becomes $a_k h(t)$, where a_k is an improvement factor, when it was h(t) in period k of PM.

Preventive maintenance policy has been applied to various fields of industries including semiconductor manufacturing company, public transportation company and so on. Lai et. al. (2000) conduct a case study of Kowloon Motor Bus Company Limited (KMB) in which they study the policies of PM and replacement for an engine based on the minimization of cost due to stoppage of an engine. And Charles et. al. (2003) address the problem of preventive maintenance (PM) strategy optimization in a semiconductor manufacturing environment, with the objective of minimizing maintenance costs.

In this paper, we consider a periodic PM policy in which each PM reduces the hazard rate but the pattern of hazard rate remains unchanged. The system is preventively maintained at periodic times kx and is replaced by a new system at the Nth PM, where $k = 1, 2, \cdots, N$. It is assumed that the system undergoes only minimal repair at any failure between PM's and hence, the hazard rate remains unchanged by any of minimal repairs. The expected cost rate per unit time is obtained. The optimal number N of the periodic PM and the optimal period x, which minimize the expected cost rate per unit time, are discussed.

In Section 2, we describe the periodic PM model and its assumptions. The expression for the expected cost rate for the policy is obtained in Section 3. Section 4 is devoted to discuss the optimal period and the optimal number of PM's for policy. In Section 5, the optimal schedules are computed explicitly when the failure time follows Weibull distribution.

Notation

$h(t)$	hazard rate without PM
$h_{pm}(t)$	hazard rate with PM
x	period of PM
N	number of PM's where the system is replaced.
p	improvement factor in hazard rate, $0 \le p \le 1$.
C_{mr}	cost of minimal repair at failure
C_{pm}	cost of PM
C_{re}	cost of replacement
$C(x,N)$	expected cost rate per unit time.

2. Model and Assumptions

We consider a periodic PM model with an improvement factor which reduces the hazard rate of the system after PM. The followings are assumed :

(1) The system begins to operate at time $t=0$.

(2) The PM is done at periodic time kx (k = 1, 2, ⋯) where $x>0$, and is replaced by new one at the Nth PM.

(3) The hazard rate $h(t)$ after PM k is reduced to $ph(t)$ when it was $h(t)$ in period k of PM where $0 \le p \le 1$. When $p=0$, the system after PM is as good as new one while when $p=1$, the system right after PM has the same hazard rate as that just prior to PM but has the same degradation pattern as new one.

(4) The system undergoes only minimal repair at failures between PM's.

(5) The repair and PM times are negligible.

(6) $h(t)$ is monotone increasing.

3. Expected Cost Rate Per Unit Time

In this paper, we propose a periodic PM model with an improvement factor in which the hazard rate $h(t)$ after PM k becomes $ph(t)$ when it was $h(t)$ in the period k of PM. Under this model, the hazard rate $h_{pm}(t)$ is given by

$$h_{pm}(t) = ph_{pm}(kx) + h(t-kx) \qquad (1)$$

for $k = 0, 1, 2, \cdots$ where $kx < t \le (k+1)x$, $h_{pm}(0) = h(0)$ and x is the time interval between PM interventions.

Substituting $h_{pm}(kx)$ in the equation (1) recursively, the equation (1) can be rewritten as

$$h_{pm}(t)=\begin{cases} h(t) & \text{if } 0< t\le x \\ \sum_{i=1}^{k} p^i h(x)+h(t-kx) & \text{if } kx< t\le (k+1)x, \\ & k=1,2,\cdots,N. \end{cases} \quad (2)$$

Since it is well-known from Lemma 1.1 in Fontenot and Proschan(1984) that the number of minimal repairs during the period k of PM is nonhomogeneous Poisson process(NHPP) with intensity function $\int_{kx}^{(k+1)x} h_{pm}(t)\,dt$, the expected cost rate per unit time can be obtained in the following manner:

Expected Cost Rate Per Unit Time
= [(expected cost of minimal repairs in [0, Nx))
+ (expected cost of PM in [0, Nx))
+ (expected cost of replacement)]/ Nx.

Each expected cost given in the expected cost rate per unit time is obtained as follows:

(i) Expected cost of minimal repairs in [0, Nx) = $C_{mr}\left(\sum_{k=0}^{N-1}\int_{kx}^{(k+1)x} h_{pm}(t)\,dt\right)$,

where $h_{pm}(t)$ is given in the equation (2).

(ii) Expected cost of PM in [0, Nx) = $(N-1)\,C_{pm}$..

(iii) Expected cost of replacement = C_{re}..

Using (i), (ii) and (iii), the expected cost rate per unit time for running the periodic PM with improvement factor during $[0,\ Nx]$ is obtained as follows:

For $0\le p< 1$,

$$C(x,N) = [C_{mr}\left\{\sum_{k=0}^{N-1}\int_{kx}^{(k+1)x}\left(\frac{p(1-p^k)}{1-p}h(x) + h(t-kx)\right)dt + (N-1)C_{pm}+C_{re}\right]/Nx. \quad (3)$$

For $p=1$,

$$C(x,N) = [C_{mr}\left\{\sum_{k=0}^{N-1}\int_{kx}^{(k+1)x}(kh(x)+h(t-kx))dt\right\} + (N-1)\,C_{pm}+C_{re}]/Nx. \quad (4)$$

The equations (3) and (4) can be rewritten in more useful expression as follows:

For $0\le p< 1$,

$$C(x,N) = \frac{1}{Nx}\left[C_{mr}\left\{D(x,p)\left(N-\frac{1-p^N}{1-p}\right) + N\int_0^x h(t)dt\right\}+(N-1)C_{pm}+C_{re}\right] \quad (5)$$

and for $p=1$,

$$C(x,N) = [C_{mr}\left\{\frac{N(N-1)}{2}xh(x)+N\int_0^x h(t)dt\right\} + (N-1)C_{pm}+C_{re}]/Nx \tag{6}$$

, where $D(x,p)=pxh(x)/(1-p)$.

4. Optimal Schedules for the Periodic PM Policy

In this section, we find the optimal period x^* and the optimal number N^* of PM, which minimize the expected cost rate per unit time.

We first find the optimal number of PM, when the period x is fixed. To find the optimal N*. which minimizes $C(x,N)$, we form he following inequalities.

$$C(x,N+1) \geq C(x,N)$$

and

$$C(x,N) < C(x,N-1).$$

For $0 \leq p < 1$, it can be easily shown that $C(x,N+1) \geq C(x,N)$ implies

$$\frac{D(x,p)}{1-p}[p^N(Np-N-1)+1] \geq \frac{C_{re}-C_{pm}}{C_{mr}}. \tag{7}$$

Similarly, the inequality $C(x,N) < C(x,N-1)$ implies

$$\frac{D(x,p)}{1-p}[p^{N-1}(Np-N-p)+1] \leq \frac{C_{re}-C_{pm}}{C_{mr}}. \tag{8}$$

From equations (7) and (8), we have

$$L(x,N) \geq \frac{C_{re}-C_{pm}}{C_{mr}} \tag{9}$$

and

$$L(x,N-1) < \frac{C_{re}-C_{pm}}{C_{mr}}, \tag{10}$$

where

$$L(x,N) = \frac{D(x,p)}{1-p}[p^N(Np-N-1)+1]$$

and $D(x,p)=pxh(x)/(1-p)$ for $N = 1, 2, \cdots$ and $L(x,N)=0$ for $N=0$.

For p=1, $C(x,N+1) \geq C(x,N)$ and $C(x,N) < C(x,N-1)$ imply

$$\frac{N(N+1)}{2}xh(x) \geq \frac{C_{re}-C_{pm}}{C_{mr}} \tag{11}$$

and

$$\frac{N(N-1)}{2}xh(x) < \frac{C_{re}-C_{pm}}{C_{mr}} \tag{12}$$

respectively.

Combining the equations (11) and (12), we have

$$L'(x,N) \geq \frac{C_{re}-C_{pm}}{C_{mr}} \tag{13}$$

and

$$L'(x,N-1) < \frac{C_{re}-C_{pm}}{C_{mr}}, \qquad (14)$$

where $L'(x,N) = \frac{N(N+1)}{2} xh(x)$ for $N=1,2,\cdots$ and $L'(x,N)=0$ for $N=0$.

Lemma 4.1.

Let $Z(p,N) = p^N(N+1-Np)$. Then $\lim_{N\to\infty} Z(p,N)=0$ for all $0 \le p < 1$.

proof.

It is clear that $Z(p,N)=0$ for $p=0$. And then the limit follows.

For $0<p<1$, it is easy to see that

$$Z(p,N) = p^N(N(1-p)+1) \le \frac{1}{4} Np^{N-1} + p^N$$

The result immediately follows from the fact that

$$\sum_{N=1}^{\infty} Np^{N-1} = \sum_{N=1}^{\infty}\left(\frac{d}{dp}p^N\right) = \frac{d}{dp}\left(\sum_{N=1}^{\infty} p^N\right) = \frac{1}{(1-p)^2}$$

implies $\lim_{N\to\infty} Np^{N-1} = 0$ ■

Theorem 4.2. Suppose that x is sufficiently large so that

$$\frac{pxh(x)}{(1-p)^2} \ge \frac{C_{re}-C_{pm}}{C_{mr}} \quad \text{for } 0<p\le 1.$$

Then there exists a finite N^* which satisfies (9), (10), (13) and (14) and it is unique.

proof.

Suppose that $p=1$. Since $h(x) > h(0)$ for all $x>0$, we have

$$L'(x,N) - L'(x,N-1) = Nxh(x) > 0.$$

Thus $L'(x,N)$ is strictly increasing in N and tends to ∞ as $N\to\infty$. Hence the result follows.

Next we consider the case that $0 \le p < 1$. Since $h(x) > h(0)$ for all $x>0$, we have

$$L(x,N) - L(x,N-1) = \frac{D(x,p)}{1-p}[Np^{N-1}(1-p)^2] > 0,$$

where $D(x,p) = pxh(x)/(1-p)$. Thus $L(x,N)$ is strictly increasing in N.

It follows from Lemma 4.1 that

$$\lim_{N\to\infty} L(x,N) = \frac{pxh(x)}{(1-p)^2}$$

and the result follows. ■

<u>Remark</u> For $p=1$ the system returns to the state as good as new one. after every PM. Hence there is no need to replace the system by new one and no optimal N^* exists.

Next we consider the case when the number of PM, N, is fixed. To find the optimal period x^* for a given N which minimizes $C(x,N)$ in (5) and (6), we take the derivative $C(x,N)$ with respect to x and set it equal to 0. Then we have

$$\xi(p,N)x^2h'(x)+N(xh(x)-H(x)) = \frac{(N-1)C_{pm}+C_{re}}{C_{mr}}, \tag{15}$$

where

$\xi(p,N)=p[N(1-p)-1+p^N]/(1-p)^2$

for $0 \le p < 1$ and $\frac{N(N-1)}{2}$ for $p=1$..

Let $g(x)$ denotes the left-hand side of (15) and let C be the right-hand side of (15). Then

$$\frac{d}{dx}C(x,N)=g(x)-C,$$

where $g(0)=0$ and $C>0$.

Lemma 4.3. If $h(t)$ is strictly increasing and convex, then $g(x)$ is increasing in x.

proof.

It is easy to see that

$$\frac{d}{dx}g(x) = \xi(p,N)[2xh'(x)+x^2h''(x)]+Nxh'(x)>0$$

since $h(t)$ is strictly increasing and convex.

Theorem 4.4. If $h(t)$ is strictly increasing and convex function, then there exists a $x^* < \infty$ which satisfies (15) for a given integer N and it is unique.

proof.

It is shown from Lemma 4.3, that $g(x)$ is increasing in x.

Using Mean-value theorem, we have, for $t < t_0 < x$

$$g(x)=\xi(p,N)x^2h'(x)+\frac{N}{2}x^2h'(t_0)$$

which becomes ∞ as $x \to \infty$. It is also noted that $g(0)=0$.

Thus there exists a finite and unique x^* which satisfies (15) for any giver N. ■

We finally consider the problem of finding the optimal period x^* and the optimal number of PM, N^*, which minimizes $C(x,N)$ of the equation (5) and (6). In this case, neither x nore N is assumed to be fixed.

First, we can obtain x_N as a function of N which satisfies (15). Substituting x_N for x in $C(x,N)$ of the equations (5) and (6), we have the expected cost rate per unit time as follows.

For $0 \le p < 1$,

$$C(x_N,N) = \left[C_{mr}\left\{D(x_N,p)(N-\frac{1-p^N}{1-p})x_Nh(x_N) + \int_0^{x_N}h(t)dt\right\}+(N-1)C_{pm}+C_{re}\right]/Nx_N \tag{16}$$

and for $p=1$,

$$C(x_N,N) = \left[C_{mr}\left\{\frac{N(N-1)}{2}x_Nh(x_N)+N\int_0^{x_N}h(t)dt\right\} + (N-1)C_{pm}+C_{re}\right]/Nx_N \tag{17}$$

, where $D(x_{N,}p)=px_Nh(x_N)/(1-p)$.

Since the formulas (16) and (17) are function of N only, we can obtain N^* which minimizes $C(x_N, N)$.

Theorem 4.5. Suppose that the hazard rate of a life distribution F is strictly increasing and convex function. For a given N there exists a finite and unique x_N which satisfies the equation (15). And, if $px_N h(x_N)/(1-p)^2 \geq (C_{re}-C_{pm})/C_{mr}$, then the value N^* satisfying the equation (13) is the optimal number of PM which minimizes the expected cost rate per unit time $C(x,N)$ of the equations (5) and (6).

Using the difference operator, we can find N^* in (16) and (17). The difference operator is defined by

$$\Delta C(x_N, N) = C(x_{N+1}, N+1) - C(x_N, N),\quad N=1,2,\cdots \qquad (18)$$

Then, it is easy to see that N^* is the smallest integer N such that $\Delta C(x_N, N)$, which is equivalent to finding N^* minimizing (16) and (17). Once we obtain N^*, we can find the optimal period x^* from Theorem 4.4.

Hence the optimal period and the optimal number of PM which minimize $C(x,N)$ of the equations (5) and (6) are x^* and N^*, respectively.

5. Numerical Example

Suppose that the failure time distribution F is Weibull distribution with a scale parameter λ and a shape parameter β, is $h(t) = \beta\lambda^{\beta-1}t^{\beta-1}$ for β>0 and $t \geq 0$. As a special case, we take β =3 and λ=1 for $t \geq 0$.

For various values of p, we compute the optimal x^* and N^* which minimize $C(x,N)$ of (5) and (6). It can be obtained from the equations (5) and (6) and (15) that

$$x_N = \{(1-p)^2[(N-1)C_{pm}+C_{re}]\}^{\frac{1}{3}} \{C_{mr}[6p(N(1-p)-1+p^N) + 2N(1-p)^2]\}^{-\frac{1}{3}} \qquad (19)$$

and

$$C(x_N, N) = \left[C_{mr}\left(\frac{3px_N^3}{1-p}\left(N-\frac{1-p^N}{1-p}\right)+Nx_N^3\right). + (N-1)C_{pm}+C_{re}\right]/Nx_N. \qquad (20)$$

It should be noted that when $p=1$,

$$x_N = \left\{\frac{[(N-1)C_{pm}+C_{re}]}{C_{mr}(3N^2-N)}\right\}^{\frac{1}{3}} \qquad (21)$$

and

$$C(x_N, N) = \left[C_{mr}\left(\frac{N(N-1)}{2} x_N^3 + N x_N^3\right). + (N-1)C_{pm} + C_{re}\right] / N x_N. \quad (22)$$

Using (19) and (20), we determine N^* so that $C(x_N, N)$ is minimized. And we also obtain x^* by substituting N^* for N in the equation (19). When *p=1*, we obtain the optimal x^* and N^* by using (21) and (22) in similar way.

Then the expected cost rate per unit time is as follows:

For $0 \le p < 1$,

$$C(x_N, N) = \left[C_{mr}\left\{ D(x_N, p)(N - \frac{1-p^N}{1-p}) x_N h(x_N) + \int_0^{x_N} h(t)dt \right\} + (N-1)C_{pm} + C_{re} \right] / N x_N \quad (23)$$

and for $p=1$,

$$C(x^*, N^*) = [C_{mr}\left\{ \frac{N^*(N^*-1)}{2} x^{*3} + N^* x^{*3} \right\} + (N^*-1)C_{pm} + C_{re}] / N^* x^*. \quad (24)$$

Table 1 shows values of the optimal number of PM N^* for a given x. For Table 1, we take x=0.8, $C_{mr} = 1.0$, $C_{pm} = 1.5$ and C_{re}=2.0, 2.5, 3.0 and 3.5. It is noted that no optimal N^* exists for some small values of p. This results are corresponding to Theorem 4.1. It is interesting to note that as the value of C_{re} increases, the number of PM's needed to minimize the expected cost rate increases. Table 2 represents optimal period x^* and its corresponding expected cost rate $C(x^* \cdot N)$ for N= 1 to 19(2) when $C_{mr} = 1$, $C_{pm} = 1.5$ and $C_{re} = 3.0$. Table 2 shows that the value of x^* gets smaller and the expected cost rate increases as N increases and p increases. Table 3 lists the values of x^* and N^* by applying the equations (19) and (20) and its corresponding the expected cost rate for various choice of *p* and C_{re}. when $C_{mr} = 1$ and $C_{pm} = 1.5$. The pair (x^*, N^*) represents the optimal period and the number of PM's which minimize the expected cost rate. For instance, when *p*=0.4 and C_{re}=2.6, he optimal period, x^*, and the number of PM's, N^*, are 0.862 and 2, respectively and its corresponding expected cost rate is 4.872. As we expect from Theorem 4.5, the optimal period, x^*, decreases while the number of PM's, N^*, needed to minimize the expected cost rate increases as the value of C_{re} increases. And the optimal pair (x^*, N^*) dost not exits when improvement factor, *p*, is less than a certain value for given C_{re}. This result is quiet natural since the unit does not have to be replaced by new one if PM restores the unit to the state like almost new one. It is also noted that, for fixed C_{re}, the optimal PM period, x^*, becomes longer and consequently, the optimal number, N^*, gets smaller as the degree of improvement decreases.

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<Table 1> Optimal number of PM's N^* and expected cost rate $C(x, N^*)$ with $C_{mr}=1$, $C_{pm}=1.5$ and $x=0.8$

	C_{re}							
	2.0		2.5		3.0		3.5	
	N^*	$C(x,N^*)$	N^*	$C(x,N^*)$	N^*	$C(x,N^*)$	N^*	$C(x,N^*)$
0.1	-	-	-	-	-	-	-	-
0.2	-	-	-	-	-	-	-	-
0.3	2	3.115	-	-	-	-	-	-
0.4	1	3.140	2	3.524	4	3.744	-	-
0.5	1	3.140	2	3.620	2	3.933	3	4.148
0.6	1	3.140	2	3.716	2	4.029	2	3.341
0.7	1	3.140	1	3.765	2	4.125	2	4.437
0.8	1	3.140	1	3.765	2	4.220	2	4.533
0.9	1	3.140	1	3.765	2	4.317	2	4.629
1.0	1	3.140	1	3.765	1	4.390	2	5.015

<Table 2> Optimal period x* and expected cost rate C(x*, N) with $C_{mr}=1$, $C_{pm}=1.5$ and $C_{re}=3.0$

N		p									
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1	x*	1.1447	1.1447	1.1447	1.1447	1.1447	1.1447	1.1447	1.1447	1.1447	1.1447
	C(x*, N)	3.9311	3.9311	3.9311	3.9311	3.9311	3.9311	3.9311	3.9311	3.9311	3.9311
3	x*	0.9384	0.8855	0.8395	0.7991	0.7631	0.7310	0.7020	0.6758	0.6519	0.6300
	C(x*, N)	3.1968	3.3877	3.5734	3.7544	3.9311	4.1039	4.2732	4.4392	4.6021	4.7622
5	x*	0.8941	0.8320	0.7769	0.7272	0.6820	0.6405	0.6024	0.5672	0.5347	0.5047
	C(x*, N)	3.0199	3.2451	3.4753	3.7128	3.9591	4.2154	4.4823	4.7602	5.0493	5.3495
7	x*	0.8748	0.8095	0.7510	0.6976	0.6480	0.6015	0.5577	0.5164	0.4774	0.4409
	C(x*, N)	2.9395	3.1767	3.4239	3.6860	3.9680	4.2747	4.6107	4.9800	5.3862	5.8321
9	x*	0.8640	0.7970	0.7370	0.6818	0.6299	0.5804	0.5327	0.4867	0.4424	0.4002
	C(x*, N)	2.8936	3.1366	3.3921	3.6669	3.9690	4.3075	4.6929	5.1366	5.6506	6.2467
11	x*	0.8571	0.7892	0.7282	0.6719	0.6187	0.5673	0.5170	0.4674	0.4186	0.3712
	C(x*, N)	2.8639	3.1103	3.3706	3.6530	3.9673	4.3266	4.7477	5.2518	5.8642	6.6129
13	x*	0.8522	0.7837	0.7222	0.6652	0.6111	0.5585	0.5063	0.4539	0.4012	0.3490
	C(x*, N)	2.8432	3.0918	3.3552	3.6424	3.9648	4.3384	4.7856	5.3386	6.0400	6.9428
15	x*	0.8487	0.7797	0.7178	0.6604	0.6057	0.5522	0.4987	0.4440	0.3879	0.3313
	C(x*, N)	2.8278	3.0780	3.3436	3.6342	3.9623	4.3460	4.8129	5.4056	6.1871	7.2442
17	x*	0.8460	0.7767	0.7144	0.6567	0.6016	0.5475	0.4929	0.4365	0.3775	0.3167
	C(x*, N)	2.8160	3.0673	3.3346	3.6277	3.9600	4.3512	4.8331	5.4582	6.3115	7.5224
19	x*	0.8438	0.7743	0.7118	0.6538	0.5984	0.5439	0.4885	0.4306	0.3690	0.3044
	C(x*, N)	2.8067	3.0588	3.3273	3.6224	3.9579	4.3549	4.8486	5.5002	6.4180	7.7815

<Table 3> Optimal period x^*, number of PM's N^* and its expected cost rate $C(x, N^*)$ with $C_{mr}=1$, $C_{pm}=1.5$

p	C_{re}								
	2.0			2.2			2.4		
	x^*	N^*	$C(x^*, N^*)$	x^*	N^*	$C(x^*, N^*)$	x^*	N^*	$C(x^*, N^*)$
0.2	0.876	2	4.279	-	-	-	-	-	-
0.3	1.000	1	5.25	1.032	1	5.376	-	-	-
0.4	1.000	1	5.25	1.032	1	5.376	1.063	1	5.505
0.5	1.000	1	5.25	1.032	1	5.376	1.063	1	5.505
0.6	1.000	1	5.25	1.032	1	5.376	1.063	1	5.505
0.7	1.000	1	5.25	1.032	1	5.376	1.063	1	5.505
0.8	1.000	1	5.25	1.032	1	5.376	1.063	1	5.505
0.9	1.000	1	5.25	1.032	1	5.376	1.063	1	5.505
1.0	1.000	1	5.25	1.032	1	5.376	1.063	1	5.505

p	C_{re}								
	2.6			2.8			3.0		
	x^*	N^*	$C(x^*, N^*)$	x^*	N^*	$C(x^*, N^*)$	x^*	N^*	$C(x^*, N^*)$
0.4	0.862	2	4.872	-	-	-	-	-	-
0.5	1.091	1	5.635	1.119	1	5.766	0.863	2	5.214
0.6	1.091	1	5.635	1.119	1	5.766	1.145	1	5.897
0.7	1.091	1	5.635	1.119	1	5.766	1.145	1	5.897
0.8	1.091	1	5.635	1.119	1	5.766	1.145	1	5.897
0.9	1.091	1	5.635	1.119	1	5.766	1.145	1	5.897
1.0	1.091	1	5.635	1.119	1	5.766	1.145	1	5.897

p	C_{re}								
	4.0			5.0			6.0		
	x^*	N^*	$C(x^*, N^*)$	x^*	N^*	$C(x^*, N^*)$	x^*	N^*	$C(x^*, N^*)$
0.7	0.875	2	5.998	-	-	-	-	-	-
0.8	1.260	1	6.548	0.744	3	6.386	0.624	5	6.487
0.9	1.260	1	6.548	0.884	2	6.785	0.746	3	7.036
1.0	1.260	1	6.548	1.357	1	7.184	0.909	2	7.429