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Moment Inequalities of NBURFR and NBARFR Classes with Hypotheses Testing Applications

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Abstract. Nonparametric families of aging distributions have been the subject of investigation from long period of time and still. Both probabilistic and statistical properties of these distributions were studied for such families as new better than used renewal failure rate (NBURFR) and new better average renewal failure rate (NBARFR) classes. They have been studied by Abouammoh and Ahmed (1992). In the present work, moment inequalities are derived for the above mentioned families that demonstrate that if the mean life is finite for any of them then all higher order moments exist. Next, based on these inequalities, new test procedures for exponentiality against these families are studied showing that it is simple and hold high relative efficiency for some commonly used alternatives. Dealing with censored data case also studied.

1. INTRODUCTION

Testing exponentiality against various classes of life distributions has got a good deal of attention. With respect to testing against IFR, see Proschan and Pyke (1967), Barlow (1968), and Ahmed (1975) among others. For testing against IFRA, see Deshpande (1983), Linmk (1989), Aly (1989), and Ahmed (1994). For testing against NBU, see Hollander and Proschan (1972), Koul (1977), Kumazawa (1983) and Ahmed (1994). For testing against NBUE, NBUFR and NBAFR classes, we refer to Klefsjo (1981 and 1982), Deshpande et al. (1986), Abouammoh and Ahmed (1988), Loh (1984) and Hendi et al. (2000). Recently Mahmoud and Abdul Alim (2002) studied testing exponentiality against NBURFR based on a U-statistic for censored and non censored data. Ahmed (2001) proposed moment inequalities for studying hypotheses testing problems in the case of IFR, NBU, NBUE and HNBUE classes. Recently Ahmed (2003) used a new techniques to address other three classes, the increasing failure rate average, the new better than used in convex ordering, and the decreasing mean residual life time for completing the study of perennial group of life distributions.

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Aging is characterized by a non negative random variable T with distribution function $F(t) = P(T \le t)$ and a survival function $\overline{F}(t) = 1 - F(t)$. For practicalities, T is often assumed (but need not be) continuous with probability density function $f(t) = F'(t)$. Aging distributions are divided many non-parametric classes according to the type of aging that take place. For example we mention here NBURFR (new better than used renewal failure rate) and NBARFR (new better average renewal failure rate) and their duals NWURFR (new worth than used renewal failure rate) and NWARFR (new worth average renewal failure rate) . Formally these classes and their dual classes are defined as follows:

Definition 1. F is new better than used renewal failure rate (NBURFR) if

$$r_F(0) \le r_w(t), \quad t \ge 0 \,, \tag{1.1}$$

i.e the failure rate of a new system is less than the renewal failure rate of a used system.

Definition 2. F is new worth than used renewal failure rate (NWURFR) if

$$r_F(0) \ge r_w(t), t \ge 0 \,, \tag{1.2}$$

i.e the failure rate of a new system is greater than the renewal failure rate of a used system. For convenience, we note that (1.1) and (1.2) are equivalent to

$$\int_t^\infty \overline{F}(u)du \le \overline{F}(t)/r_F(0), t > 0 \tag{1.3}$$

$\int_t^\infty \overline{F}(u)du \ge \overline{F}(t)/r_F(0), t > 0$, see (Abouammoh and Ahmed (1992)) (1.4)

Definition 3. F is new better than average renewal failure rate (NBARFR) if

$$r_F(0) \le t^{-1} \int_0^t r_{W_F}(u)du, t > 0 \,.$$

Equivalently $r_F(0) \le t^{-1} \ln \overline{W}_F(t)$ where, $\overline{W}_F(0) = \mu_F^{-1} \int_0^\infty \overline{F}(u)du = 1$, i.e the failure rate of a new system is less than the average renewal failure rate of a used system.

Definition 4. F is new worth than used average renewal failure rate (NWARFR) if

$$r_F(0) \ge t^{-1} \int_0^t r_{W_F}(u)du, \text{t} > 0 \,.$$

Equivalently $r_F(0) \ge t^{-1} \ln \overline{W}_F(t)$, i.e the failure rate of a new system is greater than the average renewal failure rate of a used system see (Abouammoh and Ahmed (1992))

Theorem 1. The life distribution F or its survival $\overline{F}$ having NBARFR iff

$$\int_t^\infty \overline{F}(u)du \le \mu_F e^{-tr_F(0)}, \text{t} > 0.$$

Theorem 2. The life distribution F or its survival $\overline{F}$ having NWARFR iff

$$\int_t^\infty \overline{F}(u)du \geq \mu_F e^{-tr_F(0)}, \text{t} > 0.$$

Note that the mean μ of an aging random variable T is called " mean time to failure" , cf Zacks (1991), and is expected to be finite since otherwise studying aging has no meaning . Thus throughout this work, it is assumed that $\mu < \infty$.

Probabilistic properties of the above classes of aging distribution have been extensively studied by Abouammoh and Ahmed (1992). The first purpose of the current investigation is to provide some moment inequalities of the above classes that will generally assert that if $\mu < \infty$ then all moments would exist.

On the other hand, testing H_0 : is exponential against an alternative that $H_1^{(i)}$:, $i = 1,2$ belongs to NBURFR and NBARFR classes and not exponential was taken up by for example Abouammoh and Ahmed (1992) and Mahmoud and Abdul-Alim (2002). The thread that connects most work mentioned or not mentioned is that a measure of departure from H_0, which is often some weight functional of F, is developed which is strictly positive under H_1 and is zero under H_0. Then a sample version of this measure is used as test statistic and its properties are studies. In this spirit, the moment inequalities developed in section (III) is used to construct test statistics for the problems in section (II) . These tests are based on sample moments of the aging distribution. It is simple to devise, calculate and studies and have exceptionally high efficiency for some well known of alternatives relative to other more complicated tests. Using Monte Carlo methods, critical values of the test statistic presented here are easy to obtain for different choice of the orders of moments.

Note here that we derive for each problem a class of test statistics indexed by an integer valued parameter.

2. MOMENT INEQUALITIES

In this Section we derive moment inequalities for the NBURFR and NBARFR classes. For these results, moments are assumed to be exist and finite.

Theorem 3. If F is NBURFR , then for all integer $r \geq 0$

$$\frac{\mu_{(r+1)}}{(r+1)} \geq f(0)\frac{\mu_{(r+2)}}{(r+1)(r+2)}$$

Proof. Since F is NBURFR then,

$$f(0)\omega(t) \leq \overline{F}(t),$$

where,

$$\omega(t) = \int_t^\infty \overline{F}(u)du = E(T-t)I(T>t).$$

Thus for all integer $r \geq 0$,

$$\int_0^\infty t^r \overline{F}(t)dt \geq \int_0^\infty t^r f(0)\omega(t)dt.$$

Since

$$\int_0^\infty t^r \overline{F}(t)dt = E\int_0^\infty t^r I(T > t)dt = \frac{1}{r+1}E(T^{r+1}) = \frac{\mu_{(r+1)}}{r+1}.$$

And

$$\int_0^\infty t^r \omega(t)dt = E\int_0^\infty t^r (T-t)I(T > t)dt = E\left(T\int_0^T t^r dt - \int_0^T t^{r+1}dt\right)$$

$$= E(T^{r+2})\left(\frac{1}{r+1} - \frac{1}{r+2}\right) = \frac{\mu_{(r+2)}}{(r+1)(r+2)}.$$

The result follows.

Theorem 4. If F is NBARFR , then for all integer $r \geq 0$

$$\frac{r!\mu}{(f(0))^{r+1}} \geq \frac{\mu_{(r+2)}}{(r+1)(r+2)}.$$

Proof. Since F is NBARFR then,

$$\omega(t) \leq \omega(0)e^{-tf(0)},$$

where $\omega(t)$ is as in previous theorem thus for all integer $r \geq 0$,

$$\int_0^\infty t^r \omega(0)e^{-tf(0)}dt \geq \int_0^\infty t^r \omega(t)dt$$

equivalently

$$\mu\int_0^\infty t^r e^{-tf(0)}dt \geq \int_0^\infty t^r \omega(t)dt.$$

Since

$$\int_0^\infty t^r \omega(t)dt = \frac{\mu_{(r+2)}}{(r+1)(r+2)},$$

and

$$\int_0^\infty t^r e^{-tf(0)}dt = \frac{r!}{(f(0))^{r+1}}.$$

Then the result follows.

3. HYPOTHESIS TESTING PROBLEMS FOR NON-CENSORED DATA

In this section we consider two hypothesis tests the first one concerns NBURFR class and the other concerns NBARFR class.

3.1. Testing Against NBURFR Class

For the problem in life testing is to test H_0 : is exponential against an alternative that $H_1^{(1)}$: belongs to NBURFR class and not exponential. Using theorem 3., we can propose the following measure of departure from H_0 in favor of $H_1^{(1)}$: F is NBURFR and not exponential:

$$\delta_{r+2}^{(1)} = \frac{\mu_{(r+1)}}{(r+1)} - f(0)\frac{\mu_{(r+2)}}{(r+1)(r+2)}.$$

Which is estimated by $\hat{\delta}_{r+2}^{(1)}$ where

$$\hat{\delta}_{r+2}^{(1)} = \sum_{i=1}^{n} \frac{T_i^{r+1}}{n(r+1)} - \sum_{i=1}^{n} \hat{f}_n(0)\frac{T_i^{r+2}}{n(r+1)(r+2)}$$
$$= \frac{1}{n^2}\sum_{j=1}^{n}\sum_{i=1}^{n} \frac{T_i^{r+1}}{(r+1)} - \frac{T_i^{r+2}}{a(r+1)(r+2)}K\left(\frac{-T_j}{a}\right),$$

and

$$\hat{f}_n(0) = \frac{1}{na}\sum_{j=1}^{n} K\left(\frac{-T_j}{a}\right).$$

Where K(.) be a known pdf, symmetric and bounded with 0 mean and variance $\sigma_k^2 > 0$. For more details about K (.) and the sequence $\{a_n\}$ See Hardle (1991).
Now choosing

$$\phi(T_1,T_2) = \frac{T_1^{r+1}}{(r+1)} - \frac{T_1^{r+2}}{a(r+1)(r+2)}K\left(\frac{-T_2}{a}\right),$$

then $\hat{\delta}_{r+2}^{(1)}$ equivalents U-statistic cf Lee(1989). The proof of the following theorem follows from the standard theory of U-statistic.

Theorem 5. As $n \to \infty$, $\sqrt{n}\left(\hat{\delta}_{r+2}^{(1)} - \delta_{r+2}^{(1)}\right)$ is asymptotically normal with 0 mean and variance given by (3.1). Under H_0, the value of $\delta_{r+2}^{(1)} = 0$ and the variance is given by (3.2) and $\sigma_0^2 = 2$ when r=0.

Proof. From U-statistic theory, $\sqrt{n}\left(\hat{\delta}_{r+2}^{(1)} - \delta_{r+2}^{(1)}\right)$ is asymptotically normal with mean 0 and variance $\sigma^2 \doteq Var(\eta)$, where $\eta = \left(E\{\phi(T_1,T_2 \mid T_1) + \phi(T_1,T_2 \mid T_2)\}\right)$

So

$$\eta = \frac{T_1^{r+1}}{(r+1)}\int_0^\infty dF(T_2) - \frac{T_1^{r+2}}{(r+1)(r+2)}\int_0^\infty \frac{1}{a}K\left(\frac{-T_2}{a}\right)dF(T_2)$$

$$+\int_0^\infty \frac{T_1^{r+1}}{(r+1)} dF(T_1) - \frac{1}{a} K\left(\frac{-T_2}{a}\right) \int_0^\infty \frac{T_1^{r+2}}{(r+1)(r+2)} dF(T_1)$$

then

$$\sigma^2 = Var\left\{ \frac{T_1^{r+1} + \mu_{(r+1)}}{(r+1)} - \frac{T_1^{r+2} - f(0)\mu_1^{r+2}}{(r+1)(r+2)} \right\}. \tag{3.1}$$

Under H_0 (exponentiality)

$$\sigma_0^2 = \frac{\mu_{(2r+2)}^2 - \mu_{(r+1)}^2}{(r+1)^2} - \frac{2\mu_{(2r+3)} + 2\mu_{(r+1)}\mu_{(r+2)}}{(r+1)^2(r+2)} + \frac{\mu_{(2r+4)} - \mu_{(r+2)}^2}{(r+1)^2(r+2)^2},$$

which leads to

$$\sigma_0^2 = \left\{ \frac{(2r+2)!}{(r+1)^2} - 2\frac{(2r+3)!+(r+1)!(r+2)!}{(r+1)^2(r+2)} + \frac{(2r+4)!-((r+2)!)^2}{(r+1)^2(r+2)^2} \right\}. \tag{3.2}$$

When r=0, σ_0^2 =2 and the test statistic is given by

$$\hat{\delta}^{(1)} = \frac{1}{n^2} \sum_{j=1}^{n} \sum_{i=1}^{n} T_i - \frac{T_i^2}{2a} K\left(\frac{-T_j}{a}\right).$$

critical values for $\hat{\delta}^{(1)}$ with respect to n can be observed through Figure 1. To use above test, calculate $\sqrt{n}\hat{\delta}_{r+2}^{(1)} / \sigma_0$ and reject H_0 if this exceeds the normal variate $Z_{1-\alpha}$.

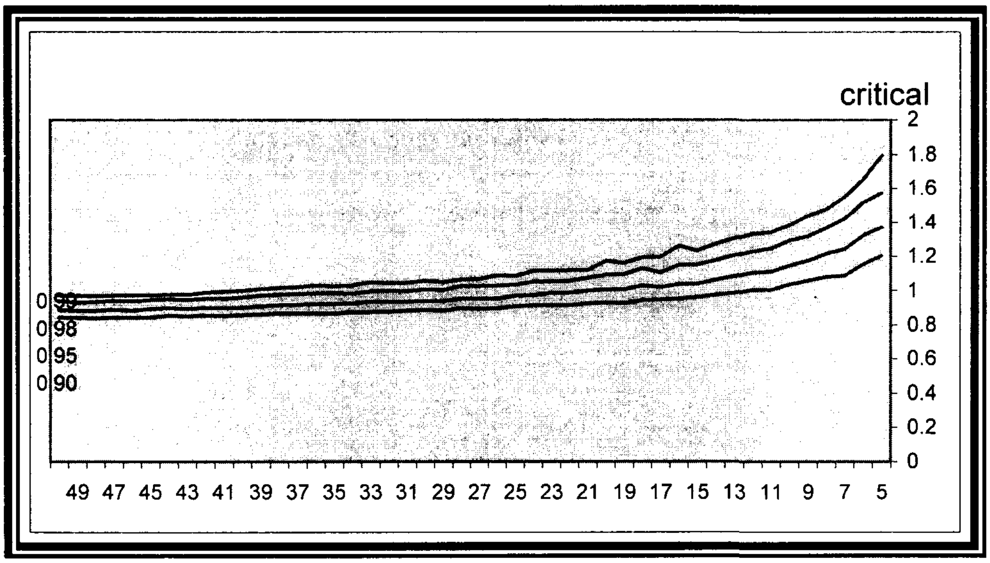

Figure 1. Relation between critical values and sample size for complete data

3.2. Testing Against NBARFR Class

Another problem in life testing is to test H_0 against $H_1^{(2)}$: F is NBARFR and not exponential.

Using Theorem 4., we propose the following measure of departure from H_0

$$\delta_{r+2}^{(2)} = r!\mu - (f(0))^{r+1} \frac{\mu_{(r+2)}}{(r+1)(r+2)}.$$

which is estimated by $\hat{\delta}_{r+2}^{(2)}$ where

$$\hat{\delta}_{r+2}^{(2)} = r!\sum_{i=1}^{n} \frac{T_i}{n} - \frac{1}{(r+1)(r+2)} \left(\sum_{j=1}^{n} \frac{1}{na_n} K\left(\frac{-T_j}{a_n} \right) \right)^{r+1} \sum_{i=1}^{n} \frac{T_i^{r+2}}{n}$$

$$= \frac{(r+1)!}{n(n-1)(n-2)\ldots(n-r)} \sum_{i=1}^{n} r!T_{i_1} - \frac{T_{i_1}^{r+2}}{(r+1)(r+2)} \prod_{z=1}^{r+1} \frac{1}{a} K\left(\frac{-T_{i_z}}{a} \right),$$

where summation extends over all $1 \le i_1 \le \ldots \le i_{r+2} \ne n$.
If we choose

$$\phi(T_{i_1}, T_{i_z}) = r!T_{i_1} - \frac{T_{i_1}^{r+1}}{(r+1)(r+2)} \prod_{z=1}^{r+1} \frac{1}{a_n} K\left(\frac{-T_{i_z}}{a_n} \right),$$

then $\hat{\delta}_{r+2}^{(2)}$ equivalents U-statistic cf Lee(1989). As previous in the case of dealing with NBURFR class the following theorem can be proved.

Theorem 6. As $n \to \infty$ $\sqrt{n}\left(\hat{\delta}_{r+2}^{(2)} - \delta_{r+2}^{(2)}\right)$ is asymptotically normal with 0 mean and variance given by

$$\sigma^2 = Var\left\{ r!(T_{i_1} + \mu) - \frac{T_{i_1}^{r+2} + (f(0))^{r+1} \mu_{(r+2)}}{(r+1)(r+2)} \right\},$$

under H_0, the value of $\delta_{r+2}^{(2)} = 0$ and the variance is

$$\sigma_0^2 = \left\{ 2(r!)^2 - r! + \frac{(2r+4)! - ((r+2)!)^2}{(r+1)^2 (r+2)^2} - 2\frac{r!(r+3)! + r!(r+2)!}{(r+1)(r+2)} \right\}$$

when r=0 the statistic $\hat{\delta}_{r+2}^{(2)}$ is equivalent to $\hat{\delta}^{(1)}$.

3.3. Asymptotic Relative Efficiencies And Powers

We will discuss in this section the powers and asymptotic efficiencies of the two tests which are the same when r=0. Since above tests are new, so we will compare our tests to smaller classes and choose the NBU (Ahmed (1994)) , NBUFR (Hendi et al (2000)) and also expected departure tests for the same classes of Mahmoud and Abdul Alim(2002). We choose the following alternatives:

(i) F_1 Linear failure rate family : $\overline{F}(t) = \exp\left(-t - \theta t^2 / 2\right)$, t > 0, $\theta \geq 0$

(ii) F_2 Makeham family $\overline{F}(t) = e^{-t-\theta(t+e^{-t}-1)}$

(iii) F_3 Weibull family $\overline{F}(t) = e^{-t^{\theta}}$, t>0, $\theta \geq 0$

(iv) F_4 Gamma family $\overline{F}(t) = \int_t^{\infty} e^{-u} u^{\theta-1} du / \Gamma(\theta)$, t > 0, $\theta \geq 0$

(V) F_5 Pareto family $\overline{F}(t) = (1-\theta t)^{1/\theta}$

Note that H_0 is attained at $\theta = 1$, in (iii) and (v), is attained at $\theta = 0$ in (i) and (ii), and is attained when $\theta \rightarrow 0$ in (v).

Direct calculations of the asymptotic efficiencies of our tests here compared with NBU (Ahmed(1994)), NBUFR (Hendi et al (2000)) and NBARFR tests (Mahmoud and Abdul Alim (2002)) is in table 4.1. For the previous alternatives, the powers for the proposed tests are computed as in table 4.2 using simulated number of sample 5000 for sample sizes 10,20 and 30 and θ values 2,3 and 4.

Table 4.2 Asymptotic efficiency of our two tests when r=0 relative to $\hat{\delta F}_n^{(1)}$ Ahmed (1994), $\hat{\delta F}_n^{(2)}$, Hendi et al (2000), $\hat{\delta F}_n^{(3)}$ Mahmoud and Abdul Alim (2002)

Statistics	F_1	F_2	F_3	F_4	F_5
$\hat{\delta F}_n^{(1)}$	0.8056	0.2854	....	....	....
$\hat{\delta F}_n^{(2)}$	0.433	0.289	0.1880	....	....
$\hat{\delta F}_n^{(3)}$	1.2990	0.5774	0.4330	0.9699	0.5196
$\hat{\delta F}_n$	1.414	.53	1.006	.35	1.414
$e(\hat{\delta F}_n, \hat{\delta F}_n^{(1)})$	1.755	1.857	....	....	....
$e(\hat{\delta F}_n, \hat{\delta F}_n^{(2)})$	3.266	1.834	5.351	....	....
$e(\hat{\delta F}_n, \hat{\delta F}_n^{(3)})$	1.089	0.918	2.323	2.721	0.361

From Table 4.1, it is clear that our test performs well for F1,F3 and F5 for all previous tests and the efficiency of our test is less with respect to $\hat{\delta F}_n^{(3)}$ for F4 family.

Table 4.2 Powers of $\hat{\delta F}_n$ test (r=0)

N	θ	F_1	F_3	F_5
10	2.000	1.000	1.000	1.000
20		1.000	1.000	1.000
30		1.000	.999	1.000
10	3.000	1.000	1.000	1.000
20		1.000	1.000	1.000
30		1.000	1.000	1.000
10	4.000	1.000	1.000	1.000
20		1.000	1.000	1.000
30		1.000	1.000	1.000

It is clear from this table that our test satisfies a very good powers.

4. HYPOTHESES TESTING PROBLEMS FOR CENSORED DATA

In this Section, a test statistic is proposed to test H_0 versus $H_1^{(i)c}$:, $i = 1,2$ with randomly right censored samples. In the censoring model, instead of dealing with $X_1, X_2, X_3, ...X_n$, we observe the pair (Z_i, δ_i), i=1,2,3,...,n, where $Z_i = \min(X_i, Y_i)$ and $\delta_i = 1$ if $Z_i = X_i$ and $\delta_i = 0$ if $Z_i = Y_i$, where $X_1, X_2, X_3, ...X_n$ denote their true life time from a distribution F and $Y_1, Y_2, Y_3, ...Y_n$ be i.i.d according to censored distribution G. Also we assume X's and Y's are independent. Let $Z_{(0)} = 0 \le Z_{(1)} \le Z_{(2)} \le Z_{(3)} ... \le Z_{(n)}$ denote the ordered Z's and $\delta_{(i)}$ is the δ_i corresponding to $Z_{(i)}$, respectively. In the case of the censored data, (Z_i, δ_i), i=1,2,3,...,n, we will use Kaplan and Meier (1958) estimator,

$$\hat{\bar{F}}_n(X) = 1 - \hat{F}_n(X) = \prod_{(i < Z_{(i)} \le X)} \left[\frac{n-i}{n-i+1}\right]^{\delta_{(i)}}, X \in [0, Z_{(n)}]$$

and the kernel estimator of the hazard rate in the censored case

$$\hat{r}_n(t) = \frac{1}{2R_k} \sum_{i=1}^{n} \left[\frac{\delta_{(i)}}{n-i+1} K\left(\frac{t - Z_{(i)}}{2R_k}\right)\right] \text{ Tanner (1983),}$$

where

R_k distance between point t and its k th nearest failure point

K(.) a function of bounded variation with compact support on the interval [-1,1].

Then the proposed test statistics are

$\hat{\delta}_{F_n}^{c(1)}$

$$= \sum_{i=1}^{n} Z_{(i)}^{r} \prod_{j=1}^{i-1} \left(C_{(j)}\right)^{\delta_{(j)}} \left(Z_{(i)} - Z_{(i-1)}\right) - \sum_{i=1}^{n} Z_{(i)}^{r} \hat{r}(0) \sum_{j=i}^{n} \prod_{k=1}^{j-1} \left(C_{(k)}\right)^{\delta_{(k)}} \left(Z_{(j)} - Z_{(j-1)}\right)\left(Z_{(i)} - Z_{(i-1)}\right)$$

$$= \sum_{i=1}^{n} Z_{(i)}^{r} \left(Z_{(i)} - Z_{(i-1)}\right) \left[\prod_{j=1}^{i-1} \left(C_{(j)}\right)^{\delta_{(j)}} - \hat{r}(0) \sum_{j=i}^{n} \prod_{k=1}^{j-1} \left(C_{(k)}\right)^{\delta_{(k)}} \left(Z_{(j)} - Z_{(j-1)}\right) \right],$$

and

$\hat{\delta}_{F_n}^{c(2)}$

$$= \sum_{i=1}^{n} Z_{(i)}^{r} \left[\sum_{j=1}^{n} \prod_{k=1}^{j-1} \left(C_{(k)}\right)^{\delta_{(k)}} \left(Z_{(j)} - Z_{(j-1)}\right)\left(Z_{(i)} - Z_{(i-1)}\right) e^{-Z_{(i)}\hat{r}(0)} - \sum_{j=i}^{n} \prod_{k=1}^{j-1} \left(C_{(k)}\right)^{\delta_{(k)}} \left(Z_{(j)} - Z_{(j-1)}\right) \right],$$

where

$C_k = (n-k)/(n-k+1)$ and $dF_n(Z_j) = \bar{F}_n(Z_{j-1}) - \bar{F}_n(Z_j)$.

Using IMSL sub-routines HAZARD & HAZST, the percentiles of the two tests in this section are computed when r=0 which give the same values in this special case. These percentiles are given in appendix but here we present the trends of percentiles with respect to sample size from Figure 2.

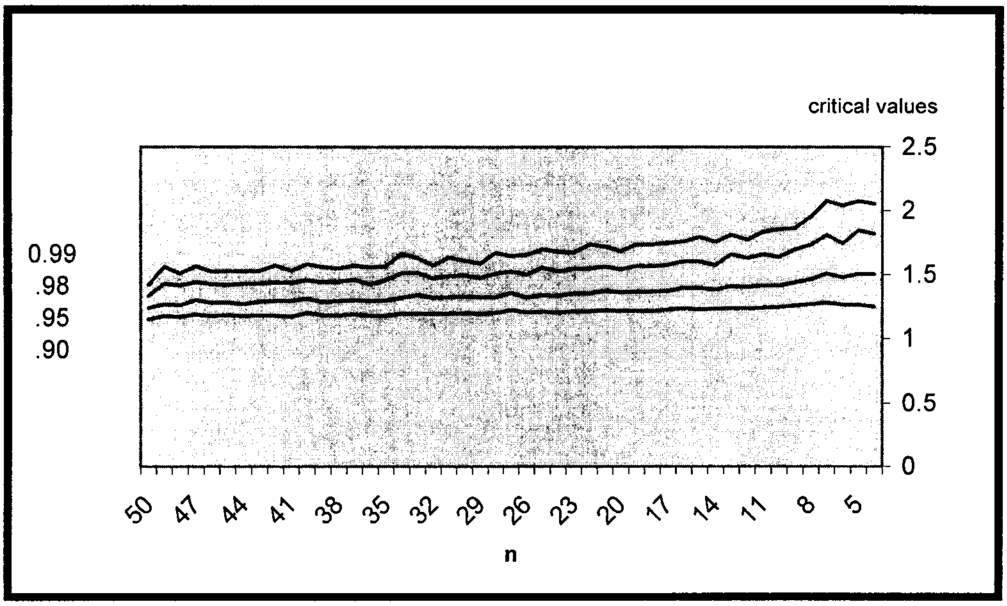

Figure 2. Relation between critical values and sample size for censored data

5. APPLICATIONS

(1) Consider the data in Abouammoh et al. (1994) . these data represent set of 40 patients suffering from blood cancer (Leukemia) from one of Ministry of Health Hospitals in Saudi Arabia and the ordeıcd values are :

115	181	255	418	441	461	516	739	743	789
807	865	924	983	1024	1062	1063	1165	1191	1222
1222	1251	1277	1290	1375	1369	1408	1455	1478	1549
1578	1599	1603	1605	1696	1735	1799	1815	1852	1599

It was found that the test statistic for the data set, by formula (3.2) is given by $\hat{\delta}^{(1)}$=1145.025 which exceeds the critical value of table 2.1. Then we reject the null hypothesis of exponentiality.

(2) In an experiment at Florida state university to study the effect of methyl mercury poising on the life lengths of fish, goldfish were subjected to various dosages of methyl mercury (see, Kochar (1985)). At one dosage level, the ordered times to death in days were

42	43	51	61	66	69	71	81	82	82

We use these data to illustrate NBURFR (NBARFR) tests based on (3.2) statistic. The calculated value of $\hat{\delta}^{(1)}$=64.8 is greater than the critical value of table 3.1. Then we accept the alternative hypothesis of NBURFR (NBARFR) property.

(3) Consider the data in susarla and vanryzin (1978). These data represent 81 survival times of patients of melanoma. Of them 46 represent whole life times (non-censored data) and the ordered values are :

13	14	19	19	20	21	23	23	25	26	26	27
27	31	32	34	34	37	38	38	40	46	50	53
54	57	58	59	60	65	65	66	70	85	90	98
102	103	110	118	124	130	136	138	141	234		

The ordered censored observations are:

16	21	44	50	55	67	73	76	80	81	86	93
100	108	114	120	124	125	129	130	132	134	140	147
148	151	152	152	158	181	190	193	194	213	215	

A simple computer program is written to calculate the value of statistic for these data (censored) and the value we get leads to a 102.176 greater than the critical value in table 3.2 at 95% upper percentile. Then we accept alternative which states that the set of data have NBURFR (NBARFR) property.

Appendix

Table 3.1 Percentiles for $\hat{\delta}^{(1)}_{(r+2)}$ and $\hat{\delta}^{(2)}_{(r+2)}$ tests (r=0)

N	.01	.05	.10	.90	.95	.98	.99
5	.2211	.3366	.4017	1.2033	1.3695	1.5716	1.7911
6	.2493	.3543	.4178	1.1532	1.3236	1.5168	1.6517
7	.2614	.3684	.4297	1.0888	1.2435	1.4228	1.5497
8	.2747	.3888	.4455	1.0803	1.2159	1.3665	1.4762
9	.2960	.4042	.4633	1.0605	1.1747	1.3194	1.4387
10	.3195	.4192	.4706	1.0357	1.1469	1.2954	1.3824
11	.3232	.4259	.4800	1.0029	1.1107	1.2483	1.3420
12	.3459	.4402	.4909	1.0027	1.1045	1.2292	1.3316
13	.3607	.4522	.5011	.9850	1.0835	1.2072	1.3066
14	.3477	.4495	.4978	.9738	1.0629	1.1772	1.2730
15	.3771	.4591	.5077	.9600	1.0385	1.1518	1.2318
16	.3854	.4691	.5157	.9507	1.0354	1.1493	1.2630
17	.3802	.4793	.5238	.9449	1.0187	1.1044	1.1964
18	.3887	.4724	.5238	.9414	1.0290	1.1302	1.1927
19	.3954	.4809	.5267	.9257	1.0063	1.0963	1.1612
20	.4087	.4837	.5266	.9283	1.0031	1.0912	1.1725
21	.4074	.4899	.5337	.9223	.9989	1.0743	1.1194
22	.4206	.4964	.5390	.9135	.9868	1.0590	1.1173
23	.4268	.5057	.5448	.9124	.9844	1.0546	1.1184
24	.4354	.5075	.5475	.9134	.9718	1.0527	1.1148
25	.4358	.5120	.5551	.9055	.9679	1.0337	1.0859
26	.4475	.5179	.5580	.8946	.9515	1.0289	1.0908
27	.4402	.5159	.5575	.8907	.9529	1.0259	1.0694
28	.4455	.5216	.5581	.8952	.9539	1.0254	1.0666
29	.4534	.5257	.5623	.8822	.9354	1.0017	1.0501
30	.4512	.5256	.5615	.8858	.9379	1.0061	1.0585
31	.4553	.5300	.5691	.8823	.9340	.9952	1.0456
32	.4593	.5272	.5633	.8769	.9332	.9988	1.0440
33	.4633	.5301	.5650	.8752	.9346	.9950	1.0492
34	.4592	.5318	.5672	.8742	.9250	.9810	1.0299
35	.4741	.5352	.5734	.8656	.9217	.9855	1.0236
36	.4741	.5377	.5742	.8672	.9212	.9834	1.0302
37	.4746	.5377	.5719	.8649	.9159	.9803	1.0179
38	.4795	.5429	.5772	.8661	.9130	.9749	1.0154
39	.4855	.5465	.5795	.8614	.9072	.9691	1.0068
40	.4853	.5494	.5835	.8568	.8983	.9588	.9985

41	.4824	.5520	.5826	.8519	.8961	.9521	.9926
42	.4946	.5519	.5837	.8560	.8994	.9528	.9868
43	.4934	.5519	.5840	.8506	.8919	.9435	.9740
44	.4891	.5517	.5846	.8528	.8974	.9486	.9761
45	.5080	.5558	.5867	.8452	.8919	.9429	.9706
46	.4999	.5586	.5874	.8446	.8827	.9365	.9720
47	.5093	.5590	.5905	.8437	.8866	.9404	.9697
48	.5011	.5578	.5877	.8392	.8810	.9318	.9662
49	.4987	.5626	.5924	.8426	.8815	.9285	.9698
50	.5078	.5665	.5942	.8445	.8852	.9373	.9705

Table 3.2 Critical values of $\hat{\delta}^{c(1)}_{F_n}$ statistic when (r=0)

n	.01	.05	.10	..90	.95 .
5	.1598	.2611	.3274	1.2497	1.5022
6	.1924	.2880	.3578	1.2656	1.5041
7	.2274	.3260	.3874	1.2647	1.4768
8	.2450	.3453	.4178	1.2803	1.5060
9	.2643	.3790	.4371	1.2670	1.4620
10	.3003	.4061	.4632	1.2588	1.4356
11	.3173	.4191	.4811	1.2439	1.4120
12	.3301	.4338	.4940	1.2392	1.4144
13	.3348	.4435	.5018	1.2351	1.4016
14	.3529	.4513	.5178	1.2379	1.4066
15	.3749	.4712	.5325	1.2336	1.3815
16	.3637	.4832	.5412	1.2274	1.3925
17	.4031	.4970	.5588	1.2367	1.3961
18	.3957	.4932	.5568	1.2243	1.3719
19	.4154	.5077	.5685	1.2159	1.3654
20	.4350	.5177	.5798	1.2171	1.3639
21	.4320	.5268	.5866	1.2136	1.3605
22	.4394	.5293	.5907	1.2211	1.3722
23	.4526	.5423	.6000	1.2130	1.3530
24	.4610	.5468	.6055	1.2116	1.3506
25	.4708	.5621	.6166	1.2013	1.3320
26	.4810	.5687	.6217	1.2098	1.3390
27	.4783	.5657	.6232	1.2039	1.3211
28	.4776	.5726	.6247	1.2213	1.3562
29	.4974	.5834	.6376	1.2005	1.3233
30	.4983	.5889	.6415	1.1920	1.3232
31	.4969	.5859	.6394	1.1982	1.3279
32	.5110	.5947	.6470	1.1952	1.3238
33	.5062	.5974	.6484	1.1954	1.3178
34	.5086	.5983	.6490	1.1954	1.3372
35	.5283	.6069	.6590	1.1948	1.3189
36	.5300	.6102	.6598	1.1776	1.2968
37	.5299	.6080	.6575	1.1780	1.2931

38	.5264	.6124	.6712	1.1904	1.3014
39	.5381	.6180	.6713	1.1786	1.2916
40	.5383	.6239	.6729	1.1808	1.2864
41	.5351	.6339	.6765	1.1988	1.3111
42	.5439	.6263	.6789	1.1705	1.2966
43	.5520	.6311	.6799	1.1757	1.2916
44	.5477	.6335	.6867	1.1800	1.2871
45	.5604	.6391	.6859	1.1749	1.2664
46	.5552	.6394	.6883	1.1809	1.2795
47	.5656	.6442	.6917	1.1745	1.2770
48	.5765	.6514	.6969	1.1867	1.3012
49	.5668	.6429	.6916	1.1693	1.2602
50	.5795	.6522	.6948	1.1739	1.2672
81	.6404	.7103	.7512	1.1492	1.2386

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