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Moment Inequalities for Testing New Renewal Better Than Used and Renewal New Better Than Used Classes of Life Distributions

M. A. W. Mahmoud*
Mathematics Department, Faculty of Science
Al-Azhar University, Nasr city, 11884, Cairo, EGYPT

S. M. EL-Arishy and L. S. Diab
Math. Department, Faculty of Science, AL-Azhar University
(Girls branch) Nasr-city, 11884.Egypt

Abstract. Based on moments inequalities new testing procedures are derived for testing exponentiality against new renewal better than used (NRBU) and renewal new better than used (RNBU). These classes play an important role in formulating repair or replacement policies. The asymptotic Pitman efficiency of (NRBU) and (RNBU) testes are studied. Selected critical values are tabulated for sample sizes n=5(1) 50. The power estimates for some commonly used life distributions in reliability are also calculated. Some set of real data is used as an example to elucidate the use of the proposed test statistic for practical reliability analysis. The problem in case of right-censored data is also handled.

Key Words : *moments inequalities, new renewal better than used (NRBU), renewal new better than used (RNBU), U-test, reliability, life testing, exponential distribution, hypothesis testing, monte carlo methods*

1. INTRODUCTION

Many reliability applications, various classes of life distributions, and their duals have been introduced to describe several types of deterioration or improvement that accompany aging. Aging distributions are divided into many nonparametric classes according to the type of aging that take place of the vastly used classes over the past several decades. Among the well known families are classes of increasing failure rate (IFR), increasing failure rate average(IFRA),new better than used(NBU), new better than used in expectation(NBUE), harmonic new better than used in expectation(HNBUE), The

* Corresponding Author
E-mail address: mawmahmoud11@hotmail.com

implications among these classes as easily seen, cf. Barlow and Prochan (1981) are as follow:

$$\text{IFR} \Rightarrow \text{IFRA} \Rightarrow \text{NBU} \Rightarrow \text{NBUE} \Rightarrow \text{HNBUE}$$

Probabilistic properties of the above four classes of aging distributions as well as many others have been extensively studied in the literature. For the IFR class, see Barlow, Marshall (1964a,b) and Barlow and Marshall (1965) for the basic early results. On the other hand, testing exponential against IFR, was first discussed by Proschan and Pyke (1967) followed by Barlow and Prochan (1969), Bickel and Doksum (1969), Bickel (1969), Ahmad (1975), Matthews, etall (1985), Epps and Pulley (1986),and Ebrahim(2000),among others.For testing exponential against IFRA, we refer to Ahmad (1974), Deshpande (1983), Link (1989), Aly (1989) and Ahmad (1994).For NBU and NBUE classes, see Bryson and Siddiqui (1969) and Marshall and Proshan(1972) for early fundamental results. While testing exponential against NBU was developed first by Hollander and Proschan (1972), then by Koul (1977), Alam and Basu (1990), Ahmad (1994) and Ebrahim (2000), several others followed. Testing exponential against NBUE was developed first by Hollander and Proschan (1975), then by Koul and Susarla (1980), Borges, etall (1984) and Ebrahim (2000), followed by others. Finally, for the HNBUE see Rolski (1975), Basu and Kirmani (1986) and Basu and Bhattacharjee (1984) for basic results. Whereas testing exponential against HNBUE was done by Klefsjoe (1983), Basu and Ebrahimi (1985) and Bergman, Klefsjoe (1987) and Handietall (1998) among others. Different authors have introduced other relevant aging criteria.

Recently, Ahmed (2001) studied the testing the exponentility against IFR, NBU, NBUE and HNBE based on the moments inequalities.

Let X be a nonnegative with distribution function F and survival function $\overline{F} = 1 - F$. Assume also that X is absolutely continuous with probability density function f and has mean μ and variance σ^2.

Consider a device with life length X and life distribution F . The device is replaced instantly upon failure by a sequence of mutually independent devices. These devices are independent of the first unit and identically distributed with the same life distribution F . When the renewal of the system is continued in definitely, the stationary life distribution of a device in operation at time t is $W_F(t) = \frac{1}{\mu}\int_0^t \overline{F}(u)du, \; t \geq 0.$

And the survival of a device in operation at any time $t \geq 0$ is given by the stationary renewal distribution,

$$\overline{W_F}(t) = \mu_F^{-1}\int_t^{\infty} \overline{F}(u)du, \qquad \text{for } 0 \leq t \leq \infty,$$

where $\mu = \mu_F = \int_0^{\infty} \overline{F}(x)dx < \infty$, is the mean of life distribution F .

Abouammoh etal (2000) introduced the NRBU, RNBU, NRBUE, HNRBUE classes of life distributions and they studied the relation between them. Abouammoh and Khalique (1998) are investigated a test statistic for testing exponentiality versus NRBU based on total time on test(TTT)-transform empirically. The asymptotic distribution of this test is established theoretically by El-arishy and Diab (1998) .They showed that the distribution of the test statistic is asymptotically normal with mean zero and variance 1/48. Hendi and Abouammoh (2001) are investigated the two test statistics for testing exponentiality versus (NBRUE),(HNBRUE) classes of life distribution based on U-statistic.

Mahmoud etal (2002, a,b) investigated the two test statistics for testing exponentiality versus NRBU and RNBU classes of life distribution based on U-statistic.

In fact , stochastic comparison between the random variable T with distribution F and its renewal random variable T_{W_F} with life distribution W_F , for which $W_F(0-)=0$, density function w_F and renewal survival function $\overline{W_F}$, leads to the following Definition.

Definition 1.1. A random variable T or its distribution F is said to have new renewal better than used, denoted by (NRBU) property, if $T_t \leq^{st} T_{W_F}$, where T_t is the conditional variable of T given t with distribution, $\overline{F_y}(t)=p(T\leq t\mid T\geq y)$

This definition means that, T is NRBU, if

$$\overline{F_y}(t)\leq\overline{W_F}(t), \qquad \forall y,t\geq 0 \qquad (1.1)$$

Relation (1.1) can have the form

$$\overline{F}(y+t)\leq\overline{F}(y)\overline{W_F}(t), \qquad \forall y,t\geq 0 \qquad (1.2)$$

Integrating both sides of relation (1.2) w.r.t y over $[x,\infty]$, gives

$$\overline{W_F}(x+t)\leq\overline{W_F}(x)\overline{W_F}(t), \qquad (1.3)$$

where $\overline{W_F}(x+t)=\dfrac{1}{\mu}\int_{x+t}^{\infty}\overline{F}(y)dy$, and $\overline{W_F}(x)=\dfrac{1}{\mu}\int_{x}^{\infty}\overline{F}(y)dy$

i.e . the renewal distribution is NBU and is denoted by RNBU

The corresponding dual classes of life distribution are new renewal worse than used and renewal new worse than used parent property, denoted by NRWU and RNWU are defined by reversing the inequality sign of (1.1) and (1.3).

The first purpose of the current investigation is to provide the moments inequalities of the above two classes that will generally assert that if $\mu<\infty$ then all moments would exist for any member of any of these two classes.

In this spirit, the moment inequalities developed in section 2, are used to construct test statistics for the two problems. These test statistics are, for the first time, based on sample moments of the aging distributions. They are simple to devise, calculate, study and have exceptionally privet the asymptotic Pitman efficiency (APE), In section 3, Monte Carlo null distribution critical points are simulated for sample sizes n=5(1) 50.The power estimates of the second test are also calculated. An example using real data representing 40 patients suffering from blood cancer from one of ministry of Health hospitals in Saudi

Arabia is given as an application. In section 4, we consider the problem of dealing with right-censored data and selected critical values are tabulated. An example using data from Susarla and Vanryzin (1978) is also used as an application in medical sciences in both complete and incomplete data.

2. MOMENT INEQUALITIES

2.1 Moments Inequalities For NRBU, RNBU Alternatives

The first result provides moments inequality for the NRBU distributions. In this, as well as subsequent results all moments are assumed to exist and are finite.

Theorem 2.1. If F is NRBU, then for all integers $r \geq 0, s \geq 0$

$$\frac{\mu_{(r+1)}\mu_{(s+2)}}{(r+1)(s+1)(s+2)} \geq \mu \sum_{i=0}^{r} (-1)^i \frac{\binom{r}{i}}{(i+s+1)} \frac{\mu_{(r+s+2)}}{(r+s+2)}. \tag{2.1}$$

Proof. Since F is NRBU, then $\overline{F}(x+t) \leq \overline{F}(x)\overline{W}(t)$, thus for all integers $r \geq 0$, $s \geq 0$,

$$\int_0^\infty \int_0^\infty x^r t^s \overline{F}(x+t)dxdt \leq \int_0^\infty \int_0^\infty x^r t^s \overline{F}(x)\overline{W}(t)dxdt .$$

It is easily to show that

$$\int_0^\infty x^r \overline{F}(x)dx = E\int_0^\infty x^r I(X > x)dx = \frac{1}{r+1}E(X^{r+1}) = \frac{\mu_{(r+1)}}{r+1}. \tag{2.2}$$

$$\int_0^\infty x^r \overline{W}(x)dx = \frac{1}{\mu}\int_0^\infty \frac{x^{r+1}}{r+1}\overline{F}(x)dx = \frac{1}{r+1}E(X^{r+2}) = \frac{1}{\mu}\left\{\frac{\mu_{(r+2)}}{(r+1)(r+2)}\right\} \tag{2.3}$$

Using (2.2) and (2.3) yields the result (2.1).

When $r = 1, s = 0$ $\quad \dfrac{\mu_{(2)}^2}{4} \geq \dfrac{\mu\mu_{(3)}}{6}.$ (2.4)

Theorem 2.2. If F is RNBU, then for all integers $r \geq 0, s \geq 0$

$$\frac{\mu_{(r+2)}\mu_{(s+2)}}{(r+1)(r+2)(s+1)(s+2)} \geq \mu \sum_{i=0}^{r} (-1)^i \frac{\binom{r}{i}}{(i+s+1)} \frac{\mu_{(r+s+3)}}{(r+s+2)(r+s+3)}, \tag{2.5}$$

Proof. Since F is RNBU, then $\overline{W_F}(x+t) \le \overline{W_F}(x)\overline{W_F}(t)$, thus for all integers $r \ge 0$, $s \ge 0$,

$$\int_0^\infty \int_0^\infty x^r t^s \overline{W}(x+t)dxdt \le \int_0^\infty \int_0^\infty x^r t^s \overline{W}(x)\overline{W}(t)dxdt,$$

By using Theorem 2.1 the result follows directly.

When $r = 0$, $s = 0$, we obtain the same form for special case of NRBU in this case too which that.

$$\frac{\mu_{(2)}^2}{4} \ge \frac{\mu\mu_{(3)}}{6}. \tag{2.6}$$

2.2 On Testing Exponentially Against NRBU Alternative

One of the oldest problems in life testing is to test $H_0 : F$ is exponential against $H_1 : F$ is NRBU. Using Theorem 2.1 with $r \ge 0, s \ge 0$

We may use as a measure of departure.

$$\delta_1 = \frac{\mu_{(r+1)}\mu_{(s+2)}}{(r+1)(s+1)(s+2)} - \mu\sum_{i=0}^{r}(-1)^i \frac{\binom{r}{i}}{(i+s+1)}\frac{\mu_{(r+s+2)}}{(r+s+2)}, \tag{2.7}$$

To make the test scale invariant we take $\Delta_1 = \dfrac{\delta_1}{\mu^2_{(r+s+3)}}$, which is estimated by

$$\hat{\Delta}_1 = \frac{\hat{\delta}_1}{\left[\overline{X}\right]^{r+s+3}}, \text{ where} \tag{2.8}$$

$$\hat{\delta}_1 = \frac{1}{(r+1)(s+1)(s+2)}\left\{\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} X_i^{r+1}X_j^{s+2}\right\} - \left\{\frac{1}{n^2}\sum_{i=1}^{n} X_i \sum_{i=0}^{r}(-1)^i \frac{r!}{i!(r-i)!(i+s+1)} \frac{1}{(r+s+2)}\sum_{j=1}^{n} X_j^{r+s+2}\right\}, \tag{2.9}$$

At $r = 1$, $s = 0$,

$$\delta_1 = \frac{1}{24n^2}\sum_{i}\sum_{<\, j}\left[6X_i^2X_j^2 - 4X_iX_j^3\right], \tag{2.10}$$

Note that $\qquad \varphi\,(X_1, X_2) = 6X_1^2X_2^2 - 4X_1X_2^3, \qquad (2.11)$

Then $\hat{\delta}_1$ is a classical U-statistic.

We state and prove the following Theorem.

Theorem 2.3. (i) As $n \to \infty$, $n^{\frac{1}{2}}(\hat{\Delta}_1 - \Delta_1)$ is asymptotically normal with mean 0 and variance,

$$\sigma^2 = \text{var}\left\{\frac{1}{2}X^2\int_0^\infty u^2 dF(u) - \frac{1}{3}X\int_0^\infty u^3 dF(u)\right\},$$

(ii) Under H_0, the variance reduces to,

$$\sigma_0^2 = Var\left[X^2 - X - \frac{1}{6}X^3\right] = 2.$$

Proof. (i), (ii) follow from the standard Theorem of U-statistic cf. Lee (1989),yields

$$\sigma^2 = \text{var}\left\{\sum_{i=1}^{2}\eta_i\right\},$$

where $\eta_i = E(\phi(X_1, X_2 \mid X_i))$.

$$\eta_1 = \frac{1}{4}X^2\int_0^\infty u^2 dF(u) - \frac{1}{6}X\int_0^\infty u^3 dF(u),\quad \eta_2 = \frac{1}{4}X^2\int_0^\infty u^2 dF(u) - \frac{1}{6}X^3\int_0^\infty u\, dF(u)$$

and by direct calculations respectively the result follows.

2.3 On Testing Exponentially Against RNBU Alternative

Another problem is to test $H_0 : F$ is exponential against $H_1 : F$ is RNBU. Using Theorem 2.2 with $r \geq 0, s \geq 0$ we can propose the following measure of departure.

$$\delta_2 = \frac{\mu_{(r+2)}\mu_{(s+2)}}{(r+1)(r+2)(s+1)(s+2)} - \mu\sum_{i=0}^{r}(-1)^i \frac{\binom{r}{i}}{(i+s+1)}\frac{\mu_{(r+s+3)}}{(r+s+2)(r+s+3)}. \quad (2.12)$$

To make the test scale invariant we take $\Delta_2 = \dfrac{\delta_2}{\mu^2_{(r+s+4)}}$, which is estimated

by $\hat{\Delta}_2 = \dfrac{\hat{\delta}_2}{\left[\overline{X}\right]^{r+s+4}}$, where

$$\hat{\delta}_2 = \frac{1}{(r+1)(r+2)(s+1)(s+2)}\left\{\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}X_i^{r+2}X_j^{s+2}\right\}$$

$$-\left\{\frac{1}{n^2}\sum_{i=1}^{n}X_i\sum_{i=0}^{r}(-1)^i\frac{r!}{i!(r-i)!(i+s+1)}\frac{1}{(r+s+2)(r+s+3)}\sum_{j=1}^{n}X_j^{r+s+3}\right\}, \quad (2.13)$$

When $r = 0,\ s = 0$, we given the equivalent form of $\hat{\delta}_1$, i.e $\hat{\delta}_1 = \hat{\delta}_2$

$$\delta_2 = \frac{1}{24n^2} \sum_{i} \sum_{< j} \left[6X_i^2 X_j^2 - 4X_i X_j^3\right]. \tag{2.14}$$

The following result is proving in a fashion similar to that used in Theorem 2.3 and hence it is only stated.

Theorem 2.4. (i) As $n \to \infty$, $n^{\frac{1}{2}}(\hat{\Delta}_2 - \Delta_2)$ is asymptotically normal with mean 0 and variance,

$$\sigma^2 = \text{var}\left\{\frac{1}{2} X^2 \int_0^\infty v^2 dF(v) - \frac{1}{3} X \int_0^\infty v^3 F(v)\right\},$$

(ii)Under H_0, the variance reduces to,

$$\sigma_0^2 = Var\left[X^2 - X - \frac{1}{6} X^3\right] = 2.$$

It is clear from (2.9) and (2.13) that the tests based on the moment inequalities contains two variables which are simpler than those given by Mahmoud etal (2002, a, b) .

2.4 The Pitman Asymptotic Efficiency

In this section, we shall calculate the Pitman asymptotic efficiencies of the above NRBU, RNBU tests.

Note that the Pitman asymptotic efficiency of $\Delta_1(\theta)$ is given by:

$$\text{PAE}(\Delta(\theta)) = \frac{\left\{\left|\frac{\partial}{\partial \theta} \Delta(\theta)\right|_{\theta \to \theta_0}\right\}}{\sigma_0}, \tag{2.15}$$

where, $\mu_{(r+1)}(\theta) = (r+1) \int_0^\infty w^r \bar{F}_\theta(w) dw$.

Using the most commonly used alternatives:

(i) Weibull family: $\overline{F_1}(x) = \exp(-x^\theta), x > 0, \theta \geq 1$.

(ii) Linear failure rate family: $\overline{F_2}(x) = \exp(-x - \theta x^2 / 2), x > 0, \theta \geq 0$.

(iii) Gamma family: $\overline{F_4}(x) = \int_x^\infty e^{-u} u^{\theta - 1} du / \Gamma(\theta), x > 0, \theta \geq 0$.

Note that H_0 (the exponential) is attained at $\theta = 1$, in (i) and (iii), and is attained at $\theta = 0$ in (ii).

Direct calculations of the asymptotic efficiencies of the NRBU and RNBU tests are summarized in Table 2.1.

Table 2.1. Asymptotic efficiencies of Δ_1, Δ_2 tests for the four alternatives.

Efficiency	Weibull	Linear failure rate	Gamma
$\hat{\Delta}_i, i=1,2$	.3571	.7143	.1191

3. MONTE CARLO NULL DISTRIBUTION CRITICAL POINTS

In practice, simulated percentiles for small samples are commonly used by applied statisticians and reliability analyses. We have simulated the upper percentile points for 90 %,95% ,98 %,99 %.Table 3.1 gives these percentile points of the statistics $\hat{\Delta}_1$ that given in (2.8) and the calculation are based on 5,000 simulated samples of sizes n=5(1)50.

Table 3.1 Critical values for the upper percentiles of $\hat{\Delta}_1$.

n	%90	%95	%98	%99
5	.2353	.3072	.4206	.4944
6	.2201	.2710	.3600	.4452
7	.2024	.2433	.3047	.3588
8	.1978	.2339	.2938	.3503
9	.1895	.2241	.2706	.3274
10	.1808	.2091	.2582	.2949
11	.1774	.2050	.2443	.2807
12	.1725	.1964	.2337	.2726
13	.1658	.1903	.2239	.2559
14	.1648	.1857	.2167	.2472
15	.1592	.1799	.2098	.2336
16	.1578	.1801	.2089	.2307
17	.1552	.1758	.2010	.2188
18	.1523	.1720	.1972	.2155
19	.1498	.1676	.1905	.2083
20	.1504	.1677	.1896	.2057
21	.1471	.1647	.1875	.2008
22	.1468	.1638	.1822	.1999
23	.1439	.1596	.1819	.1978
24	.1434	.1582	.1763	.1878
25	.1420	.1563	.1720	.1864
26	.1406	.1557	.1727	.1870
27	.1388	.1523	.1680	.1785
28	.1370	.1521	.1700	.1855
29	.1371	.1507	.1674	.1784
30	.1358	.1483	.1620	.1752

31	.1356	.1471	.1637	.1748
32	.1333	.1471	.1643	.1751
33	.1327	.1468	.1616	.1733
34	.1329	.1449	.1597	.1699
35	.1314	.1426	.1579	.1670
36	.1298	.1408	.1576	.1662
37	.1295	.1414	.1559	.1669
38	.1294	.1403	.1556	.1663
39	.1279	.1396	.1537	.1659
40	.1279	.1403	.1510	.1603
41	.1261	.1378	.1521	.1622
42	.1258	.1379	.1505	.1584
43	.1248	.1357	.1477	.1572
44	.1260	.1373	.1505	.1577
45	.1251	.1341	.1481	.1570
46	.1229	.1329	.1451	.1532
47	.1233	.1341	.1472	.1547
48	.1229	.1336	.1450	.1548
49	.1224	.1324	.1444	.1530
50	.1203	.1309	.1420	.1501

It is noticed from Table 3.1 and Figure1 that the critical values are increasing as the confidence level increasing, and decreasing as the sample size increasing.

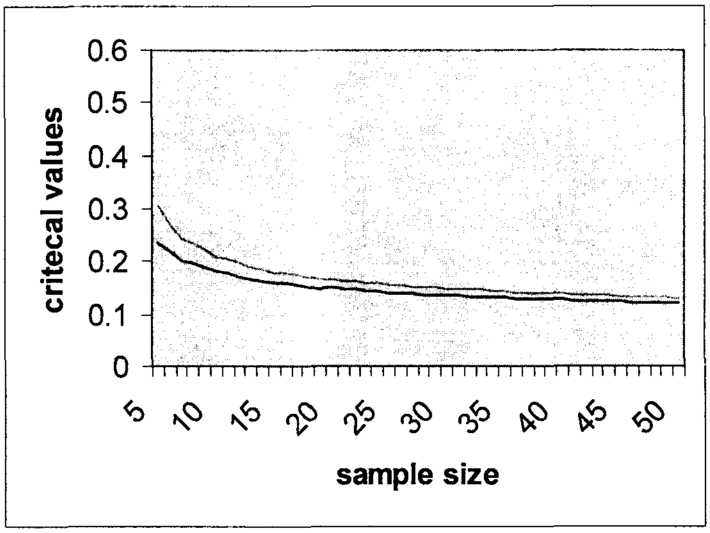

Figure 1. Relation between critical values, sample size and confidence levels.

3.1 The power estimates

In this section, we present an estimation of the power for testing exponentially versus NRBU property using significance level $\alpha = 0.05$ and for commonly used distributions in reliability .These distributions are Weibull , linear failure rate and gamma alternatives .The estimates are based on 5000 replications. The corresponding distribution function are given respectively by:

Table 3.2 below includes the power estimate using $\alpha = 0.05$ with parameter values 0.6, 1.2 and 1.8 at n=10, 20 and 30.

Table 3.2. Power estimates using $\alpha = .05$

Distribution	**Parameter** θ	**Sample size** 10	20	30
F_1	0.6	0.981	0.812	0.736
	1.2	0.968	0.981	0.989
	1.8	0.996	1.000	1.000
F_2	0.6	0.980	0.996	0.997
	1.2	0.987	0.997	0.999
	1.8	0.988	0.999	1.000
$\overline{F}_3$	0.6	0.997	1.000	1.000
	1.2	0.998	1.000	1.000
	1.8	0.997	1.000	1.000

Table 3.2 shows that our test has good power, and the power is getting smaller as NRBU approaches the exponential distribution.

3.2 Application

Consider the data in Abouammoh etal (1994). These data represent 40 patients suffering from blood cancer from one of Ministry of Health Hospitals in Saudi Arabia and the ordered life time (in days) are recorded:

115,181,255,418,441,461,516,739,743,789,807,865,924,983,1024,106,1063,1165,1191, 1222,1222,1251,1277,1290,1357,1369,1408,1455,1478,1549,1578,1578,1599,1603,1605, 1696,1735,1799,1815,1852.

It is found that the test statistic $\hat{\Delta}_2$ for the set of data, by using Equation (2.8) $\hat{\Delta}_2 = .0969$

It is clear from Table 3.1 the computed value of the test statistic $\hat{\Delta}_2$, smaller than the tabulated value, that we reject H_1 which states that the set of data have NRBU (RNBU) property under significant level at 95% upper percentile.

4. ON TESTING EXPONENTIALITY AGAINST NRBU (RNBU) ALTERNATIVES IN THE CENSORED CASE

Test for NRBU RNBU in case of right censored data

In this section a test statistic is proposed to test H_0 versus H_1 with randomly right-censored data. Such a censored data is usually the only information available in a life-testing model or in a clinical study where patients may be lost (censored) before the completion of a study. This experimental situation can formally be modeled as follows. Suppose n objects are put on test, and $X_1, X_2, ..., X_n$ denote their true lifetime .We assume that $X_1, X_2, ..., X_n$ be independent, identically distributed (i.i.d.) according to a continuous life distribution F. Let $Y_1, Y_2, ..., Y_n$ be (i.i.d.) according to a continuous life distribution G. Also we assume that X's and Y's are independent. In the randomly right-censored model, we observe the pairs $(Z_j, \delta_j), j = 1......n$ where $Z_j = \min(X_j, Y_j)$ and

$$\delta_j = \begin{cases} 1 & if \quad Z_j = X_j \quad (j^{th} observn\ is\ uncensored) \\ 0 & if \quad Z_j = Y_j \quad (j^{th} observn\ is\ censored) \end{cases}$$

Let $Z(0) = 0 < Z(1) < Z(2) < ...... < Z(n)$ denote the ordered Z's and $\delta_{(j)}$ is the δ_j corresponding to $Z_{(j)}$ respectively.

Using the censored data $(Z_j, \delta_j), j = 1, ..., n$,Kaplan and Meier (1958) proposed the product limit estimator.

$$\overline{F_n}(X) = 1 - F_n(X)$$
$$= \prod_{[j:Z_{(j)} \le X]} \{(n-j)/(n-j+1)\}^{\delta_{(j)}}, \quad X \in [0, Z_{(n)}] \ . \tag{4.1}$$

Now for testing $H_0 : \Delta_2 = 0$, against $H_1 : \Delta_2 > 0$, using the randomly right censored data, we propose the following test statistic.

$$\Delta_2^C = \frac{1}{\mu_{(r+s+3)}} \left\{ \left\{ \frac{1}{(r+1)} \int_0^\infty x^{r+1} \overline{F}_n(x) dx \right\} \left\{ \frac{1}{(s+1)} \int_0^\infty x^{s+1} \overline{F}_n(t) dt \right\} - \left\{ \mu_n \sum_{i=0}^{r} (-1)^i \binom{r}{i} \frac{1}{(i+s+1)(r+s+2)} \int_0^\infty t^{r+s+2} \overline{F}_n(t) dt \right\} \right\}. \tag{4.2}$$

Where, $r = 0, s = 0$ and $\overline{F_n}$ is the product limit estimator, given in (4.1).

For computational purpose, $\hat{\Delta}_2^C$ in (4.2) may be rewritten as

$$\hat{\Delta}_2^C = \frac{1}{\mu^4}\left\{\psi^2 - \frac{1}{2}\Re\Omega\right\} \tag{4.3}$$

where

$$\psi = \left\{\sum_{i=1}^{n}\prod_{V=1}^{i-1} z(i) C_V^{\delta_{(V)}}(Z_{(i)} - Z_{(i-1)})\right\}, \qquad \Re = \sum_{K=1}^{n}\prod_{m=1}^{k-1} C_m^{\delta_{(m)}}(z_{(K)} - z_{(K-1)})$$

$$\Omega = \sum_{j=1}^{n}\prod_{Q=1}^{j-1}\left[z_{(j)}\right]^2 C_Q^{\delta_{(Q)}}(z_{(j)} - z_{(j-1)})\Bigg\}, \qquad \mu = \left\{\sum_{K=1}^{n}\prod_{m=1}^{K-1} C_m^{\delta_{(m)}}(Z_{(K)} - Z_{(K-1)})\right\}$$

$C_K = [n-k][n-k+1]^{-1}$, and $dx = (z_{(i)} - z_{(i-1)})$

Table 4.1 gives the critical values percentiles of $\hat{\Delta}_2^C$ test for sample sizes n=5(1)50.(10)81 based on 5,000 replications.

Table 4.1. Critical values for percentiles of $\hat{\Delta}_2^C$

n	%90	%95	%98	%99
5	.5104	.6741	.9796	1.2540
6	.4797	.6172	.8696	1.1238
7	.4527	.5772	.7913	1.0050
8	.4129	.5205	.6965	.8359
9	.3897	.4982	.6674	.8017
10	.3648	.4610	.6238	.7751
11	.3533	.4287	.5414	.6454
12	.3345	.4076	.5332	.6487
13	.3234	.4012	.5320	.6192
14	.3142	.3768	.4929	.6192
15	.3081	.3675	.4721	.5669
16	.2963	.3650	.4762	.5816
17	.2890	.3549	.4670	.5545
18	.2772	.3331	.4439	.5332
19	.2697	.3265	.4031	.4865
20	.2612	.3104	.3957	.4555
21	.2572	.3012	.3892	.4753
22	.2595	.3068	.3870	.4623
23	.2479	.2944	.3802	.4522
24	.2467	.2894	.3639	.4418
25	.2478	.2879	.3576	.4154
26	.2433	.2836	.3500	.4029
27	.2422	.2892	.3570	.4132

28	.2329	.2739	.3409	.4019
29	.2323	.2679	.3298	.3794
30	.2278	.2680	.3295	.3952
31	.2192	.2500	.3083	.3666
32	.2237	.2553	.3187	.3888
33	.2191	.2536	.3091	.3759
34	.2176	.2543	.3027	.3452
35	.2145	.2473	.3076	.3678
36	.2145	.2425	.2950	.3467
37	.2120	.2408	.2892	.3472
38	.2041	.2321	.2794	.3264
39	.2092	.2363	.2924	.3408
40	.2033	.2343	.2950	.3517
41	.2043	.2311	.2791	.3472
42	.2032	.2285	.2743	.3174
43	.2013	.2274	.2734	.3106
44	.1992	.2288	.2735	.3193
45	.1978	.2202	.2670	.2997
46	.1986	.2244	.2663	.3153
47	.1964	.2211	.2693	.3215
48	.1953	.2200	.2639	.3075
49	.1938	.2197	.2557	.2995
50	.1939	.2169	.2607	.3023
60	.1837	.2045	.2334	.2630
70	.1754	.1934	.2191	.2494
81	.1688	.1819	.2134	.2438

Table 4.1 and Figure 2 show that the critical values are increasing as confidence level increasing and decreasing as sample size increasing.

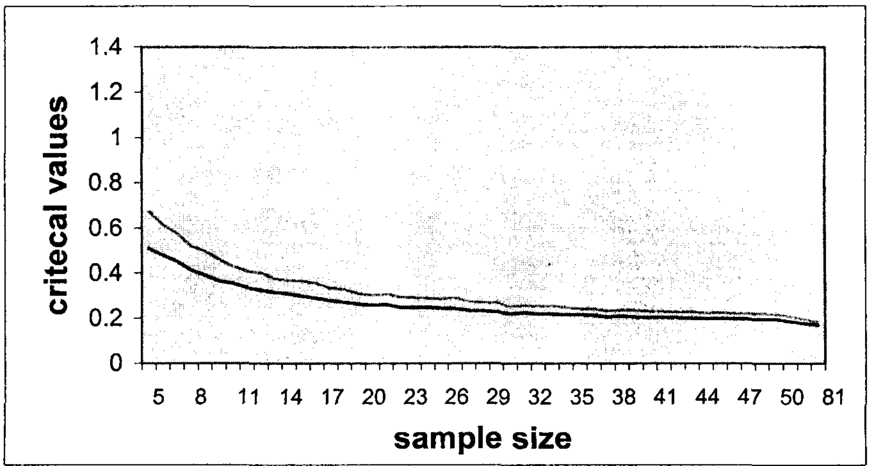

Figure 2. Relation between critical values, sample size and confidence levels.

Table 4.1 and Figure 2 show that the critical values are increasing as confidence level increasing and decreasing as sample size increasing.

A numerical example for NRBU(RNBU) test in censored sample

Consider the data Susarla and Vanryzin (1978), which represent 81 survival times (in weeks) of patients of melanoma. Out of these 46 represents non-censored data, and the ordered values are: 13,14,19,19,20,21,23,23,25,26,26,27,27,31,32,34,34,37,38,38,40,46, 50,53,54,57,58,59,60,65,65,66,70,85,90,98,102,103,110,118,124,130,136,138,141,234.

The ordered censored observations are: 6,21,44,50,55,67,73,80,81,86,93,100,108, 114,120,124,125,129,130,132,134,140,147,148,151,152,152,158,181,190,193,194,213, 215.

Now, taking into account the whole set of survival data (both censored and uncensored), and computing the statistic from (4.3) censored data, we get $\hat{\Delta}_2^C = 0.1418$, which is less than the critical value in Table 4.1 at %95 upper percentile, then, we accept H_0 which states that the set data have not NRBU (RNBU) property.

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