

Analysis of Leaky Modes on Circular Dielectric Rods using Davidenko's Method

Davidenko 방법을 이용한 원형 유전체 봉의 누설 모드 해석

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Abstract

Leaky modes on a circular dielectric rod are investigated from the precisely determined normalized complex propagation constants using Davidenko's complex root finding technique. Below the cutoff frequency of the guided mode, distinct frequency regions that have unique properties are observed, such as nonphysical region, antenna mode region, reactive mode region, and spectral gap region.

요약

Davidenko 방법으로 정확하게 구하여진 정규화 복소 전파 상수로부터 원형 유전체 봉의 누설 모드를 조사하였다. 도파 모드의 차단 주파수 아래에서 nonphysical 영역, 안테나 모드 영역, Reactive 모드 영역, spectral gap 영역 등의 독특한 주파수 영역이 관찰되었다.

Key words : Leaky Mode, Circular Dielectric Rod, Complex Propagation Constant, Davidenko's Method

I. Introduction

A circular dielectric rod is one of the simplest structures that can be used as either a waveguide or an antenna. Above the guided mode cutoff, the rod operates as a waveguide and its modal characteristics have been studied extensively for a long time^{[1],[2]}, and even nowadays^[3]. Below the cutoff frequency of the guided modes, in radiation mode region^[4], the propagation constants become complex and make up another class of discrete sets of eigenvalue solutions which represent leaky modes. The major feature of

the leaky modes is that the amplitude of the waves is growing along the transverse direction, so it is called the improper waves^[5]. The leaky mode dispersion characteristics of the circular dielectric rod are little known, in spite of that its characteristic equation has been revealed long time ago^[6]. In 1969, Ambak determined the complex propagation constants of the leaky modes of the circular dielectric rod and demonstrated the existence of the leaky modes in a circular dielectric rod by applying an approximate analysis to the characteristic equation^[7]. However, it is focused on finding the complex

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propagation constants and more detailed discussions about the leaky modes of the dielectric rod were not available at that time.

In this paper, we investigated the leaky modes of the dielectric rod in more detail such as the nonphysical mode, the antenna mode, the reactive mode, and the spectral gap for the lowest several TM modes from the precisely determined normalized complex propagation constants by Davidenko's method. The effects of the two design parameters, the radius of the dielectric rod and the dielectric constant of the rod material, to the dispersion characteristics are also considered.

II. Characteristic Equation

Fig. 1 shows the structure of the circular dielectric rod employed in this work. The axial component of the electric and the magnetic fields can be expressed as follows.

$$E_{zd} = A_{mn} J_m(k_d \gamma) \exp[j(\omega t - m\theta - \gamma z)] \quad (1)$$

$$H_{zd} = B_{mn} J_m(k_d \gamma) \exp[j(\omega t - m\theta - \gamma z)] \quad (2)$$

$$E_{zf} = C_{mn} H_m^{(2)}(k_f \gamma) \exp[j(\omega t - m\theta - \gamma z)] \quad (3)$$

$$H_{zf} = D_{mn} H_m^{(2)}(k_f \gamma) \exp[j(\omega t - m\theta - \gamma z)] \quad (4)$$

J_m and $H_m^{(2)}$ are the m th order Bessel and Hankel functions of the second kind; m is the azimuthal eigenvalue; A_{mn} , B_{mn} , C_{mn} and D_{mn} are complex constants corresponding to the modes; k_d and k_f are the complex transverse propagation constants in the dielectric region and the free space region, respectively, and are related with the material con-

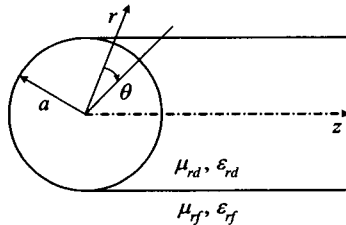


Fig. 1. Circular dielectric rod.

stants and the complex axial propagation constants as

$$k_i^2 = k_0^2 \mu_{ri} \epsilon_{ri} - \gamma^2 = k_0^2 (\mu_{ri} \epsilon_{ri} - \bar{\gamma}^2), (i = d, f) \quad (5)$$

Here, k_0 is the free space wave number, and $\bar{\gamma}$ is the normalized complex axial propagation constant, which is composed of the normalized phase and the attenuation constants, *i.e.*,

$$\bar{\gamma} = \frac{\gamma}{k_0} = \frac{\beta + j\alpha}{k_0} = \frac{\beta}{k_0} + j \frac{\alpha}{k_0} = \bar{\beta} + j\bar{\alpha} \quad (6)$$

Substituting (6) into (5) for the complex waves ($\bar{\alpha} \neq 0$), we have the following relationships^[5]

$$\begin{cases} (Re\{k_i\})^2 - (Im\{k_i\})^2 = k_0^2 (\mu_{ri} \epsilon_{ri} - \bar{\beta}^2 + \bar{\alpha}^2) \\ Re\{k_i\} Im\{k_i\} = -k_0^2 \bar{\alpha} \bar{\beta} \end{cases} \quad (7)$$

where $Re\{k_i\}$ and $Im\{k_i\}$ are the real and the imaginary parts of the transverse propagation constants, respectively. In addition, the leaky waves (Here, forward leaky waves) are the complex waves having the properties of $\bar{\beta} > 0$, $\bar{\alpha} < 0$, $Re\{k_i\} > 0$, and $Im\{k_i\} > 0$.

Applying the boundary conditions at the radius of the circular dielectric rod $r = a$ to the axial and the azimuthal components of the fields in the dielectric and the free space region yields the 4×4 coefficient matrix, and the determinant of the matrix should be zero to avoid nontrivial solutions. This is the characteristic equation of the circular dielectric rod and is expressed as follows.

$$P^2 - QR = 0 \quad (8)$$

$$P = m \left(\frac{\bar{\gamma}}{\alpha} \right) \left(\frac{1}{k_d^2} - \frac{1}{k_f^2} \right) \quad (9)$$

$$Q = \frac{\epsilon_{rd}}{k_d} \frac{J_m'(k_d a)}{J_m(k_d a)} - \frac{\epsilon_{rf}}{k_f} \frac{H_m^{(2)'}(k_f a)}{H_m^{(2)}(k_f a)} \quad (10)$$

$$R = \frac{\mu_{rd}}{k_d} \frac{J_m'(k_d a)}{J_m(k_d a)} - \frac{\mu_{rf}}{k_f} \frac{H_m^{(2)'}(k_f a)}{H_m^{(2)}(k_f a)} \quad (11)$$

For $m = 0$, the characteristic equation (8) is decoupled to the characteristic equation of the TM_{0n} mode ($Q = 0$), and the TE_{0n} mode ($R = 0$), respectively. As seen in (10) and (11), the characteristic

equations of the TM_{0n} and the TE_{0n} mode are identical except for the material constants and the hybrid mode such as the HE_{mn} and EH_{mn} modes are linear combinations of each transverse mode, thus, the TM_{0n} mode represents the most general feature of the circular dielectric rod in our case. So, we focus on our attention to the characteristics of the TM_{0n} modes. The characteristic equation of the TM_{0n} mode can be expressed as follows.

$$Q = \frac{\epsilon_{rd}}{k_d} \frac{J_1(k_d a)}{J_0(k_d a)} - \frac{\epsilon_{rf}}{k_f} \frac{H_1^{(2)}(k_f a)}{H_0^{(2)}(k_f a)} = 0 \quad (12)$$

III. Davidenko's Method

Below the cutoff frequency of the guided modes, the propagation constants become complex, and the normalized phase and attenuation constants in (7) are two of the most important parameters in analyzing leaky mode. Thus, normalized complex propagation constants should be determined precisely. The complex roots (the complex propagation constants) of (12) are determined with Davidenko's method^[8]. Davidenko's method is known to be superior to another complex roots finding methods such as the Muller's method^[9] or the Newton-Raphson method in its insensitiveness of initial guess and high speed of root search. We will briefly review Davidenko's method^[8].

Let $F(x) = 0$ be a nonlinear algebraic equation and x is the root of this equation. In the Newton-Raphson method^[9], $(n+1)$ th approximation to the root x of the equation $F(x) = 0$ can be in the form as follows.

$$x_{n+1} = x_n - \frac{F(x_n)}{\frac{dF(x_n)}{dx}} \quad (13)$$

Equation (13) can be written as

$$\frac{dF(x_n)}{dx} = - \frac{F(x_n)}{x_{n+1} - x_n} = - \frac{F(x_n)}{\Delta x_n} \quad (14)$$

where $\Delta x_n = x_{n+1} - x_n$ is the n th correction term

between the $(n+1)$ th and n th approximations.

If the $dF(x_n)/dx$ is too small, n th correction term may diverge, so the Newton-Raphson method fails. It is the problem of the Newton-Raphson method, if the value of initial guess (x_0) is set up far from the root x , of the given equation. Equation (14) may be modified to include a factor of small positive quantity, ζ ($0 < \zeta < 1$) to avoid the failure of the Newton-Raphson method,

$$\frac{dF(x_n)}{dx} = - \frac{F(x_n)}{\Delta x_n} \zeta \quad (15)$$

Then, the small value of the right hand side of (15) caused by the small value of $dF(x_n)/dx$, is mainly weighted to the factor ζ , thereby the correction term Δx_n may not have large value, so the Newton-Raphson method would not fail.

Taking the limit of both sides in (15) as $\zeta \rightarrow 0$, the n th correction term Δx_n and the factor ζ will change into dx and dt , respectively. Thus the equation (15) becomes

$$\frac{dF(x)}{dx} = - \frac{F(x)}{dx} dt \quad (16)$$

where t is a scalar dummy variable independent of x . Rearranging equation (16) as

$$\frac{dx}{dt} = - \frac{F(x)}{\frac{dF(x)}{dx}} = - \frac{dx}{d[\ln F(x)]} \quad (17)$$

Then, the denominators of both sides in (17) are

$$dt = d[\ln F(x)] \quad (18)$$

Integrating both sides of (18),

$$\int dt = - \int d[\ln F(x)] = \ln F(x) + C_1 \quad (19)$$

$$\ln F(x) = - t + C_2 \quad (20)$$

where C_1 and C_2 are arbitrary integration constants. Finally, we have

$$F(x) = Ce^{-t} \quad (21)$$

where C is a constant. Therefore, we have $F(x) = 0$

as the independent scalar dummy variable t approaches infinity.

Generally, this procedure can be applied for the two dimensional case, which corresponds to our TM_{0n} mode characteristic equation (eq. (12)) having two unknowns of $\bar{\beta}$ and $\bar{\alpha}$.

Interchanging the function $F(x)$ with the TM_{0n} mode characteristic function $Q(\bar{\gamma})$ with $\bar{\gamma} = \bar{\beta} + j\bar{\alpha}$, the first two terms of the equation (17) can be written as

$$\frac{d\bar{\gamma}}{dt} = -J^{-1}Q(\bar{\gamma}) \quad (22)$$

where J is the Jacobian matrix of the form

$$J = \begin{bmatrix} \operatorname{Re}\left\{\frac{\partial Q(\bar{\gamma})}{\partial \beta}\right\} & \operatorname{Re}\left\{\frac{\partial Q(\bar{\gamma})}{\partial \alpha}\right\} \\ \operatorname{Im}\left\{\frac{\partial Q(\bar{\gamma})}{\partial \beta}\right\} & \operatorname{Im}\left\{\frac{\partial Q(\bar{\gamma})}{\partial \alpha}\right\} \end{bmatrix} \quad (23)$$

Since the characteristic function $Q(\bar{\gamma})$ is analytic, the derivative of the $Q(\bar{\gamma})$ with respect to $\bar{\gamma}$ can be expressed using Cauchy-Riemann relation as follows.

$$\begin{aligned} Q_{\bar{\gamma}} &\equiv \frac{\partial Q(\bar{\gamma})}{\partial \bar{\gamma}} = \operatorname{Re}\left\{\frac{\partial Q(\bar{\gamma})}{\partial \beta}\right\} + j\operatorname{Im}\left\{\frac{\partial Q(\bar{\gamma})}{\partial \beta}\right\} \\ &= \operatorname{Re}\left\{\frac{\partial Q(\bar{\gamma})}{\partial \beta}\right\} - j\operatorname{Re}\left\{\frac{\partial Q(\bar{\gamma})}{\partial \alpha}\right\} \end{aligned} \quad (24)$$

Thus, the following relations are obtained.

$$\begin{cases} \operatorname{Re}\{Q_{\bar{\gamma}}\} = \operatorname{Re}\left\{\frac{\partial Q(\bar{\gamma})}{\partial \beta}\right\} \\ \operatorname{Im}\{Q_{\bar{\gamma}}\} = -\operatorname{Re}\left\{\frac{\partial Q(\bar{\gamma})}{\partial \alpha}\right\} \end{cases} \quad (25)$$

Using (23), (24) and (25), the inverse form of the Jacobian matrix in (22) is

$$J^{-1} = -\frac{1}{\det J} \begin{bmatrix} \operatorname{Re}\{Q_{\bar{\gamma}}\} & -\operatorname{Im}\{Q_{\bar{\gamma}}\} \\ \operatorname{Im}\{Q_{\bar{\gamma}}\} & \operatorname{Re}\{Q_{\bar{\gamma}}\} \end{bmatrix} \quad (26)$$

with

$$\det J = \left[\operatorname{Re}\left\{\frac{\partial Q(\bar{\gamma})}{\partial \beta}\right\} \right]^2 + \left[\operatorname{Re}\left\{\frac{\partial Q(\bar{\gamma})}{\partial \alpha}\right\} \right]^2 = |Q_{\bar{\gamma}}|^2 \quad (27)$$

Since the real imaginary terms of the normalized propagation constants $\bar{\gamma} (= \bar{\beta} + j\bar{\alpha})$ and the charac-

teristic function $Q(\bar{\gamma}) (= \operatorname{Re}\{Q(\bar{\gamma})\} + j\operatorname{Im}\{Q(\bar{\gamma})\})$ can be expressed as matrix forms as follows,

$$\bar{\gamma} = \begin{bmatrix} \bar{\beta} \\ \bar{\alpha} \end{bmatrix} \quad (28)$$

$$Q(\bar{\gamma}) = \begin{bmatrix} \operatorname{Re}\{Q(\bar{\gamma})\} \\ \operatorname{Im}\{Q(\bar{\gamma})\} \end{bmatrix} \quad (29)$$

the equation (22) can be expressed as follows.

$$\begin{aligned} \frac{d}{dt} \begin{bmatrix} \bar{\beta} \\ \bar{\alpha} \end{bmatrix} &= \frac{1}{|Q_{\bar{\gamma}}|^2} \begin{bmatrix} \operatorname{Re}\{Q_{\bar{\gamma}}\} & \operatorname{Im}\{Q_{\bar{\gamma}}\} \\ \operatorname{Im}\{Q_{\bar{\gamma}}\} & \operatorname{Re}\{Q_{\bar{\gamma}}\} \end{bmatrix} \\ &\times \begin{bmatrix} \operatorname{Re}\{Q(\bar{\gamma})\} \\ \operatorname{Im}\{Q(\bar{\gamma})\} \end{bmatrix} \end{aligned} \quad (30)$$

Finally, we have Davidenko's expression of two coupled first ordinary differential equation with an independent scalar variable t as follows.

$$\begin{cases} \frac{d\bar{\beta}}{dt} = -\frac{\operatorname{Re}\{Q(\bar{\gamma})\}\operatorname{Re}\{Q_{\bar{\gamma}}\} + \operatorname{Im}\{Q(\bar{\gamma})\}\operatorname{Im}\{Q_{\bar{\gamma}}\}}{|Q_{\bar{\gamma}}|^2} \\ \frac{d\bar{\alpha}}{dt} = -\frac{\operatorname{Re}\{Q(\bar{\gamma})\}\operatorname{Im}\{Q_{\bar{\gamma}}\} - \operatorname{Im}\{Q(\bar{\gamma})\}\operatorname{Re}\{Q_{\bar{\gamma}}\}}{|Q_{\bar{\gamma}}|^2} \end{cases} \quad (31)$$

As the dummy variable goes to infinity, each variable approaches to true values. Equation (31) is implemented with the software package MATHEMATICA 4.0 and numerically solved for large t . The obtained normalized phase and attenuation constants are substituted to the original TM mode characteristic equation (12) and are checked the accuracy under 10^{-10} for both the real parts and the imaginary parts. The obtained normalized complex phase and attenuation constants are also substituted in (7), and have checked that the modes have these values are the (forward) leaky modes^[5].

IV. Numerical Results

At first, we consider the normalized complex propagation constants of the circular dielectric rod for the three lower order TM modes. The dielectric constant and the radius of the dielectric rod are arbitrarily chosen to be 5.0 and 5.0 mm, respectively, and the rod is embedded in free space. Below the

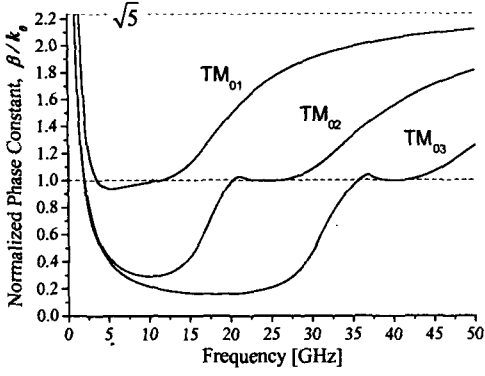


Fig. 2. Normalized phase constants.

cutoff frequencies of the guided modes, several kinds of distinct frequency regions such as nonphysical mode regions, reactive mode regions, antenna mode regions, and spectral gap regions are observed. Fig. 2 shows the normalized phase constants of the leaky modes for TM_{01} , TM_{02} , and TM_{03} modes as well as the guided modes and Fig. 3 shows the corresponding normalized attenuation constants. (In Fig. 3, we have changed the sign of the normalized attenuation constants, *i.e.*, $\bar{\gamma} = \bar{\beta} - j\bar{\alpha}$.) The cutoff frequencies of the guided modes for the TM_{01} , TM_{02} , and TM_{03} modes are 11.48, 26.35, and 41.32 GHz, respectively and below these cutoff frequencies, the nonzero value of the normalized attenuation constants are introduced as seen from Fig. 3.

As the operating frequency approaches to zero, the

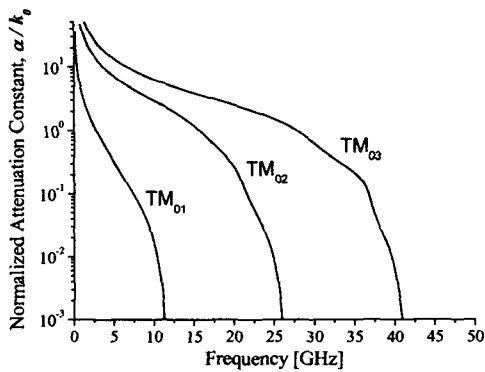


Fig. 3. Normalized attenuation constants.

normalized phase constants exceed the unity and approach the infinities. This frequency range does not have physical meaning^[10]. The upper limits of these nonphysical mode regions for the TM_{01} , TM_{02} , and TM_{03} are 3.51, 1.98, and 1.95 GHz, respectively. Above these limit frequencies, the normalized phase constants decrease to the minimum values and then increase to the unity again. This frequency region ($\bar{\beta} < 1$) corresponds to the physical leaky mode regions^[11]. This region can be divided into two distinct regions as the reactive mode regions ($\bar{\beta} < 1, \bar{\beta} < \bar{\alpha}$) and the antenna mode regions ($\bar{\beta} < 1, \bar{\beta} > \bar{\alpha}$), respectively^[12]. TM_{01} mode does not have the reactive mode region, since the frequency that the normalized phase constants and the normalized attenuation constants are same lies on the nonphysical mode region. The reactive mode regions for the TM_{02} and TM_{03} modes are ranging from 1.98 to 17.15 GHz and from 1.95 to 30.57 GHz, respectively; the antenna mode regions for the TM_{02} and TM_{03} modes are from 17.15 GHz to 20.27 GHz and 30.57 to 35.76 GHz, respectively. Both the spectral widths of the reactive mode region and the antenna mode region are increased as higher the modes.

Fig. 4 shows the normalized phase constants near the unity corresponding to Fig. 2. As the frequency goes to higher than that of the antenna mode region,

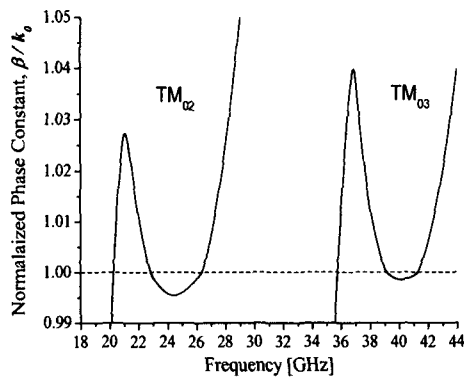


Fig. 4. Normalized phase constants with enlarged scale near the unity.

the normalized phase constants exceed the unity again, seen from Fig. 4. This region also has no physical meaning and is called the spectral gap region, ranging from 20.27 to 22.84 GHz and from 35.76 to 39.13 GHz for the TM_{02} and TM_{03} modes, respectively. The spectral width of the spectral gap region increases as higher the modes. TM_{01} mode has no spectral gap regions. The remaining portion of the frequency regions below the cutoff frequency of the guided mode is another antenna mode region above the spectral gap region in frequency, ranging from 3.51 to 11.48 GHz, from 22.84 to 26.36 GHz, and from 39.13 to 41.32 GHz for the TM_{01} , TM_{02} , and TM_{03} mode, respectively. The width of the second antenna mode shrinks as higher the modes. The upper limit frequency of this range meets the cutoff frequency of the guided mode. In other guiding structures such as the NRD guide^[11] and the partially dielectric-loaded open guiding structure^[13], the normalized attenuation constant becomes zero at the frequency with maximum normalized phase constants within the spectral gap region. The normalized attenuation constant of our structure becomes zero at this cutoff frequency of the guided mode, however, outside the spectral gap region, implying that the spectral gap region is not always consistent with the transition region between the guided mode and the leaky mode region.

V. Conclusion

We investigated the leaky modes on a circular dielectric rod structure from the precisely determined normalized phase and attenuation constants by Davidenko's method. In the frequency region below the cutoff of the guided modes, distinct frequency regions such as the nonphysical region, the antenna mode region, the reactive mode region, the spectral gap regions are observed.

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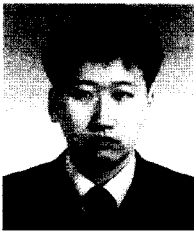
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References

- [1] E. Snitzer, "Cylindrical dielectric waveguide modes", *Journal of Optical Society of America*, vol. 51, no. 5, pp. 491-498, May 1961.
- [2] A. Safaai-Jazi, G. L. Yip, "Classification of hybrid modes in cylindrical dielectric optical waveguides", *Radio Science*, vol. 12, no. 4, pp. 603-609, July-August 1977.
- [3] Achint Kapoor, G. S. Singh, "Mode classification in cylindrical dielectric waveguides", *Journal of Lightwave Technology*, vol. 18, no. 5, pp. 849-852, May 2000.
- [4] Dietrich Marcuse, *Light Transmission Optics*, Van Nostrand Reinhold Company, 1982.
- [5] Akira Ishimaru, *Electromagnetic Wave Propagation, Radiation, and Scattering*, Prentice Hall, Inc., 1991.
- [6] Julius Adams Stratton, *Electromagnetic Theory*, McGraw-Hill, 1941.
- [7] J. Ambak, "Leaky modes on a dielectric rod", *Electronics Letters*, vol. 5, no. 3, pp. 41-42, 1969.
- [8] Hassan A. N. Hejase, "On the use of Davidenko's method in complex root search", *IEEE Trans. Microwave Theory and Techniques*, vol. 41, no. 1, pp. 141-143, January 1993.
- [9] S. D. Conte, Carl De Boor, *Elementary Numerical Analysis: An Algorithmic Approach*, 3rd ed., McGraw-Hill, 1980.
- [10] Paolo Lampariello, Fabrizio Frezza and Arther A. Oliner, "A versatile leaky-wave antenna based on stub-loaded rectangular waveguide: Part I - Theory", *IEEE Trans. Microwave Theo-*

- ry and Techniques, vol. 46, no. 7, pp. 1032-1041, July 1998.
- [11] Carlo Di Nallo, Fabrizio Frezza, Alessandro Galli, Paolo Lampariello and Arther A. Oliner, "Properties of NRD-guide and H-guide higher-order modes: Physical and nonphysical ranges", *IEEE Trans. Microwave Theory and Techniques*, vol. 42, no. 12, pp. 2429-2434, Dec. 1994.
- [12] Xiang-yin Zeng, Shan-jia Xu, Ke Wu and Kwai-Man Luk, "Properties of guided modes on open structure near the cutoff region using a new version of complex effective dielectric constant", *IEEE Trans. Microwave Theory and Techniques*, vol. 50, no. 5, pp. 1417-1424, May 2002.
- [13] Paolo Lampariello, Fabrizio Frezza and Arther A. Oliner, "The transition region between bound-wave and leaky-wave ranges for a partially dielectric-loaded open guiding structure", *IEEE Trans. Microwave Theory and Techniques*, vol. 38, no. 12, pp. 1831-1836, Dec. 1990.

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