# Hot Gas Analysis of Circuit Breakers By Combining Partial **Characteristic Method with Net Emission Coefficient**

# Sang-Hun Park\*, Chae-Yoon Bae\* and Hyun-Kyo Jung\*

**Abstract** - This paper proposes a radiation model, which considers radiation transport as an important component in hot gas analysis. This radiation model is derived from combining the method of partial characteristics (MPC) with net emission coefficient (NEC), and it covers the drawbacks of existing models. Subsequently, using this proposed model, the arc-flow interaction in an arcing chamber can be efficiently computed. The arc is represented as an energy source term composed of ohmic heating and the radiation transport in the energy conservation equation. Ohmic heating term was computed by the electric field analysis within the conducting plasma region. Radiation transport was calculated by the proposed radiation model. Also, in this paper, radiation models were introduced and applied to the gas circuit breaker (GCB) model. Through simulation results, the efficiency of the proposed model was confirmed.

**Keywords**: arc, circuit breaker, Euler equation, method of partial characteristics (MPC), net emission coefficient (NEC), radiation transport.

#### 1. Introduction

Puffer-type circuit breakers are now widely used in high voltage power networks owing to their effective breaking capacity. However, it is necessary to evaluate the performance of circuit breakers by numerical analysis because huge costs are required in the design and analysis of GCBs.

In general, the technique combining the computation fluid dynamic tools with arc modeling is used as a numerical analysis method for GCBs [1-4]. A basic governing equation for the field flow analysis is the axisymmetric Euler equation. This equation neglects the viscous effect from the Navior-Stokes equation and was solved by the finite volume fluid in cell (FVFLIC) method in this paper [5].

In the energy conservation equation, the arc is considered as the energy source term, which is the sum of the periods of ohmic heating and radiation transport. The ohmic heating term was computed by electric field analysis within the conducting plasma region only. A radiation transport is important in calculating the arc energy because the majority of energy is exchanged through this process.

Liebermann and Lowke initiated research on arc modeling and computed NEC by evaluating radiation transport form in a broad point of view [6]. Thereafter, Fang and Zhang etc. proposed the radiation model considering reabsorption of the radiation energy according to the temperature [3, 4 & 7]. Recently, Sevast'yanenko suggested

In this paper, radiation models to solve the RTE were introduced and applied to the GCB model. In particular, this paper proposed a radiation model combining MPC with NEC, subsequently being able to cover the negative aspects of these models. Simulation results according to the radiation models were compared.

## 2. Flow Governing Equation

Generally, Euler equation neglecting the viscous term in the Navior-Stokes equation is used for flow field analysis, and this equation is represented by the conservation equations for mass, momentum and energy where it is assumed the arc plasma is in the local thermodynamic equilibrium (LTE) state. Also, the effects of the heat conduction and viscous terms are regarded as small enough to be negligible in the process of high current arc.

In this paper, the axisymmetric Euler equation is used for the governing equation of the flow field due to the shape of the circuit breaker. The conservation equations of mass, momentum and energy are expressed as follows:

Mass conservation equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u r)}{r \partial r} + \frac{\partial (\rho v)}{\partial z} = 0 \tag{1}$$

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that MPC could easily solve the radiation transport equation (RTE) by using tabled material functions [8-12]. However, there are difficulties in computing radiation transport despite the development of various radiation models.

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Momentum conservation equation (for r and z axis):

$$\frac{\partial \rho u}{\partial t} + \frac{\partial (\rho u^2 r)}{r \partial r} + \frac{\partial (\rho u v)}{\partial z} = -\frac{\partial (\rho r)}{r \partial r}$$
(2)

$$\frac{\partial \rho \, v}{\partial t} + \frac{\partial (\rho \, uvr)}{r \partial r} + \frac{\partial (\rho \, v^2)}{\partial z} = -\frac{\partial (\rho)}{\partial z} \tag{3}$$

Energy conservation equation:

$$\frac{\partial E}{\partial t} + \frac{\partial (\rho u E r)}{r \partial r} + \frac{\partial (\rho v E)}{\partial z} = -\frac{\partial (\rho u r)}{r \partial r} - \frac{\partial (\rho v)}{\partial z} + S_{\epsilon} \qquad (4)$$

Where, u, v: vector component of z and r direction,  $\rho$ : density, p: pressure, and E: specific total energy.

The influences of the arc are caused from the ohmic heating and the radiation transport. The source term,  $S_e$  in (4), represents the arc effects in the flow field.

This equation is solved by the FVFLIC method [13]. Implementation of the FVFLIC method is simple and can be applied to an unstructured grid. Furthermore, it takes small computation time and is accurate for shock waves.

In (1-4), the density  $\rho$  and the specific total energy E can be obtained. To calculate the temperature T and the pressure P, the state equation is required. For a perfect gas, the state equation is  $P = (\gamma - 1)i$ , where i: internal energy and  $\gamma$ : specific heat ratio.

However, in the case of high temperature plasma, this equation cannot express the accurate relationship between the flow state variables. Therefore, the pressure and temperature can be obtained by the data in [14]. In [14], the  $SF_6$  properties are given as the function of the pressure and temperature by:

$$\rho = \rho \left( P, T \right) \tag{5}$$

$$i = i(P,T) \tag{6}$$

Since  $\rho$  and i are known from solving the Euler equation, the following functions are needed.

$$P = P(\rho, i) \tag{7}$$

$$T = T(\rho, i) \tag{8}$$

We obtained the above functions using the Newton-Raphson method with the curve-fitted data of (5, 6).

#### 3. Arc Modeling

If there is high temperature plasma in the GCB, the main

energy transport mechanisms can be considered, such as ohmic heating, radiation transport, conduction and convection. Since the arc influences the energy balance by ohmic heating and radiation transport, these two mechanisms are considered as important terms to the analysis. Therefore, many researchers represent the effects of the arc as:

$$S_e = S_{ohm} - U_{rad} \tag{9}$$

Where,  $S_{ohm}$ : ohmic heating source and  $U_{rad}$ : radiation transport.

# 3.1 Ohmic Heating

This term represents the electric input energy by the arc current and can be calculated using

$$S_{ohm} = \sigma |\vec{E}|^2 \tag{10}$$

Where,  $\sigma$ : electrical conductivity and  $\vec{E}$ : the electric field intensity.

The electrical conductivity  $\sigma$  is a function of the local temperature T and pressure P. Therefore, if temperature and pressure are known from the flow field analysis, the electrical conductivity in each cell can be obtained from  $\sigma = \sigma(T,P)$  by using the data of Frost and Liebermann [14] for  $SF_6$  gas.

The electric field intensity  $\vec{E}$  can be obtained using (11, 12), that is, the conservation equation of the electric current and Ohm's law.

$$\nabla \cdot \vec{J} = 0 \tag{11}$$

$$\vec{J} = \sigma \ \vec{E} \tag{12}$$

Where,  $\vec{J}$ : the electric current density.

E can be expressed as a gradient of scalar potential, then the equation expressing the current distribution in the arc region becomes:

$$\nabla \cdot (\sigma \nabla \phi) = 0 \tag{13}$$

Therefore, if the arc voltage is known, a voltage difference can be obtained using (13). In the practical situation, however, the arc current is known, while the voltage difference between the electrodes is unknown. To determine a voltage difference V and an ohmic heating term  $S_{ohm}$ , the following procedures are used.

# 1) Definition of the artificial arc region.

This region may be arbitrarily defined to cover the entire arc expected and can include the electrode as shown in Fig. 1. The right boundary  $S_c$  of this region is within the electrode.

2) Analysis of electric field intensity using the finite element method with the boundary condition  $\phi_a = I[V]$ ,  $\phi_c = O[V]$ .

If the electrical conductivity is zero in some elements, then  $\sigma_{min} = 10^{-3}$  is imposed in those elements.

3) Calculation of the electric field intensity  $\vec{e}$ .

This is the field intensity when the voltage difference is 1 / V.

4) Calculation of the current flowing in the plane  $S_c$  by using the following relationship.

$$I_0 = \int_{S_0} \vec{j} \cdot ds = \int_{S_0} \sigma \cdot \vec{e} ds \tag{14}$$

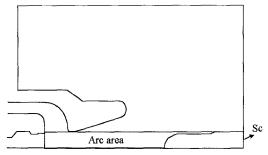
5) Calculation of the voltage difference and the electric field intensity.

$$V = I_{arc} / I_0 \tag{15}$$

$$\vec{E} = V \ \vec{e} \tag{16}$$

Where,  $I_{arc}$ : the known arc current.

6) Calculation of ohmic heating using (10).



(a) Artificial arc region for the calculation of the current density distribution.

$$\phi_a \longrightarrow Arc region Sc \longrightarrow \leftarrow \phi$$

(b) Boundary condition for the finite element analysis.

Fig. 1 Definition of the artificial arc region

# 3.2 Radiation Transport

Radiation transport is the most important energy mechanism within the arc plasma but it is difficult to compute accurately. The basic quantity in the radiation transport is the spectral intensity of radiation  $I_{\nu}(n,X)$  and this quantity represents the power per unit frequency (or wavelength) per solid angle per unit area perpendicular to the direction n at a point X [11]. Also, the radiation transport within a medium is expressed considering the radiation intensity at any position along a path through an absorbing,

emitting, and scattering medium [15].

At first, the attenuation of radiation intensity can be considered. This term describes the decrease of radiation intensity by the absorption and self-scattering within a medium. If a coefficient of proportionality  $k_v$  that depends on the local properties of the medium is introduced, then the decrease is:

$$dI_{\nu} = -k_{\nu}I_{\nu}ds \tag{17}$$

Where, ds: the path length of the radiation.

Next, the gain of radiation intensity can be considered. This term expresses the contribution by spontaneous emission and incoming scattering in the medium along the path to the intensity in an arbitrary direction as the radiation along the path is in LTE. This term is written as:

$$dI_{\nu} = k_{\nu} I_{\nu b} ds + \frac{ds \, \sigma_{\nu}}{4\pi} \int_{\omega_{i}=0}^{4\pi} I_{\nu}(\omega_{i}) \Phi(\nu, \, \omega_{i}, \, \omega) \, d\omega_{i} \quad (18)$$

Where,  $\omega_i$ : the incoming solid angle,  $\omega$ : the arbitrary solid angle,  $\Phi(v, \omega_i, \omega)$ : the phase function describing the angular distribution of the scattered intensity, and  $\sigma_v$ : the scattering coefficient.

Thus, for the change in intensity with s in the solid angle,  $\omega$  in the s direction, adding (17) to (18) gives:

$$\frac{dI_{v}}{ds} = -k_{v}(S)I_{v} + k_{v}(S)I_{vb} + \frac{\sigma_{v}}{4\pi} \int_{\omega_{i}=0}^{4\pi} I_{v}(s,\omega_{i})\Phi(v,\omega_{i},\omega) d\omega_{i}$$
(19)

These equations are from [15].

This equation is referred to as a radiation transfer equation and expresses the increment of the radiation intensity at any position along a path through an absorbing, emitting, and scattering medium.

# 4. Radiation Models

To calculate the emission and the absorption of radiation, the radiation intensity must be obtained by solving the RTE and be simultaneously integrated over all elements, all solid angles and all frequencies. Accordingly, it takes such a long computation time that it is impossible to compute radiation term.

To cover these difficulties, some approximate models were proposed. Of these models, there is NEC, which evaluates radiation transport in broad view by assuming cylindrical arc, MPC, which uses the tabled material function, and so on.

# 4.1 Net Emission Coefficient

Assuming non-scattering, stationary plasma in LTE, RTE can be similar (20).

$$n \nabla I_v + k_v I_v = k_v B_v \tag{20}$$

Where,  $k_{\nu}$ : the spectral coefficient of absorption including induced emission and  $B_{\nu}$ : the Plank function or blackbody intensity.

The coefficient of absorption  $k_v$  is from the line and continuum spectrum, and it is a function of plasma temperature and pressure.

The radiation flux density at the point X,  $\vec{F}(X)$ , is obtained by integrating the radiation intensity over all solid angles, i.e.

$$\vec{F}(X) = \int_{4\pi} I(X, \vec{n}) \ \vec{n} \ d\Omega \tag{21}$$

Where,  $\vec{n}$ : a unit vector in the particular direction and  $d\Omega$ : an element of the solid angle.

Finally, the net emission of radiation is obtained from the divergence of the radiation flux.

$$U = 4\pi\varepsilon_N = \nabla \cdot \vec{F}(X) \tag{22}$$

Where,  $\varepsilon_N$ : the radiation emission per unit solid angle at X.

NEC represents average radiation energy per volume and is a function of arc temperature, pressure and radius.

This model can be simply implemented and it takes minimal computation time as compared with any other model. Also, in the central region of the plasma with high temperature, this model shows good agreement to the experimental result [6]. In the case of the arc edge with steep temperature gradients, there can be a certain deviation from LTE, subsequently bringing an error into the final results. The reason for this is that the assumption of the existence of LTE is valid only in the central region of the plasma with high temperature and for pressures of over 1 bar. In addition, since it is difficult to consider the radiation flux in the complex nozzle geometry, this model is restricted to the geometry.

### 4.2 Method of Partial Characteristics

This model, which has been proposed by Sevast' yanenko, briefly solves RTE using the tabled material functions related to temperature, pressure and distance.

Assuming integration along the ray path  $\xi$  and non-scattering media, RTE becomes:

$$\frac{dI_{\nu}}{d\xi} + k_{\nu}I_{\nu} = k_{\nu}B_{\nu} \tag{23}$$

When assuming non-scattering media, the radiation intensity along the direction  $\xi$  from the plasma boundary R to given point X is provided by:

$$I_{\nu}(X) = \int_{0}^{\infty} \int_{X}^{R} k_{\nu}(\xi) B_{\nu}(\xi) e^{-\int_{X}^{\xi} k(\eta) d\eta} d\xi d\nu$$
 (24)

In (24), the term  $k_{\nu}B_{\nu}$  represents photons generated along the path  $\overline{XR}$ , and the exponential term accounts for self-absorption along the path  $\overline{XE}$ .

Net emission  $\varepsilon_v$  at any frequency is the difference between the local emission of radiation  $B_v k_v$  at point X in the plasma and the absorption of radiation  $I_v k_v$  at X coming from all other points in the plasma. Thus, for contributions to  $\varepsilon_v$  from the on line segment  $\overline{XR}$ :

$$\varepsilon_{\nu} = B_{\nu}k_{\nu} - \int_{X}^{R} k_{\nu}(\xi)B_{\nu}(\xi) \exp[-\int_{0}^{X} k_{\nu}(\eta)d\eta]d\xi \qquad (25)$$

Assuming that  $k_{\nu}(X) = (d/d\xi) \left[ \int_{X}^{\xi} k_{\nu}(\eta) d\eta \right]$ , the net emission  $\varepsilon_{\nu}$  can be evaluated as:

$$\varepsilon_{N}(X) = \int_{0}^{\infty} \varepsilon_{\nu}(X) d\nu$$

$$= Som(T_{X}, T_{\xi}, \xi - X) - \int_{0}^{x} \Delta Sim(T_{X}, T_{\xi}, \xi - X) d\xi \qquad (26)$$

Where the new functions Som and  $\Delta Sim$  are given by:

$$Som(T_X, T_{\xi}, x) = \int_0^\infty B_{\nu}(X) k_{\nu}(X) e^{-\int_0^x k_{\nu}(\eta) d\eta} d\nu$$
 (27)

$$\Delta Sim(T_X, T_\xi, \xi - X)$$

$$= \int_0^\infty [B_u(X) - B_u(\xi)] k_u(X) k_u(\xi) \cdot \exp(-\int_0^x k_u(\eta) d\eta) dv \quad (28)$$

These equations are referred from [11].

This model can be easily used without a deep understanding of either radiation transfer theory or atomic theory. Also, using certain assumptions, it has higher efficiency in solving the RTE directly. However, it needs tabled datum to calculate variables like Som and  $\Delta Sim$ . In addition, it requires considerable computation time, though this model is more efficient than solving the RTE directly.

#### 4.3 Proposed Model

As seen previously, both NEC and the MPC have certain limitations for solving RTE, subsequently these drawbacks

create some difficulties in the hot gas analysis of the GCB. Therefore, in this paper, a radiation model is proposed for covering these drawbacks.

The basic concept of the proposed model is to take only merits of the two previous models and properly applying their applications according to the temperature of arc cells as shown Fig. 2. Accordingly, it is possible to reduce computation time and to obtain an accurate solution.

Additionally, the criterion temperature must be determined for applying the two previous models. If the criterion temperature is so high, the computation time is increased because the effect by the MPC is dominated. In the opposite scenario, the errors of the simulation results become enlarged due to the effect of NEC. In this paper, this criterion temperature was determined as 15000K.

The algorithm of the proposed model is shown in Fig. 3. At first, to determine which model can be used, the temperature of all cells is checked. However, it takes a great deal of computation time in checking the temperature of all flow cells in the flow field and actually, there is no need to do so. The reason for this is that only the radiation within the arc is considered in solving the RTE. So, rather than examining the temperature of all flow cells in the flow field, temperatures are only checked in the artificial arc region defined in the flow analysis.

Then, of these cells, NEC is applied to cells higher than the criterion temperature, while the MPC is applied to cells lower than that. Finally, complete cell energy is computed.

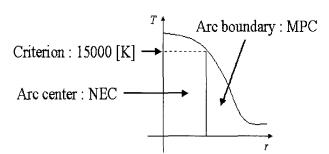


Fig. 2 Conceptual diagram of the proposed model

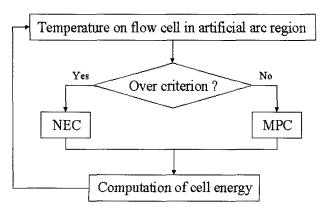


Fig. 3 Flow chart on the algorithm of the proposed model

#### 5. Simulation Results

In this section, the pressure in the cylinder and the arc voltages from each radiation model are compared with experimental values. Also, computation times according to the radiation models are compared with one another.

The pressure rises and the arc voltages are respectively compared according to the radiation models in Fig. 4 and Fig. 5. In these graphs, the simulation results from both the MPC and the proposed model show a similar trend, and relatively good agreement. However, NEC has a greater amount of errors than other models. Therefore, it can be seen that both the MPC and the proposed model evaluate the radiation transport more accurately than NEC. The deviations between the simulation and experimental results are considered due to neglecting viscous term, vapor effects, and so on.

In addition, the computation time of the MPC and the proposed model is respectively 2.5 and 1.6 times when compared with that of NEC. Therefore, with consideration given to computation time and accuracy, it is obvious that the proposed model is most efficient among the three radiation models.

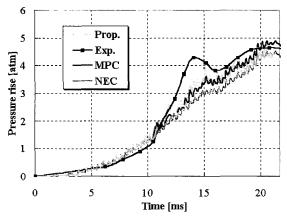


Fig. 4 Comparison of experimental data with arc voltages according to radiation models

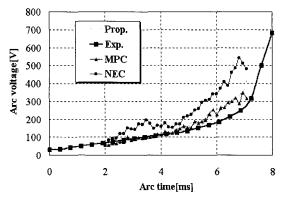


Fig. 5 Comparison of experimental data with arc voltages according to radiation models

# 6. Conclusion

This paper proposed a radiation model combining NEC with MPC, thereby being able to cover drawbacks of the two previous models. In addition, hot gas analysis was performed applying the radiation model to the GCB. The flow governing equation was the Euler equation solved by FVFLIC. Ohmic heating was calculated by the electric field analysis using FEM, and the radiation transport was computed by using NEC, MPC and the proposed model. Furthermore, simulation results were compared with the experimental values, and the proposed model showed high efficiency in relation to the accuracy and the computation time.

Our analysis model of hot gas flow is computationally economical and is able to provide a quantitative description of the aerodynamic and electrical behavior of a puffer circuit breaker. However, other effects, such as vapor, turbulence and viscosity must be considered for more exact analysis. The reason is that they alter the properties of the dielectric material and can subsequently have an effect on the performance of GCBs.

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