(s, S) 재고관리 시스템에 대한 확률최적화 기법의 응용*

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Application of Stochastic Optimization Method to (s, S) Inventory System

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Abstract

In this paper, we focus an optimal policy for a certain class of (s, S) inventory control systems. To this end, we use the perturbation analysis and apply a stochastic optimization algorithm to minimize the average cost over a period. We obtain the gradients of objective function with respect to ordering amount S and reorder point s via a combined perturbation method. This method uses the infinitesimal perturbation analysis and the smoothed perturbation analysis alternatively according to occurrences of ordering event changes.

Our simulation results indicate that the optimal estimates of s and S obtained from a stochastic optimization algorithm are quite accurate. We consider that this may be due to the estimated gradients of little noise from the regenerative system simulation, and their effect on search procedure when we apply the stochastic optimization algorithm.

The directions for future study stemming from this research pertain to extension to the more general inventory system with regard to demand distribution, backlogging policy, lead time, and review period. Another directions involves the efficiency of stochastic optimization algorithm related to searching procedure for an improving point of (s, S).

Key Words; (s, S) Inventory System, Stochastic Optimization Algorithm, Perturbation Analysis

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1. Introduction

This paper considers how simulation can be used to obtain an optimal policy for a certain class of (s, S) inventory systems. An (s, S) ordering policy specifies that an order is placed up to S when the inventory level on hand falls below the level s. The objective of inventory control system is to find an ordering policy to minimize a cost function associated with ordering, holding and shortage costs.

To specify the (s, S) system, we require the demand characteristics, lead time, back-logging policy, and associated costs. We consider the standard *infinite horizon*, single product, and periodic review inventory model with full backlogging and independent demands. Under these conditions, since Scarf [17] and Iglehart[13] showed that an optimal policy can be found within the class of (s, S) policies, a considerable amount of research has been worked for several decades, however computationally, obtaining the optimal policies is quite complex and difficult to apply in real world [16].

The usual approach for finding the optimal policies includes dynamic programming(DP) and stationary analysis(SA)[9]. The DP method uses a recursive means to find the optimal values, and the SA method uses numerical methods and can be applied to restricted cases. Given the analytical complexity of (s, S) inventory systems, one obvious way to analyze the systems is through simulation. While evaluating alternate systems through simulation is a fairly routine problem, optimization through simulation is a challenging problem in recent years. Andradottir and Azadivar provided review on simulation optimization techniques commonly found in simulation literature[1,3]. One such approach is the stochastic optimization method based on gradient search techniques. This method tries to find the sample path estimates of the derivatives of the objective function with respect to (w.r.t) parameters during the course of simulation run, and then to search an optimality of system performance via stochastic approximation method.

To find the gradients of parameters from a single run of simulation, Ho and Cao first suggested the perturbation analysis(PA) [12]. PA has been used to estimate derivative of performance measure in various simulation systems [2, 5, 6, 10]. Fu presented a method of obtaining the sample path derivatives for (s, S) inventory system [9]. He considered a periodic review system with full backlogging and holding and shortage costs, where the demands were assumed to occur once each period, to be independent and identically distributed and to have a continuous distribution function. Under these conditions, he derived the PA derivative estimators of the average cost per period w.r.t s and $q(=S-s)$, and provided strong consistency proofs of them.

The primary objective of this paper is to find the optimal policies of (s, S) inventory system by applying stochastic optimization algorithm(SOA). We consider that the inventory level is characterized by a regenerative process and obtain the gradient estimates by the PA method based on this property. We explore the efficiency of this algorithm under certain conditions and discuss the issues in implementation of the SOA.

2. PA for (s, S) Inventory System

To find the optimal policy for (s, S) inventory system via the SOA, we need the derivatives of objective function (average cost per period) w.r.t decision variables, s and S. PA provides a means to estimate the gradient of performance measure from a single run of simulation. The idea of PA is a thought experiment of introducing a perturbation ΔS (or Δs) into the sample path and tracing its effect during the course of simulation.

If an infinitesimal change of ΔS (or Δs) produces no change in the sample perturbed path during simulation run, we can apply the infinitesimal perturbation analysis(IPA). On the other hand, if such a change may cause changes in the sequence of events in the nominal sample path, then it may result in the perturbed sample path quite different from the nominal one. In this case, we can apply the smoothed perturbation analysis (SPA). In this section, we first describe the (s, S) inventory system, and then present the methodology to derive the gradient estimators of objective function for IPA and SPA.

2.1 (s, S) Inventory System

We consider a single commodity per period model. A sequence of ordering decisions is to be made periodically at the beginning of each period. These decisions result in an order up to S if the inventory level of period i, X_i falls below s. Otherwise no order is placed. The demand of period i, D_i causes a depletion of the inventory at the end of each period. So the recursive equation for X_i is given by $X_{i+1} = X_i - D_i$ if $X_i \geq s$, and $X_{i+1} = S$ if $X_i < s$. Each period demand is independently and identically distributed(i.i.d) with density function $f(d)$ and distribution function $F(d)$. Typical sample path for X_i is presented in Figure 1.

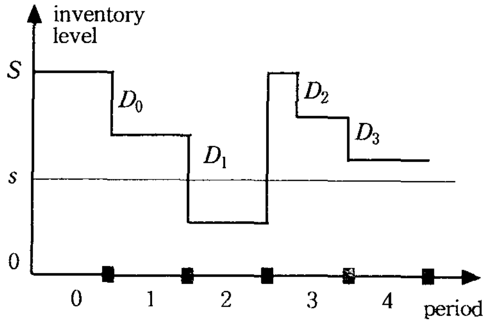

Figure 1. Typical Sample Path

There are three types of costs incurred during each period which influence the ordering decisions: a fixed ordering cost k when order is placed, a holding cost h charged for inventory on hand, and a shortage cost p associated with the failure to meet demand. Holding and shortage costs are charged at the end of each period. We assume that order delivery is instantaneous and backlogging is permitted for excess demand. Our objective is to find the optimal ordering policy minimizing the average cost over a period, $AC(s, S)$, which is the time average function of inventory level and ordering decision.

We suppose the initial inventory level, $X_0 = S$ and consider that the period inventory levels $\{X_i;\ i=1, 2, \cdots\}$ is characterized by a regenerative process with a regenerative point S. A perturbation ΔS changes the inventory level by ΔS, and Δs changes the reorder point. Thus both perturbations may change the period of ordering, and whether or not an ordering change occurs in a certain period changes the regenerative cycle lengths and their associated costs. To estimate the sample derivatives of $AC(s,S)$, we use the IPA in case of no change in cycle length, and apply the SPA in another case. As simulation advances, we apply the IPA and SPA alternatively to regenerative cycles.

2.2 Infinitesimal Perturbation Analysis

Figure 2 presents two sample paths for inventory system operating at S (nominal path) and $S+\Delta S$ (perturbed path), where the solid line shows the inventory level of each period in ordinary sample path and the dotted line presents that in the perturbed sample path. If the sequences of ordering decisions of two paths are same in regenerative cycles, then we have the same ordering cost for both paths. Merely, an infinitesimal change in S produces an infinitesimal change in the inventory and shortage costs depending on the inventory levels.

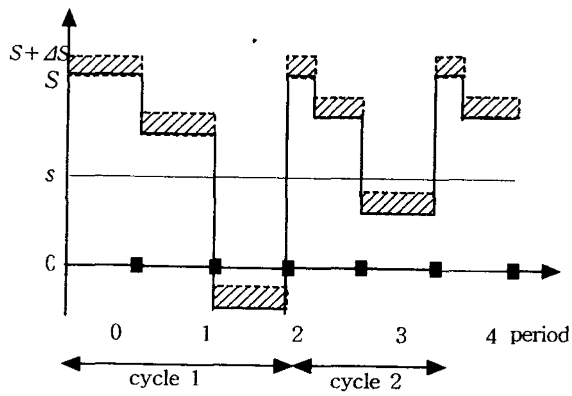

Figure 2. Infinitesimal Change between Nominal and Perturbed Paths

We let N be the number of total periods in simulation run. We also let N_1 and N_2 be the number of periods in which the inventory level is positive and negative, respectively. From the sample path, we see that the difference in two objective functions, $\Delta AC(s,S)$ is given by

$$\begin{aligned}\Delta AC(s,S) &= [\,TC(s,S)_p - TC(s,S)_n]/N \\ &= [\,h\Delta S\times N_1 - p\Delta S\times N_2]/N, \qquad (1)\end{aligned}$$

where $TC(s,S)_p$ and $TC(s,S)_n$ are the total cost of the perturbed and nominal sample paths, respectively for cycles with no order changes. Thus the sample derivative of $\Delta AC(s,S)$ w.r.t S is represented as follows:

$$\begin{aligned}\delta AC(s,S)/\delta S &= \lim_{\Delta S\to 0}\Delta AC(s,S)/\Delta S \\ &= [\,hN_1 - pN_2]/N. \qquad (2)\end{aligned}$$

In a similar way, we can obtain the gradient estimator of $AC(s,S)$ w.r.t the reorder point s. A change from s to $s+\Delta s$ does not effect on the cost function as long as the sequence of events from two sample paths remain same. Therefore we have

$$\delta AC(s,S)/\delta s = 0. \qquad (3)$$

2.3 Smoothed Perturbation Analysis

Figure 3 presents the situation where the ordering change situation occurs in the perturbed sample path at period $i-1$. We observe that in the nominal path, an order up to S is placed. However, the inventory level in the perturbed path, $X_{i-1}+\Delta S$, is greater than s and no order is placed. From i, two sample paths evolve differently until they converge at a certain period. If the inventory

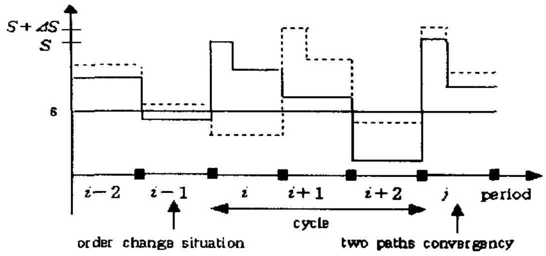

Figure 3. Order Change Situation and Convergency of two paths

levels of both sample paths are below s at period $i+2$, then orders up to S would be placed in both paths at period j. Thus, both sample converge at j and the assumption of IPA may hold from j to the instant when two paths show the different sequence of events. We denote the periods from i to j as a cycle. At j, we reinitialize the simulation and restart the sample path at the inventory level S. From i to j, a finite change in two objective functions, $\Delta AC(s,S)$ can be observed during the course of simulation. For this case, we apply the SPA suggested by Gong and Ho [4].

We now derive the expression for the sample derivative estimators under the condition that ΔS goes to 0. We first identify the situation where the ordering change potentially occurs. If the demand D_{i-1} brings the inventory level X_{i-1} to a point below s, then an order is placed in period i and $X_i=S$. Thus an potential ordering change occurs for the period i when $X_i=S$. We define such a set as $M=\{i: i\le N,\ X_i=S\}$. For the period $i\in M$ ($X_{i-1}<s$), an actual ordering change occurs if $X_{i-1}+\Delta S>s$.

For the convenience of derivation, we define a variable, $\alpha_{i-1}=s-X_{i-1}$. Then the condition for actual ordering change would be equivalent to $\alpha_{i-1}<\Delta S,\ i\in M$. When an actual ordering change occurs due to ΔS, we observe a change in two cost functions, $\Delta AC(s,S)$, from two sample paths during the cycle of ordering change. Given the ordering change, the expected ordering change effect per period is can be represented as the conditional expectation of $\Delta AC(s,S)$ multiplied by the probability of actual ordering change:

$$E=\sum_{i\in M}E[\Delta AC(s,S)\mid \alpha_{i-1}\le\Delta S]\times Pr[\alpha_{i-1}\le\Delta S]. \quad (4)$$

The first term of the above equation is the expected finite change in two cost functions caused by an ordering change. The process for obtaining the conditional expectation of $\Delta AC(s,S)$ is quite self- explanatory. During the simulation run, when the condition for ordering change occurs, we start to trace the inventory levels of nominal and perturbed paths respectively and calculate their associated costs until two inventory levels of both sample paths fall below s. At this instant, we re-initialize and restart the sample paths. As we noted, the cycles form a regenerative process. From the simulation run enough for large numbers of cycles, we estimate $E[\Delta(AC(s,S)]$ as follows:

$$\hat{E}[\Delta AC(s,S)]=[\,TC(s,S)_p-TC(s,S)_n]/N, \quad (5)$$

where $TC(s,S)_p$ and $TC(s,S)_n$ are same as given in (1) and they are obtained during cycles including order changes.

We now consider the second term in (4), the probability of actual ordering change. We define a random variable, $Z_{i-1}=X_{i-2}-s$. We note that $X_{i-1}=X_{i-2}-D_{i-1}$. Then the condition of $X_{i-1} < s$ ($i \in M$) can be expressed as $X_{i-2} - s < D_{i-1}$, which is equivalent to $Z_{i-1} < D_{i-1}$. For $i \in M$, we calculate the conditional distribution for α_{i-1}:

$$Pr(\alpha_{i-1} \le \Delta S \mid X_{i-1} < s) = Pr(\alpha_{i-1} \le \Delta S \mid D_{i-1} > Z_{i-1})$$

$$= Pr(D_{i-1} \le Z_{i-1} + \Delta S \mid D_{i-1} > Z_{i-1})$$
$$= F(Z_{i-1} + \Delta S) - F(Z_{i-1}) / [1 - F(Z_{i-1})]. \quad (6)$$

Due to the regenerative property, α_i and Z_i are i.i.d for all i, and correspond to the values in the last period of each regeneration cycle. Merely for notational convenience, we drop the subscript $i-1$ in Z. Then the probability of occurrence of actual ordering change in period i, P_i is given by

$$P_i = [F(Z + \Delta S) - F(Z)] / [1 - F(Z)]. \quad (7)$$

Hence, under the condition of $\Delta S \to 0$, the SPA estimator is expressed as

$$\delta AC(s, S) / \delta S =$$
$$\sum_{i \in M} \lim_{\Delta S \to 0} E[\Delta AC(s, S) \mid \alpha_{i-1} \le \Delta S] \lim_{\Delta S \to 0} Pr[\alpha_{i-1} \le \Delta S] / \Delta S$$
$$= \hat{E}[\Delta AC(s, S) \mid \alpha_{i-1} \le \Delta S] \times$$
$$\lim_{\Delta S \to 0} [F(Z + \Delta S) - F(Z)] / \Delta S \times 1 / [1 - F(Z)]$$
$$= \hat{E}[\Delta AC(s, S) \mid \alpha_{i-1} \le \Delta S] \times f(Z) / [1 - F(Z)]. \quad (8)$$

As we see in equation (1) and (5), $\Delta AC(s, S)$ and $\hat{E}[\Delta AC(s, S)]$ are the time average cost over a period. Therefore the total gradient $\delta AC(s, S) / \delta S$ can be estimated as the sum of the IPA and SPA components.

Next we find the $\delta AC(s, S) / \delta s$ in a similar way of obtaining the $\delta AC(s, S) / \delta S$. Figure 4 shows that a potential ordering change situation occurs at period $i-1$ if the inventory level X_{i-2} is above s and X_{i-1} is below s. If a small perturbation Δs causes that $X_{i-1} > s - \Delta s$, then no order is placed in the perturbed sample path. Thus an actual ordering change occurs at period i

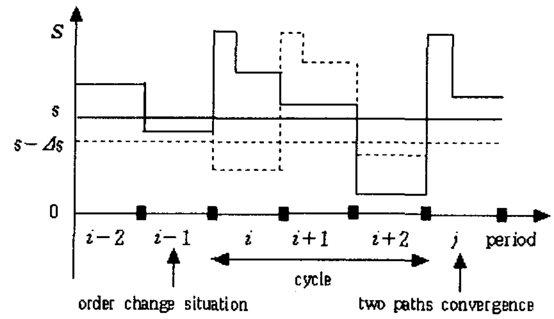

Figure 4. Order Change Situation and convergence of two paths

if $\alpha_{i-1} = s - X_{i-1} \le \Delta s$. In a similar procedure developing the equation (6), we find that P_i is given by

$$Pr(\alpha_{i-1} \le \Delta s \mid X_{i-1} < s)$$
$$= Pr(D_{i-1} \le Z_{i-1} + \Delta s \mid D_{i-1} > Z_{i-1})$$
$$= [F(Z_{i-1} + \Delta s) - F(Z_{i-1})] / [1 - F(Z_{i-1})]. \quad (9)$$

When an actual ordering change occurs due to Δs, from the simulation run, we estimate $E[\Delta(AC(s, S)]$ in the same way as given in equation (5). Finally we calculate the derivative estimate of s as follows:

$$\delta AC(s, S) / \delta s = \sum_{i \in M} \lim_{\Delta s \to 0} E[\Delta AC(s, S) \mid \alpha_{i-1} \le \Delta s]$$
$$\times \lim_{\Delta s \to 0} Pr[\alpha_{i-1} \le \Delta s] / \Delta s$$
$$= \hat{E}[\Delta AC(s, S) \mid \alpha_{i-1} \le \Delta s]$$
$$\times \lim_{\Delta s \to 0} [F(Z + \Delta s) - F(Z)] / \Delta s \times 1 / [1 - F(Z)]$$

$$= \hat{E}[\Delta AC(s,S) \mid a_{i-1} \leq \Delta s] \times f(Z)/[1-F(Z)] \quad (10)$$

This equation is given by the same form as the equation (8). From equation (3), we note the gradient for s is equal to 0 for IPA case. Hence the total gradient of $\delta AC(s,S)/\delta s$ is equivalent to SPA component.

3. Stochastic Optimization Algorithm

The stochastic optimization method can be used for simulation optimization problem by incorporating PA to estimate the derivative of performance measure w.r.t system parameters [4, 6]. Through a single simulation run, we compute the gradients of $AC(s,S)$ w.r.t s and S, and we use these gradients effectively for hill climbing of unknown cost function.

Suppose that we simulate the system with a given point (s,S) for N periods. Based on this simulation run, we compute the gradient of $AC(s,S)$ w.r.t s and S by using PA algorithm and then simulate the system with the updated s and S repeatedly. For searching the minimum $AC(s,S)$, we continuously move other points of (s,S) such that we have the improved $AC(s,S)$. In this way, an estimate of the optimum (s,S) can be obtained at the end of a simulation run. This searching scheme is similar to that of the non-linear optimization method with constraint of $0 \leq s \leq S$. The usual form of a hill climbing optimization scheme uses an iterative algorithm:

$$s_{n+1} = s_n + a_n[\delta AC(s,S)/\delta s] \text{ and}$$
$$S_{n+1} = S_n + a_n[\delta AC(s,S)/\delta s], \quad (11)$$

where s_n and S_n are the updated s and S respectively; and a_n is the moving weight at the nth iteration [15]. We now present the SOA and discuss this algorithm.

Stochastic Optimization Algorithm

(0) Set the iteration number $n=1$. Choose initial values of s_n and S_n.

(1) Simulate the system at s_n and S_n during the N periods and estimate the derivatives of $\delta AC(s,S)/\delta s$ and $\delta AC(s,S)/\delta S$ by PA presented in Section 2.

(2) Update the s and S as follows: $s_{n+1} = s_n + a_n[\delta AC(s,S)/\delta]$ where $a_n = A/n$ with $A=0.2$.

(3) Check the condition that $S_n \geq s_n \geq 0$. If $S_n \geq s_n \geq 0$ then go to step 4. Otherwise reduce the moving size in (2) by changing the value of $a_n = A/2n$.

(4) Stopping criterion: If $a_n[\delta AC(s,S)/\delta s] < \varepsilon$ and $a_n[\delta AC(s,S)/\delta S] < \varepsilon$, then stop and find estimate for the optimum(s^*, S^*). Otherwise go to step (1) with $n=n+1$.

At step (1), the gradients are stochastic and their values depend upon the simulation periods. So their estimates may have noises. Similarly to the stochastic optimization procedure [15, 18], step (2) adopts a_n to satisfy the conditions;

$$\lim_{n \to \infty} a_n = 0, \quad \sum_{n=1}^{\infty} a_n = \infty \text{ and } \sum_{n=1}^{\infty} a_n^2 < \infty; \quad (12)$$

and simply chooses a constant multiple of reciprocal of the iteration number n. Such a choice results in larger moving length of s_n and S_n in the first few steps, on the other hand, the moving length will be shorter as n increases.

Step (3) is for maintaining the condition of $S \geq s \geq 0$. When estimates of derivatives include large errors and moving sizes of first-few-stages are not small, the updated s_n may be greater than S_n. In this case, we shorten the length of moving step by half to ensure the condition $S \geq s$. The interpretation of stopping criterion is that if the maximum moving lengths for s_n and S_n are less than the prescribed value ε, we stop the algorithm. A choice of this value gives a trade-off relationship between run length and accuracy of the estimated optimum.

4. Numerical Example

We conduct a set of simulation experiments on the (s, S) inventory model to evaluate the performance of SOA and offer a summary and results. We consider that the demand per period has a standard exponential distribution. We also consider that an ordering cost is 10, and the holding and penalty costs per period are 5 and 50, respectively.

To understand a response surface of $AC(s,S)$ in two dimensional space of (s, S), we simulate this system in the range of $1 \leq S \leq 5.8$ and $0 \leq s \leq S$ for 50,000 period time. Figure 5 shows the shape of response surface of $AC(s,S)$ roughly. From a large set of simulation runs, we guess that an optimal policy which yields the minimum $AC(s,S)^*$ may happen at the point around $(s,S)=(1.2,\ 3.7)$ and $AC(s,S)^* \simeq 16$.

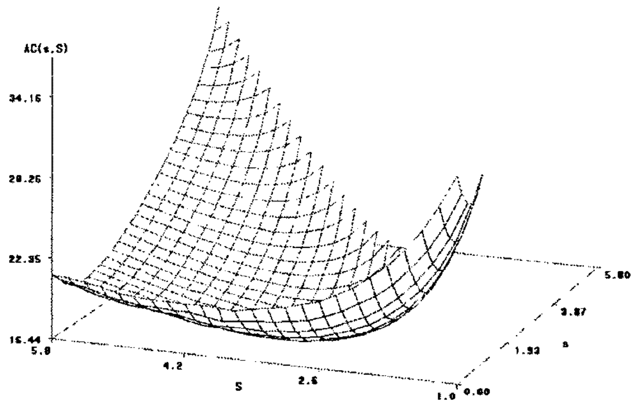

Figure 5. Response Surface of $AC(s,S)$

We simulate this model with the initial values of $(s_1, S_1)=(2,5)$. We obtain the gradient of $AC(s,S)$ w.r.t s_n and S_n from simulation run period $N=10,000$, and then we update the s_n and S_n with increasement of n by 1. We use a stopping rule of algorithm with $\varepsilon=0.01$. A summary of simulation results is presented in Table 1, where gradient estimates of $AC(s,S)$, and $AC(s,S)$ at a point (s,S) are shown in terms of iteration number n. As n increases, the absolute values of both gradients tend to be smaller. At $n=55$, the algorithm stopping criterion is satisfied. We guess that around the optimal point $(s,S)^*$, the gradient estimates of s and S may be close to 0. When we use a stopping criterion that absolute values of gradient estimates should be less than 0.01 (instead of moving length), an estimate of optimal point $(s,S)^*$ is given by (3.65, 1.18) and an estimate for $AC(s,S)^*$ is equal to 15.9. From these results, the SOA is quite accurate in the described example.

Table 1. Estimates for S and s, their gradient, and average cost over period.

n	S	s	$\delta AC(s,S)/\delta S$	$\delta AC(s,S)/\delta s$	$AC(s,S)$
1	5.000	2.000	-2.740	-1.077	18.214
5	4.086	1.571	-1.892	-.417	16.414
10	3.914	1.444	-.811	-.777	16.152
15	3.853	1.383	-.657	-.601	16.063
20	3.820	1.347	-.630	-.455	16.029
25	3.797	1.325	-.752	-.216	15.991
30	3.779	1.309	-.376	-.458	15.987
35	3.766	1.296	-.354	-.420	15.974
40	3,756	1.286	-.340	-.394	15.966
45	3.747	1.277	-.459	-.248	15.948
50	3.740	1.270	-.317	-.351	15.953
55	3.734	1.264	-.432	-.211	15.937

Figure 6 illustrates a searching procedure of SOA based on the results in Table 1. Table 2 presents simulation results obtained by 4 independent replications with simulation run period of 5,000 for updating derivatives.

Table 2. Estimates for S and s, and $AC(s,S)$

n	S	s	$AC(s,S)$
46	3.732	1.331	15.825
63	3.672	1.331	16.920
56	3.799	1.230	16.135
60	3.644	1.329	16.093

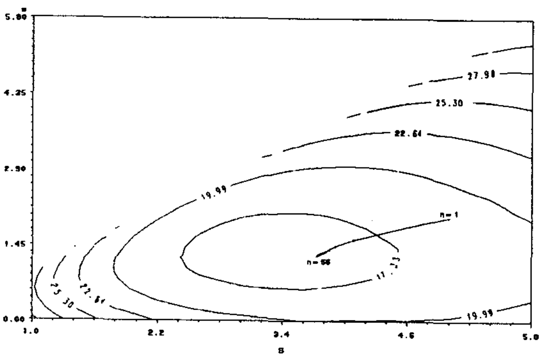

Figure 6. Searching Procedure of Stochastic Optimization Algorithm

5. Conclusion

We apply a SOA to an (s, S) inventory control system. We estimate the gradients of average cost over a period w.r.t. ordering amount S and reorder point s by a combined perturbation method of IPA and SPA. The combined method uses the IPA and SPA alternatively according to occurrences of ordering event changes. Using the estimated gradients, we search the optimum policy by the SOA. Thus far, much of research on the PA focused experimental results demonstrating its accuracy for various systems [7, 8, 14]. As a step in this direction, we here study PA applied to (s, S) inventory system.

Our simulation results show that the estimated optimum is quite accurate. We consider that this is due to little noise occurred in updating the sensitivity and the regenerative property of the (s, S) inventory system. This result is even from a simple example, but we expect that the SOA based on PA may yield very promising results in the general (s, S) inventory system.

Finally we suggest the directions for future study to extend this research to the more general inventory system with regard to demand distribution, backlogging policy, lead time, and review period having renewal arrival process. Another directions may involve the efficiency of SOA in searching procedure for an improving point of (s, S).

References

[1] Andradottir, S., "A Review of Simulation Optimization techniques," *Proceedings of the WSC* (1998), pp. 151-158.

[2] Atkinson, J. B., "A note on a Queueing Optimization Problem," *J. of the Operational Research Society*, Vol. 47 (1996), pp.463-467.

[3] Azadivar, S., "Simulation Optimization Methodology," *Proceedings of the WSC* (1999), pp. 93-100.

[4] Cao, X. R., "Perturbation Analysis of Discrete Event Systems: Concepts, Algorithm and Applications," *European J. of Operational Research*, Vol 91(1996), pp.1-13.

[5] Carson, Y. and A. Maria, "Simulation Optimization Methods and Applications," *Proceedings of the WSC* (1997), pp. 118-126.

[6] Chong, E. K .P. and P. Ramadge, "Optimization of Queues Using Infinitesimal Perturbation Analysis-Based Stochastic Algorithm with General Update Times," *SIAM J.*, Vol. 31, No. 3. (1993), pp. 698-732.

[7] L'Ecuyer, P., N. Giroux and P. W. Glynn, "Stochastic Optimization by Simulation: Numerical Experiments with the M/M/1 Queue in Steady-state," *Management Science*, Vol 40, No.10 (1994), pp. 1245-1261. [8] Ecuyer, P. L. and P. W. Glynn, "Stochastic Optimization by Simulation: Convergence Proofs for the GI/G/1 Queue in Steady-state," *Management Science*, Vol. 40, No. 11 (1994), pp. 1245-1261.

[9] Fu, M. C. "Sample Path Derivatives for (s, S) Inventory Systems," *Operations Research*, Vol. 42, No. 2 (1994), pp. 351-364.

[10]Glasserman P. and S. Tayur, "Sensitivity Analysis for Base-stock Levels in Multi echelon Production inventory Systems," *Management Science*, Vol. 41, No.2 (1995), pp. 263-281.

[12] Ho, Y. C. and X. Cao, Perturbation Analysis of Discrete Event Dynamic Systems, Kluwer Academic Publishers, Boston, 1991.

[13] Iglehart, D., "Optimality of (s, S) Policies in the Infinite Horizon Dynamic Inventory Problem," *Management Science*, Vol. 9, (1963), pp. 259-267.

[14] Lin, C., C. Madu, T.W. Chien and C. Kuei, "Queueing Models for Optimizing Sys-
tem Availability of a Flexible Manufacturing System," *J. of the Operational Research Society*, Vol. 45 (1994), pp. 1141-1155.

[15] Maryak, J. L. and D. C. chin, "Global Random Optimization by Simultaneous Perturbation Stochastic Approximation," *Proceeding of WSC* (2001), pp. 307-312.

[16] Nahmias, S. Production and Operations Analysis, Irwin Press, Boston, 1987.

[17] Scarf, H., "The Optimality of (s, S) Policies in the Dynamic Inventory Problem," Mathematical methods in the Social Sciences, Stanford University Press, Stanford, California, 1960.

[18] Yakowitz, S., L. Ecuyer and F. Vazquez -Abad, "Global Stochastic optimization with Low-dispersion Point Sets," *Operations Research* Vol. 48 (2000), pp. 939-950.

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