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Policy Capturing LP for Ranged Ratings in Performance Appraisal*

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ABSTRACT

For inferring criteria in a performance appraisal, linear programming (LP) has been utilized as an alternative to policy capturing (PC). Previous policy capturing LP (PCLP) studies were limited to the criteria of exact numerical ratings. However, under certain evaluation circumstances, a ranged rating with a lower and upper bound may be preferred over an exact numerical value. Therefore, this study introduces a new LP model that allows ranged ratings. A simple example is given to illustrate our model.

1. INTRODUCTION

Performance appraisals are based on numerical evaluations or rankings as a means of distributing rewards, or motivating and simulating the assessee's development [1]. The inference of criteria weights by an assessor for a performance appraisal is known as policy capturing (PC), where the assessor has a ranked-order of assessees. See Hopson and Gibson [5] for review of the PC literature.

Linear programming (LP) studies have resulted in various applications. LP was first identified by Srinivasan and Shocker [10] as a tool for multidimensional analysis of preferences. Since then, Pekelman and Sen [9] have applied mathematical models to measure weights of consumer preferences. Recently, LP has

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been applied as an alternative to PC for inferring criteria weights in performance appraisal: Zappe et al. [13] utilized LP to determine the post-facto consistency in a performance appraisal of major league baseball players. Horowitz and Zappe [6] proposed the LP as an inference tool for evaluating criteria weights. Furthermore, Kornbluth [7] proposed the LP approach as an interactive method of performance appraisal where a small subset of all assessees were ranked in order to induce a satisfactory ranking of the entire set. Most recently, Jung [7] proposed an LP model including ordinal data in policy capturing. See Horowitz and Zappe [6] for LP aptness as an alternative to PC. Hereafter, we call the LP for policy capturing as PCLP.

The purpose of this paper is to utilize an LP model dealing with ranged ratings (lower and upper bounds). The usefulness of ranged ratings arises when assessors are uncertain of an exact numerical rating for an evaluation criterion in a performance appraisal. Therefore, it may be convenient to assign a range of a lower and upper bound. This approach is compatible with the concept that all ratings (measurements) include some variation [2, 11, 12]. Recently, Cooper et al. [3, 4] addressed DEA (Data Envelopment Analysis) models that included imprecise data.

The remainder of this paper is organized as follows: Section 2 introduces a basic LP model of PCLP. Section 3 presents an extended PCLP model to deal with ranged ratings and then Section 4 gives an example to illustrate the extended model. Final remarks are addressed in Section 5.

2. PCLP BACKGROUND

The LP model for PC refers to the following situation: there are m assessees and n evaluation criteria. The performance measure x_{ij} is the numerical rating of the j th criterion of assessee i. The decision variable w_j is the unknown weight estimated by assessor. Thus, the overall performance of assessee j is measured by a weighted average $V_i = \sum_{j=1}^{n} x_{ij} w_j + e_i$, $i = 1, \cdots, m$, where e_i is a random-error term. In LP for PC, the decision variable w_j is estimated to maximize separation between the ranked assessees. The assessees are assumed to be listed in a non-increasing order of V_i, i.e. $V_i \geq V_{i+1}$ for $i = 1, \cdots, m-1$.

With the above notations, PCLP is formulated as follows:

$$
\text{(PCLP-1)} \quad
\begin{aligned}
& \text{Max} \quad Z \\
& \text{subject to} \\
& \sum_{j=1}^{n} (x_{ij} - x_{(i+1)j})\, w_j \ge Z\,, \quad i = 1, \cdots, m-1\,, \\
& \sum_{j=1}^{n} w_j \ge 1, \\
& Z\,, w_j \ge 0,\ j = 1, \cdots, n
\end{aligned}
$$

If the set of optimal weights of PCLP-1 is given by w_j^*, then the assessee $m+1$ who was not ranked previously can evaluated by $V_{m+1} = \sum_{j=1}^{n} x_{(m+1)j}\, w_j^*$ [13].

3. POLICY CAPTURING LP WITH RANGED RATINGS

Previous studies of PCLP application assume that all evaluation criteria are measured by exact numerical ratings [6, 8, 13]. However, some criteria can also be assessed through ranged ratings.

If criterion k of assessee i is measured with a ranged rating $[L_{ik}, U_{ik}]$, PCLP-1 can be written as:

$$
\text{(PCLP-2)} \quad
\begin{aligned}
& \text{Max} \quad Z \\
& \text{subject to} \\
& \sum_{j=1}^{k-1} (x_{ij} - x_{(i+1)j}) w_j + (x_{ik} - x_{(i+1)k}) w_k \\
& + \sum_{j=k+1}^{n} (x_{ij} - x_{(i+1)j})\, w_j \ge Z\,, i = 1, \cdots, m-1\,, \\
& \sum_{j=1}^{n} w_{ij} = 1, \\
& L_{ik} \le x_{ik} \le U_{ik}\,, \quad i = 1, \cdots, m\,, \\
& Z\,, w_j \ge 0,\ j = 1, \cdots, n\,,
\end{aligned}
$$

where L_{ik} and U_{ik} are lower and upper bounds of criterion k's rating of assessee i and they are greater than or equal to zero. The first constraint of PCLP-2 includes nonlinear terms in the multiplication of two unknown variables, i.e $(x_{ik} - x_{(i+1)k})\, w_k$, where the index k denotes a ranged rating variable. Note that for each ranged scale with m ratings, $m-1$ constraints are added.

In order to transform the nonlinear to linear, let's define t_{ik} as follows:

$$t_{ik} = x_{ik}\, w_k\,, \tag{1}$$

With (1), the nonlinear term in the first constraint of PCLP-2 is transformed as follows:

$$(x_{ik} - x_{(i+1)k})w_k = t_{ik} - t_{(i+1)k}\,, \tag{2}$$

In addition, multiplying w_k to the range constraint in PCLP-2 becomes:

$$w_k L_{ik} \le w_k t_{ik} \le w_k U_{ik}\,, \tag{3}$$

Plugging (1) into (3) generates a new range constraint as follows:

$$w_k L_{ik} \le t_{ik} \le w_k U_{ik}\,. \tag{4}$$

With (2) and (4), nonlinear LP (PCLP-2) can finally be rewritten as an LP model:

$$\text{(PCLP-3)} \qquad \begin{aligned} &\text{Max} \quad Z \\ &\text{subject to} \\ &\sum_{j=1}^{k-1}(x_{ij} - x_{(i+1)j})\, w_j + (t_{ik} - t_{(i+1)k}) \\ &+ \sum_{j=k+1}^{n}(x_{ij} - x_{(i+1)j})\, w_j \ge Z\,, i = 1,\cdots,m-1\,, \\ &\sum_{j=1}^{n} w_j = 1, \\ &w_k L_{ik} \le t_{ik} \le w_k U_{ik}\,,\ \ i = 1,\cdots,m, \\ &Z\,,\ \ w_j \ge 0,\ j = 1,\cdots,n\,. \end{aligned}$$

For generalization of PCLP-3 for more than one criterion rated in range, let Ω_N and Ω_R define index sets of numerical and range criteria, respectively. Then, PCLP-3 can be rewritten as follows:

(PCLP-4)

$$
\begin{aligned}
&\text{Max} \quad Z \\
&\text{subject to} \\
&\sum_{j\in\Omega_X}(x_{ij}-x_{(i+1)j})\,w_j+\sum_{j\in\Omega_X}(t_{ik}-t_{(i+1)k})\geq Z,\ i=1,\cdots,m\ ,\ 1, \\
&\sum_{j=1}^{n}w_j=1, \\
&w_kL_{ik}\leq t_{ik}\leq w_kU_{ik}\ ,\ \ i=1,\cdots,m\ \text{ of }\ k\in\Omega_R\ , \\
&Z\ ,w_j\geq 0,\ j=1,\cdots,n\ .
\end{aligned}
$$

In PCLP-4, if $w_k^*(k\in\Omega_R)$ is an optimal solution, then from (1) the optimal value of x_{ik} becomes:

$$x_{ik}^* = t_{ik}^* / w_k^* \tag{5}$$

In this case, it is impossible to compute an aggregated value for the assessee $(m+1)$ who was not previously ranked. Instead, this approach provides lower bound V_{m+1}^L and upper bound U_{m+1}^L of the aggregate value as follows:

$$
\begin{aligned}
V_{m+1}^L &= \sum_{j\in\Omega_X}x_{(m+1)j}w_j^* + \sum_{k\in\Omega_X}L_{(m+1)k}w_k^* \\
V_{m+1}^U &= \sum_{j\in\Omega_X}x_{(m+1)j}w_j^* + \sum_{k\in\Omega_X}U_{(m+1)k}w_k^*
\end{aligned}
$$

In case of $w_k^* = 0$, an aggregate value V_{m+1} can be computed as in Section 2.

A PCLP-4 with infeasible solution implies that assessors did not assign inconsistent values of evaluation criteria in a performance appraisal. In case of an infeasible solution, assessors are advised to reevaluate assessee's performance. Thus, PCLP can be considered as an iterative computation procedure to meet the post-facto consistency.

4. NUMERICAL EXAMPLE

This section provides a brief example to illustrate our model. Suppose that a data set is given by Table 1 according to a ranked-order of assessees. Table 1 shows six assessees and five criteria, where criterion 2 is assessed by ranged ratings.

Table 1. Data set for example

Ranked-assessee	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5
1	8	[9.5, 10]	7	8	7
2	7	[9.2, 9.7]	8	6	5
3	5	[7.5, 8.0]	7	3	6
4	5	[7.7, 8.1]	8	2	4
5	2	[4.0, 4.5]	8	4	3
6	1	[1.0, 2.0]	7	3	2

With the data in Table 1, the LP formulation of PCLP-4 becomes

$$
\begin{aligned}
&\text{Max } Z \\
&\text{subject to} \\
&w_1 + t_{12} - t_{22} - w_3 + 2w_4 + 2w_5 \geq Z, \\
&2w_1 + t_{22} - t_{32} + w_3 + 3w_4 - w_5 \geq Z, \\
&t_{32} - t_{42} - w_3 + w_4 + 2w_5 \geq Z, \\
&3w_1 + t_{42} - t_{52} - 2w_4 + w_5 \geq Z, \\
&w_1 + t_{52} - t_{52} - w_3 + w_4 + w_5 \geq Z, \\
&9.5w_2 \leq t_{12} \leq 10w_2, \\
&9.2w_2 \leq t_{22} \leq 9.7w_2, \\
&7.5w_2 \leq t_{32} \leq 8.0w_2, \\
&7.7w_2 \leq t_{42} \leq 8.1w_2, \\
&4.0w_2 \leq t_{52} \leq 4.5w_2, \\
&1.0w_2 \leq t_{62} \leq 2.0w_2, \\
&w_1 + w_2 + w_3 + w_4 + w_5 = 1, \\
&Z,\ w_j \geq 0, \quad j = 1, 2, 3, 4, 5.
\end{aligned}
$$

Solving the problem gives the optimal objective function value $Z^* = 1.107$ and the optimal solutions are as follows: $w_1^* = 0$, $w_2^* = 0.357$, $w_3^* = 0$, $w_4^* = 0.286$, $w_5^* = 0.357$, $t_{12}^* = 3.393$, $t_{22}^* = 3.464$, $t_{32}^* = 2.857$, $t_{42}^* = 2.750$, $t_{52}^* = 1.429$, and $t_{62}^* = 0.357$.

From (5), the optimal values of ranged variable x_{ik} become such as $x_{12}^* = 9.5$, $x_{22}^* = 9.7$, $x_{32}^* = 8.0$, $x_{42}^* = 8.0$, $x_{52}^* = 4.0$, and $x_{62}^* = 1.0$. Thus, we conclude that the ratings of our example are consistent on performance appraisals.

5. FINAL REMARKS

Linear programming has been utilized as an alternative to PC for identifying criteria weights in a performance appraisal. This study has presented an LP model to deal with ranged ratings in PC when assessors are uncertain of an exact numerical rating for an evaluation criterion. For each criterion rated in range, nonlinear terms in model are changed to linear terms through a simple transformation. Our PCLP model extends the applicability of the LP as an alternative to PC.

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