Robust and Non-fragile H_∞ State Feedback Controller Design for Time Delay Systems

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Abstract: This paper describes the synthesis of robust and non-fragile H_∞ state feedback controllers for linear varying systems with time delay and affine parameter uncertainties, as well as static state feedback controller with structural uncertainty. The sufficient condition of controller existence, the design method of robust and non-fragile H_∞ static state feedback controller, and the region of controllers satisfying non-fragility are presented. Also, using some change of variables and Schur complements, the obtained conditions can be rewritten as parameterized Linear Matrix Inequalities (PLMIs), that is, LMIs whose coefficients are functions of a parameter confined to a compact set. We show that the resulting controller guarantees the asymptotic stability and disturbance attenuation of the closed loop system in spite of time delay and controller gain variations within a resulted polytopic region.

Keywords: Non-fragile control, robust H_∞ control, time delay, state feedback, Parameterized Linear Matrix Inequality.

1. INTRODUCTION

It is generally known that feedback systems designed for robustness with respect to plant parameters, or for optimization of a single performance measure, may require very accurate controllers [1]. An implicit assumption that is inherent to those control methodologies is that the controller is designed to be implemented precisely. However, the controller implementation is subject to A/D conversion, D/A conversion, finite word length and round-off errors in numerical computations, in addition to the requirement of providing the practicing engineer with safe-tuning margins. Therefore, it is necessary that any controller should be able to tolerate some uncertainty in the controller as well as in the plant [1-9].

In a recent paper, Keel *et al.* [8] have shown that the resulting controllers exhibit a poor stability margin if not implemented exactly. Consequently, some researchers have developed non-fragile controller design algorithms. Dorato *et al.* [1] proposed a non-fragile controller design method via symbolic quantifier elimination. And Haddad *et al.* [5] proposed a robust resilient dynamic controller via quadratic Lyapunov bounds. Famularo *et al.* [4] and Jadbabie *et al.* [6] considered an LQ robust and non-fragile state feedback controller. However, recent researchers have not taken into account the structure of controller gain variations, the value of non-fragility, or the effect of disturbances, non-fragility, and time delay occurring simultaneously. Therefore, our objective is to extend the non-fragile control problem into a robust and non-fragile H_∞ controller design [10] case considering time delay and to obtain a set of controllers that satisfies non-fragility.

In this paper, we propose the synthesis of robust and non-fragile H_∞ state feedback controllers for linear systems with affine parameter uncertainties and time delay in state, as well as a static state feedback controller with polytopic uncertainty. Further, the sufficient condition of controller existence, the design method of the robust and non-fragile H_∞ static state feedback controller, and the region of controllers that satisfies non-fragility are presented. The sufficient condition is presented using PLMIs, that is, LMIs whose coefficients are functions of a parameter confined to a compact set. However, in contrast to LMIs, PLMI feasibility problems involve an infinite number of LMIs, hence are inherently difficult to

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solve numerically. Therefore, PLMIs are transformed into a finite number of LMI problems through the use of relaxation techniques [11, 12].

The paper is structured as follows. The definition of PLMI and basic lemma are described in section 2 while Section 3 discusses robust and non-fragile H^{∞} controller synthesis. A numerical example illustrating robustness and disturbance attenuation is given in Section 4 and our conclusions are discussed in Section 5.

2. PRELIMINARIES

We consider parameterized LMIs (PLMIs), that is, LMIs depending on a parameter θ evolving in a compact set. The parameter θ can designate parameter uncertainties or system operating conditions but virtually appears. Here, a special emphasis is put on PLMIs of the form able to designate parameter uncertainties or system operating conditions but virtually appears. In this case, a particular emphasis is placed on PLMIs of the form

$$M_0(z)+\sum_{i=1}^{L}\theta_i M_i(z)+\sum_{1\le i\le j\le L}\theta_i\theta_j M_{ij}(z)<0, \quad (1)$$

where z is the decision variable, $M_i(z)$, $M_{ij}(z)$ are affine symmetric matrix-valued functions of z and θ is a parameter confined to either the polytope

$$\theta\in\Gamma:=\left\{\theta=(\theta_1,\theta_2,\cdots,\theta_L): \sum_{i=1}^{L}\theta_i=1,\ \theta_i\ge 0,\ i=1,2,\cdots,L\right\} \quad (2)$$

or the parameter hyper-rectangle

$$\theta\in\Gamma:=[p,q];\ p\in\mathbf{R}^L,\ q\in\mathbf{R}^L, \quad p\ge 0,\ q>0,\ p_i\ge 0,\ q_i>0,\ \ i=1,2,\cdots,L, \quad (3)$$

where p_i and q_i are elements of vector p, q each other.

However, PLMI feasibility problems involve an infinite amount of LMIs according to the variations of parameters, hence are very difficult to solve numerically. Computational efforts for locating feasible points are expected to be much greater than those of LMIs. In this paper, we use relaxation techniques where PLMIs are replaced by a finite number of LMIs. Such approaches are potentially conservative but often provide practically exploitable solutions of difficult problems with a reasonable computational effort.

Lemma 1: [12] The PLMI problem (1) and (2) has a solution z whenever the following quadratic conditions hold,

$$x^T M_0(z)x+\sum_{i=1}^{L}\theta_i x^T M_i(z)x + \sum_{1\le i\le j\le L}\max\Big\{-x^T M_{ij}(z)x \cdot\left(\frac{\theta_i^2+\theta_j^2}{2}-\frac{\theta_i+\theta_j}{2}+0.125\right), \quad (4)$$

$$x^T M_{ij}(z)x\frac{\theta_i^2+\theta_j^2}{2}<0,$$

$$\forall\ \|x\|=1,\ \alpha\in \text{vert}\,\Gamma.$$

The latter conditions are readily rewritten as LMIs and can be easily expressed as an LMI feasibility problem.

Remark 1: It should be noted that

$$\max\left\{-x^T M_{ij}(z)x\left(\frac{\theta_i^2+\theta_j^2}{2}-\frac{\theta_i+\theta_j}{2}+0.125\right),\ x^T M_{ij}(z)x\frac{\theta_i^2+\theta_j^2}{2}\right\} \quad (5)$$

is a tight upper bound of $\theta_i\theta_j x^T M_{ij}(z)x$ with $\theta_i+\theta_j\le 1$. Therefore, if the set Γ is alternatively defined as

$$\Gamma:=\left\{\theta=(\theta_1,\theta_2,\cdots,\theta_L): \sum_{i=1}^{L}\theta_i=a,\ \theta_i\ge 0,\ i=1,2,\cdots,L\right\} \quad (6)$$

with $a>1$, one can use the change of variable $\overline{\theta}_i=\theta_i/a$ to recover the case $\overline{\theta}_i+\overline{\theta}_j\le 1$. Analogously, applying the change of variable $\overline{\theta}_i=(\theta_i-p)/(q-p)$ to the constraint (3) yields the relation

$$\overline{\alpha}\in[0,\ \ 1]^L. \quad (7)$$

3. ROBUST AND NON-FRAGILE H^{∞} CONTROLLER DESIGN

Consider a linear time variance uncertain system

$$\dot{x}(t)=A(t,\alpha)x(t)+A_d(t,\alpha)x(t-d) + B_1(t,\alpha)w(t)+B_2(t,\alpha)u(t) \quad (8)$$
$$z(t)=C(t,\alpha)x(t)$$

where $x(t) \in \mathbf{R}^n$ is the state, $u(t) \in \mathbf{R}^m$ is the control input, $w(t) \in \mathbf{R}^r$ is the square integral disturbance input, $z(t) \in \mathbf{R}^q$ is the controlled output, and matrices $A(t,\alpha), A_d(t,\alpha), B_1(t,\alpha), B_2(t,\alpha),$ and $C(t,\alpha)(t \geq 0)$ contain affine uncertainties of the form

$$\begin{aligned} &A(t,\alpha)A_0 + \sum_{i=1}^{L} \alpha_i(t) A_i, \\ &A_d(t,\alpha)A_{d0} + \sum_{i=1}^{L} \alpha_i(t) A_{di}, \\ &B_1(t,\alpha)B_{10} + \sum_{i=1}^{L} \alpha_i(t) B_{1i}, \\ &B_2(t,\alpha)B_{20} + \sum_{i=1}^{L} \alpha_i(t) B_{2i}, \\ &C(t,\alpha)C_0 + \sum_{i=1}^{L} \alpha_i(t) C_i. \end{aligned} \tag{9}$$

and it is assumed that

(A1) the state-space data $A(t,\alpha), A_d(t,\alpha),$ are bounded continuous functions of α

(A2) the time-varying parameter $\alpha(t)$:

$$\underline{\alpha_i} \leq \alpha_i(t) \leq \overline{\alpha_i}, \quad 1 \leq i \leq L, \quad \forall t \geq 0. \tag{10}$$

The assumption (A1) and (A2) are general and they secure existence and uniqueness of the solutions.

Although one finds the robust H state feedback controller $u(t) = Kx(t)$, the actual controller implemented is assumed as

$$\begin{aligned} &u(t) = K(t,\beta)x(t), \; K(t,\beta) = \sum_{j=1}^{M} \beta_j(t) K_j, \\ &\sum_{j=1}^{M} \beta_j(t) = 1, \quad \beta_j(t) \geq 0 \end{aligned} \tag{11}$$

where $K(t,\beta)$ is the region of controller variations and K_j is the vertices of polytope. Here, we choose the center of polytope

$$K_0 = \frac{1}{M} \sum_{j=1}^{M} K_j \tag{12}$$

as nominal controller gain. And the region of controller variations is rewritten as

$$\begin{aligned} &K(t,\beta) = K_0 + \sum_{j=1}^{M} \beta_j(t) K_j, \\ &\sum_{j=1}^{M} \beta_j(t) = 1, \quad K_j = K_j - K_0. \end{aligned} \tag{13}$$

Here, the values of K_j indicate the measure of non-fragility against controller gain variations. Now, the closed loop system from (8) and (11) is given by

$$\begin{aligned} x(t) &= [A(t,\alpha) + B_2(t,\alpha)K(t,\beta)]x(t) \\ &\quad + A_d(t,\alpha)x(t-d) + B_1(t,\alpha)w(t), \\ z(t) &= C(t,\alpha)x(t), \\ x(t) &= 0, \qquad t \leq 0. \end{aligned} \tag{14}$$

Our controller design objective is described as follows:

) The closed loop system (14) is asymptotically stable.

) The closed loop system guarantees, under zero initial conditions, $\|z(t)\|_2 \leq \gamma \|w(t)\|_2$ for all non-zero $w(t) \in L_2[0,\infty)$, affine parameter uncertainty of systems satisfying (9) and polytopic uncertainty of controller satisfying (11) or (13) and time delay d.

Therefore, the objective of this paper is to design a robust and non-fragile H state feedback controller K_0 in the presence of time delay and affine parameter uncertainty of system, and polytopic uncertainty of controller. Also the controller guarantees disturbance attenuation of the closed loop system from $w(t)$ to $z(t)$.

Lemma 2: Consider a closed loop system (14) and suppose that the disturbance input is always zero. If there exists positive definite matrix P and controller gain K satisfying

$$\begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix}^T \begin{bmatrix} Z & PA_d(t,\alpha) \\ Ad(t,\alpha)^T P & -I \end{bmatrix} \cdot \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix} < 0, \tag{15}$$

then the closed loop system is asymptotically stable. Here, Z is defined by

$$\begin{aligned} Z &= A(t,\alpha)^T P + PA(t,\alpha) \\ &\quad + I + PB_2(t,\alpha)K(t,\beta) \\ &\quad + K(t,\beta)^T B_2(t,\alpha)^T P. \end{aligned} \tag{16}$$

Proof: The Lyapunov derivative corresponding to the closed loop system with Lyapunov functional $V(x(t),t)=x(t)^T Px(t)+\int_{t-d}^{t} x(t)^T x(t)\,d\tau$ is represented as

$$\begin{aligned}\frac{d}{dt}V(x(t),t)=x(t)^T\Big\{& A(t,\alpha)^T P \\ &+PA(t,\alpha)+I+PB_2(t,\alpha)K(t,\beta) \\ &+K(t,\beta)^T B_2(t,\alpha)^T P\Big\}x(t) \\ &+x(t-d)^T A_d(t,\alpha)^T Px(t) \\ &+x(t)^T PA_d(t,\alpha)x(t-d) \\ &-x(t-d)^T x(t-d).\end{aligned} \tag{17}$$

Therefore, when $\frac{d}{dt}V(x(t),t)<0$, the closed loop system is asymptotically stable. □

Lemma 3: If there exists positive definite matrix P and the vertices of the controller variation polytope $K_j(\,j=0,1,2,\cdots,M\,)$ such that

$$\begin{bmatrix} U & PA_d(t,\alpha) & PB_1(t,\alpha) \\ A_d(t,\alpha)^T P & -I & 0 \\ B_1(t,\alpha)^T P & 0 & -\gamma_2 I \end{bmatrix} < 0, \tag{18}$$

then the closed loop system is asymptotically stable with disturbance attenuation γ and non-fragility. Here,

$$\begin{aligned}U = A(t,\alpha)^T P + PA(t,\alpha) + I \\ +C(t,\alpha)^T C(t,\alpha) + PB_2(t,\alpha)K(t,\beta) \\ +K(t,\beta)^T B_2(t,\alpha)^T P.\end{aligned} \tag{19}$$

Proof: It is noticed that (19) implies (15). Therefore, (19) ensures asymptotic stability of the closed loop system. Under zero initial condition, let us introduce

$$J=\int_0^\infty \left[z(t)^T z(t)-\gamma^2 w(t)^T w(t)\right]dt. \tag{20}$$

Then performance measure (20) for any nonzero $w(t)\in L_2[0,\infty)$

$$\begin{aligned}J &< \int_0^\infty \Big(z(t)^T z(t)-\gamma^2 w(t)^T w(t) \\ &\qquad +\frac{d}{dt}V(x(t),t)\Big]dt \\ &= \int_0^\infty \zeta(t)^T \Psi \zeta(t)\,dt \quad ,\end{aligned} \tag{21}$$

where $\zeta(t)$ and Ψ are defined as

$$\zeta(t)=\begin{bmatrix} x(t)^T & x(t-d)^T & w(t)^T \end{bmatrix}^T,$$

$$\Psi=\begin{bmatrix} U & PA_d(t,\alpha) & PB_1(t,\alpha) \\ A_d(t,\alpha)^T P & -I & 0 \\ B_1(t,\alpha)^T P & 0 & -\gamma^2 I \end{bmatrix}. \tag{22}$$

This $\Psi<0$ implies $\|z(t)\|_2 \le \gamma\|w(t)\|_2$ for any $w(t)\in L_2[0,\infty)$. Therefore, when $\Psi<0$, the closed loop system is asymptotically stable with disturbance attenuation γ and non-fragility. □

Theorem 1: Consider a linear time varying system with affine parameter uncertainties (8). If there exists matrix $Y_j(\,j=0,1,2,\cdots,M\,)$, positive definite matrix Q, and positive constant ρ satisfying

$$\begin{bmatrix} W & A_d(t,\alpha) & B_1(t,\alpha) & QC(t,\alpha)^T & Q \\ A_d(t,\alpha)^T & -I & 0 & 0 & 0 \\ B_1(t,\alpha)^T & 0 & -\rho I & 0 & 0 \\ C(t,\alpha)Q & 0 & 0 & -I & 0 \\ Q & 0 & 0 & 0 & -I \end{bmatrix} < 0, \tag{23}$$

then the closed loop system is asymptotic stable with disturbance attenuation γ and non-fragility. Some variables are defined as follows:

$$\begin{aligned}W &= A(t,\alpha)Q + QA(t,\alpha)^T \\ &+B_0(t,\alpha)Y_0 + Y_0^T B_2(t,\alpha)^T \\ &+\sum_{j=1}^{M}\beta_j(t)\left[B_2(t,\alpha)Y_j + Y_j^T B_2(t,\alpha)^T\right],\end{aligned} \tag{24}$$

$$\rho=\gamma^2, \qquad Y_0 = K_0 Q, \qquad Yj=\tilde{K}_j Q.$$

Proof: Using change of variable $Q=P^{-1}$, (18) is equivalent to

$$\begin{bmatrix} \tilde{U} & A_d(t,\alpha) & B_1(t,\alpha) \\ A_d(t,\alpha)^T & -I & 0 \\ B_1(t,\alpha)^T & 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{25}$$

where $\tilde{U}$ is described as

$$\begin{aligned}\tilde{U} &= A(t,\alpha)Q + QA(t,\alpha)^T \\ &\quad + QQ + QC(t,\alpha)^T C(t,\alpha)Q \\ &\quad + B_2(t,\alpha)K(t,\beta)Q + QK(t,\beta)^T B_2(t,\alpha)^T.\end{aligned} \tag{26}$$

Also (25) can be transformed to (23) using Schur complements and (24). □

Theorem 2: With the assumptions (A1) and (A2), the linear parameter uncertain system (8) is asymptotically stable with disturbance attenuation γ and non-fragility whenever there exists matrix $Y_j(\, j = 0,1,2,\cdots,M\,)$, positive definite matrix Q, and positive constant ρ such that

$$\begin{aligned}&x^T M_0(z)x + \sum_{i=1}^{L}\alpha_i(t)x^T M_i(z)x \\ &+\sum_{j=1}^{M}\beta_j(t)x^T N_j(z)x \\ &+\sum_{i=1}^{L}\sum_{j=1}^{M}\max\Bigg\{-x^T M_{ij}(z)x\left(\frac{\alpha_i(t)^2+\beta_j(t)^2}{2}\right. \\ &\qquad \left.-\frac{\alpha_i(t)+\beta_j(t)}{2}+0.125\right), \\ &\qquad x^T M_{ij}(z)x\frac{\alpha_i(t)^2+\beta_j(t)^2}{2}\Bigg\}, \\ &\qquad \forall \|x\| = 1,\ \alpha(t), \beta(t) \in \operatorname{vert}\Gamma\end{aligned} \tag{27}$$

holds for $z, M_i(z), N_j(z)$, and $M_{ij}(z)$ defined below:

$$M_0(z) = \begin{bmatrix} A_0Q+QA_0^T+B_{20}Y_0+Y_0^TB_{20}^T & A_{d0} & B_{10} & QC_0^T & Q \\ A_{d0}^T & -I & 0 & 0 & 0 \\ B_{10}^T & 0 & -\rho I & & 0 \\ C_0Q & 0 & 0 & -I & 0 \\ Q & 0 & 0 & 0 & -I \end{bmatrix},$$

$$M_i(z) = \begin{bmatrix} A_iQ+QA_i^T+B_{2i}Y_0+Y_0^TB_{2i}^T & A_{di} & B_{1i} & QC_i^T & 0 \\ A_{di}^T & 0 & 0 & 0 & 0 \\ B_{1i}^T & 0 & 0 & 0 & 0 \\ C_iQ & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$N_j(z) = \begin{bmatrix} B_{20}Y_j+Y_j^TB_{20}^T & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{28}$$

$$M_{ij}(z) = \begin{bmatrix} B_{2i}Y_j+Y_j^TB_{2i}^T & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Proof: Using the modified PLMI form

$$\begin{aligned}&M_0(z) + \sum_{i=1}^{L}\alpha_i(t)M_i(z) + \sum_{j=1}^{M}\beta_j(t)N_j(z) \\ &+\sum_{i=1}^{L}\sum_{j=1}^{M}\alpha_i(t)\beta_j(t)M_{ij}(z) < 0\end{aligned} \tag{29}$$

and applying lemma 1, proof is easily obtained. □

Remark 2: (27) is converted to a finite number of LMI problems in terms of $Y_j(\, j = 0,1,2,\cdots,M\,)$, Q, ρ. Therefore, the robust and non-fragile H^∞ state feedback controller K_0 and the region of controllers that satisfy non-fragility can be calculated from the $K_0 = Y_0Q^{-1}$ and $\tilde{K}_j = Y_jQ^{-1}$ after determining the LMI solutions from (27). In addition, the value of disturbance attenuation γ, can be obtained by $\gamma = \sqrt{\rho}$ in (24).

4. NUMERICAL EXAMPLE

Consider a linear time-varying system (8) with affine parameter uncertainty satisfying

$$
\begin{aligned}
A(t) &= \begin{bmatrix} 2 & 2 \\ 1 & -3 \end{bmatrix} + \alpha_1(t)\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + \alpha_2(t)\begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, \\
A_d(t) &= \begin{bmatrix} 1 & 0.3 \\ 0 & -1 \end{bmatrix} + \alpha_1(t)\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} + \alpha_2(t)\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \\
B_1(t) &= \begin{bmatrix} -1 \\ 2 \end{bmatrix} + \alpha_1(t)\begin{bmatrix} 0 \\ -0.5 \end{bmatrix} + \alpha_2(t)\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\
B_2(t) &= \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} + \alpha_1(t)\begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} + \alpha_2(t)\begin{bmatrix} 0.5 & 0 \\ 0 & -2 \end{bmatrix}, \\
C(t) &= \begin{bmatrix} 1 & 1 \end{bmatrix} + \alpha_2(t)\begin{bmatrix} 1 & 0 \end{bmatrix}
\end{aligned} \tag{30}
$$

and parameter $\alpha_1(t), \alpha_2(t)$ is

$$
\begin{aligned}
\alpha_1(t) &= 1.25 + 0.25\sin 2\pi t, \\
\alpha_2(t) &= 1.5 + 0.5\cos 2\pi t
\end{aligned} \tag{31}
$$

therefore $\alpha_1(t), \alpha_2(t)$ satisfying

$$
\alpha_1(t) \in \begin{bmatrix} 1 & 1.5 \end{bmatrix}, \qquad \alpha_2(t) \in \begin{bmatrix} 1 & 2 \end{bmatrix}. \tag{32}
$$

In Theorem 2, all solutions are obtained simultaneously as follows:

$$
\begin{aligned}
Y_0 &= \begin{bmatrix} -212.7792 & -399.1352 \\ -135.6586 & 6.9712 \end{bmatrix}, \\
Y_1 &= \begin{bmatrix} -132.2097 & -198.3016 \\ -80.9713 & 4.4951 \end{bmatrix}, \\
Y_2 &= \begin{bmatrix} -59.5698 & -194.7484 \\ -55.7981 & 1.4157 \end{bmatrix}, \\
Y_3 &= \begin{bmatrix} 191.7795 & 393.0500 \\ 136.7695 & -5.9108 \end{bmatrix}, \\
Q &= \begin{bmatrix} 15.4163 & -38.6545 \\ -38.6545 & 97.4774 \end{bmatrix}, \\
\rho &= 0.0158.
\end{aligned} \tag{33}
$$

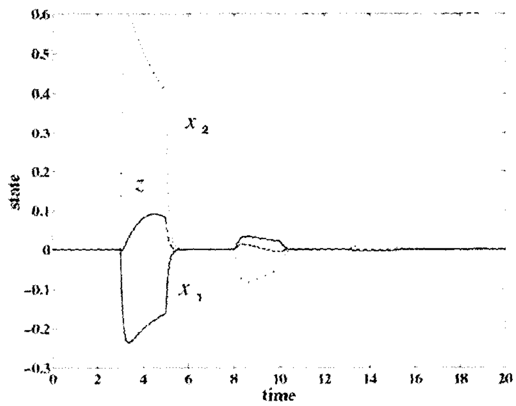
Therefore, the robust and non-fragile H^{∞} state feedback gain, vertex of perturbation satisfying non-fragility, and the value of disturbance attenuation in a closed loop system are represented from the changes of variables (24) as follows:

$$
\begin{aligned}
K_0 &= 10^3 \times \begin{bmatrix} -4.2218 & -1.6782 \\ -1.5121 & -0.5995 \end{bmatrix}, \\
\tilde{K}_1 &= 10^3 \times \begin{bmatrix} -2.3990 & -0.9533 \\ -0.9010 & -0.3572 \end{bmatrix}, \\
\tilde{K}_2 &= 10^3 \times \begin{bmatrix} -1.5565 & -0.6192 \\ -0.6285 & -0.2492 \end{bmatrix}, \\
\tilde{K}_3 &= 10^3 \times \begin{bmatrix} 3.9554 & 1.5726 \\ 1.5295 & 0.6064 \end{bmatrix}, \\
\gamma &= 0.1257.
\end{aligned} \tag{34}
$$

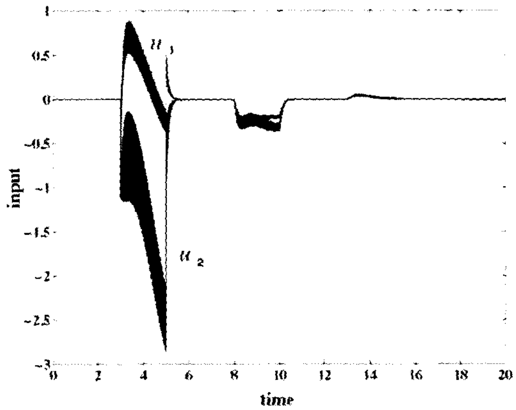
For computer simulation, $d = 5$ and the value of $w(t)$ is defined by

$$
w(t) = \begin{cases} 5, & 3\,\text{sec} \le t \le 5\,\text{sec} \\ 0, & \textit{otherwise} \end{cases} \tag{35}
$$

When nominal controller K_0 is applied, the trajectories of states, controlled output, and control input are provided in Fig. 1. And when the vertices of controller polytope K_2 are applied, the responses are given in Fig. 2. This example shows that the

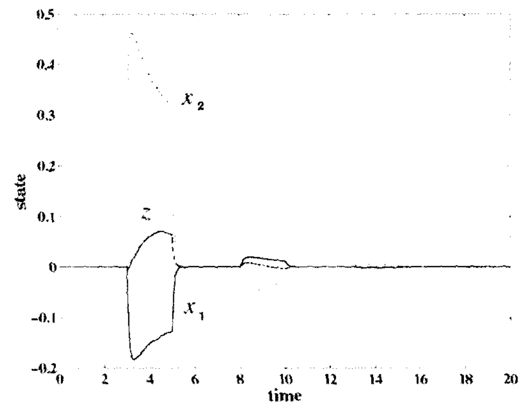


(a) The trajectories of states and controlled output.

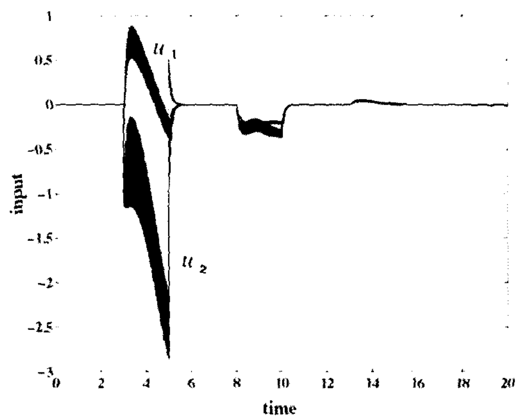


(b) The trajectories of controlled input.

Fig. 1. The case of nominal controller K_0.



(a) The trajectories of states and controlled output.



(b) The trajectories of controlled input.

Fig. 2. The case of vertex K_2.

vertices of controller polytope guarantee the asymptotic stability and disturbance attenuation γ of a closed loop system. Therefore, we conclude that the obtained robust and non-fragile H^∞ controller guarantees the asymptotic stability and disturbance attenuation $\|z(t)\|_2 \leq 0.1257\|w(t)\|_2$ for any $w(t) = L_2[0, \infty)$, in spite of the controller gain variations with the resulted polytopic region.

5. CONCLUSIONS

In this paper, we presented the robust and non-fragile H^∞ controller design method for linear varying systems with time delay as well as the affine parameter uncertainties and state feedback controller with polytopic uncertainty. Moreover, the robust and non-fragile controller, the level of disturbance attenuation, and the region of controllers that satisfy non-fragility were calculated using the PLMI approach. In spite of the controller gain variations within the resulted polytopic region, the obtained robust and non-fragile H^∞ controller guaranteed the asymptotic stability and disturbance attenuation γ of the closed loop system. The area of future research is extension of output feedback case.

REFERENCES

[1] P. Dorato, C. T. Abdallah, and D. Famularo, "On the design of non-fragile compensators via symbolic quantifier elimination," *World Automation Congress in Anchorage, Alaska*, pp. 9-14, 1998.

[2] J. R. Corrado and W. M. Haddad, "Static output feedback controllers for systems with parametric uncertainty and controller gain variation," *Proc. American Control Conference in San Diego, California*, pp. 915-919, 1999.

[3] P. Dorato, "Non-fragile controller design: An overview," *Proc. American Control Conference in Philadelphia, Pennsylvania*, pp. 2829-2831, 1998.

[4] D. Famularo, C. T. Abdallah, A. Jadbabaie, P. Dorato, and W. M. Haddad, "Robust non-fragile LQ controllers: The static state feedback case," *Proc. American Control Conference in Philadelphia, Pennsylvania*, pp. 1109-1113, 1998.

[5] W. M. Haddad and J. R. Corrado, "Robust resilient dynamic controller for systems with parametric uncertainty and controller gain variations," *Proc. American Control Conference in Philadelphia, Pennsylvania*, pp. 2837-2841, 1998.

[6] A. Jadbabie, C. T. Abdallah, D. Famularo, and P. Dorato, "Robust, non-fragile and optimal controller via linear matrix inequalities," *Proc. American Control Conference in Philadelphia, Pennsylvania*, pp. 2842-2846, 1998.

[7] L. H. Keel and S. P. Bhattacharyya, "Digital implementation of fragile controllers," *Proc. American Control Conference in Philadelphia, Pennsylvania*, pp. 2852-2856, 1998.

[8] L. H. Keel and S. P. Bhattacharyya, "Robust, fragile, or optimal," *IEEE Trans. Automatic Control*, vol. 42, no. 8, pp. 1098-1105, 1997.

[9] J. H. Kim, S. K. Lee, and H. B. Park, "Robust and non-fragile H^∞ control of parameter uncertain time-varying delay systems," *SICE in Morioka*, pp. 927-932, July 1999.

[10] S. H. Cho, K. T. Kim, and H. B. Park, "Robust and non-fragile H^∞ controller design for affine parameter uncertain systems," *Proc. IEEE Conference on Decision and Control in Sydney*, pp. 3224-3229, 2000.

[11] P. Apkarian and H. D. Tuan, "Parameterized LMIs in control theory," *Proc. IEEE Conference on Decision and Control in Florida*, pp. 152-157, 1998.

[12] H. D. Tuan and P. Apkarian, "Relaxations of parameterized LMIs with control applications," *International Journal of Robust Nonlinear*

Control, vol. 9, pp. 59-84, 1999.
[13] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, 1994.
[14] P. Gahinet and P. Apkarian, "A linear matrix inequality approach to H^∞ control," *International Journal of Robust Nonlinear Control*, vol. 4, pp. 421-448, 1994.

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