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Professional Development for Teachers of Mathematics[1]

Donaldson, John D.
School of Mathematics and Physics, Faculty of Science, Engineering and Technology, University of Tasmania, Australia; E-mail: john.donaldson@utas.edu.au
Current Address: Division of Applied Mathematics, School of Natural Science, KAIST, Guseong-dong, Yuseong-gu, Daejeon 305-701, Korea

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At a time when mathematics is becoming more important in our everyday lives and more relevant in applications in industry and the emerging technologies, there are signs of a decrease in numbers of students and their interest in the subject. Teachers must be encouraged to take a new approach to generating enthusiasm amongst students by showing them that mathematics is an integral part of the future. To achieve this, opportunities for renewal of teachers' knowledge and updating of skills should be made available. In this paper, emphasis is placed on mathematics in the real world and how it can be used to develop the more general skills such as self-teaching and communication which are an essential part of preparation for entry into higher education or the workplace.

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ZDM classification: B59, C79
MSC2000 classification: 97B50, 97C70

1. INTRODUCTION

As teachers, one of our main purposes is to hand down knowledge to our students. But, we must take an approach to this task which will enthuse our students to take a lasting interest in our subject so that the process will continue from generation to generation. To achieve this we, ourselves, must have enthusiasm, an enthusiasm that is continually revitalized.

At universities and institutes of higher learning, this renewal occurs naturally through

[1] This paper will be presented as an Invited Lecture at the Eighth International Seminar on Education of Talented Children in Mathematics at the Daegu National University of Education, Daegu, Korea, October 3, 2003.

the daily contact with colleagues, in our and related disciplines, who are keenly pushing back the frontiers of knowledge.

There is a need for a corresponding environment for teachers of pre-university students, if not on a daily basis, then often enough to have a continuing effect on their approach to the teaching of their discipline. The motivated teacher will be better prepared to motivate the students.

With this aim, the Departments of Education and Mathematics at the University of Tasmania, Australia, combined to create such an environment by developing a Professional Development Programme for teachers of Mathematics, in which the teacher would acquire new knowledge and skills that could be effective in challenging students in the classroom. A primary requirement of the new knowledge was that it should have relevance and interest. The programme included contributions from 15 academics and teachers of mathematics on a variety of topics from chaos to music and art (Chick & Watson 1994).

In this talk, consideration is given to the issues facing the teaching of the discipline of mathematics and the problems confronting both teachers and students in relation to these issues. We will then consider how we can respond to the challenges posed, by asking what our society and employers want from education. The skills required of today's students are delineated and how teachers can be assisted in imparting these skills is outlined.

Analysing the approach that mathematicians take when attempting to solve problems in the real world, it is evident that mathematics can and must play a significant role in developing the more general education required of our students. This is illustrated by following through an example of mathematics in nature. Shin & Han (2000) state that 'the curriculum for the Education of Gifted Students in Science should include an encouragement of students to recognize mathematics as an integral part of (the) nature and (the) everyday life…'. We should aim at this for all students.

2. MOTIVATING THE MOTIVATORS

In today's complex society where logical decision making is part of every day life, one could expect that mathematics would be more important rather than less. Unfortunately this is not reflected in a growth in numbers or quality of students. Certainly, there are many extremely talented students who are involved and participate in the furthering of the subject and its applications. But there are many more talented and capable students who have not been given the opportunity to take part. Even more of a problem is the number of students who regard mathematics as irrelevant. Students must be motivated to realize the position of mathematics as essential to the future of our world and that they, the students, are that future. Our governments and industries need people with the skills that come from studying mathematics at all levels.

It is a challenge that our teachers have confronted, are confronting and will continue to confront. They must be given every assistance and appropriate preparation to meet the challenge.

2.1. Problems for Teachers

It will be argued that our teachers are under enough pressure without giving them more. However this task has its natural rewards in watching students progress and take an interest in the subject. To enable this, efforts to alleviate other problems should be made.

The problems facing the teaching of mathematics in Australia, UK and US, and we suspect in other countries like South Korea, include class sizes which are too big, lack of discipline in the classroom, less time being spent on mathematics, the strict adherence to the curriculum and, because of the reduced numbers of graduates, the use of teachers without appropriate qualifications or experience. Not all of these problems belong in a Professional Development Programme, but after attendance teachers should be more prepared to confront them and gain more satisfaction from having greater confidence in their approach to teaching.

2.2. Incentives

Teachers cannot be expected to take on new roles without incentives. In UK, special allowances are being paid to enter the teaching profession in areas of need, in particular in mathematics. There should be additional incentives such as smaller classes and free time to attend development or refresher programmes.

2.3. Student Problems

Mathematics is regarded by students as one of the difficult subjects. The relative rewards do not bear close relationship with the efforts needed to obtain good results. To gain a high university entry score is considered much easier in other disciplines unless one is already accomplished in mathematics. The student who is good at mathematics is considered to be out of mainstream, often described as a 'nerd'. In the community being good at mathematics is seen to be extremely unusual. How often do we hear 'I was never any good at maths? What use can it possibly be?'

Mathematics is subject to more and more competition from other new disciplines such as drama, environmental studies, civics and management. These subjects compete for time during the school day resulting in a decrease in time for mathematics. Other disciplines like biology, psychology and finance are more glamorous and fashionable, being featured prominently in our daily newspapers and television news programmes. Disciplines such as medicine and law offer high returns in salary and students will take those courses, which will prepare them for entry to the corresponding university course. It is notable that even though engineering still requires mathematics for entry there is a reduced amount needed to graduate. Strangely, in some universities no mathematics is required for computing, a course requiring so much logic.

Many disciplines have updated to the computer age and are appealing to students through the use of a plethora of wonderful modern imagery available on the World Wide Web as illustrations. Students spend considerable extra time in drama and music rehearsing for music festivals and eisteddfods. Do mathematics teachers take such an approach or is a computer merely a high powered calculator? A computer can do wonderful arithmetic and draw graphs very quickly, but much more.

Why do we not have as many mathematics festivals?

Adding that mathematically trained people are finding it more difficult to obtain employment outside education, the difficulties that teachers of mathematics have in convincing students of its usefulness and rewards are considerable.

3. RESPONDING TO THE CHALLENGES

Different students react to different approaches. Some students see the patterns in the mathematics itself and are content to work as if in a closed field. However many more want to know why they are doing mathematics and what is its relevance for the practical world and for them. How can computers in conjunction with mathematics be used to solve real problems? In what other disciplines is mathematics used and what advantages does it have?

In experimenting with the population and chaos examples[2] mentioned later in section 10, Pennel (2003)[3] uses a simple computer programme to illustrate the 'Unreasonable Effectiveness of Mathematics' and Vivaldi (2001)[4] suggests that

> 'Computers have added a new dimension to the experimental side of mathematics and have made mathematical experimentation as fruitful and as tangible as that of the physical

[2] For a simple population experiment leading to chaos, try http://www.geocities.com/athens/aegean/9116/chaos.html.

[3] See also Pennel, S.: *The Unreasonable Effectiveness of Mathematics* (2003), http://www.faculty.uml.edu/spennel/59.260/Math.Modeling.pdf.

[4] See also Vivaldi, F.: *An Experiment with Mathematics* (2003), http://www.fortunecity.com/emachines/e11/86/expmaths.html.

world.'

Students are naturally inquisitive and this should be exploited by allowing them to explore. As early as possible, they should be teaching themselves, with guided help from the teacher. At some stage, self-teaching will be an essential attribute in their lives.

4. RELEVANCE

In today's world, nearly all decisions that are made must have some relevance to creating more wealth in the community. This wealth may be measured in terms of more material goods, better education, better health systems or a better environment amongst other things. What we teach in the classroom has to have more relevance. Civics is now taught in schools because politicians want their future constituents to be more aware of government: in universities, engineers are taught management skills, medical practitioners are taught bedside manner.

What we do and how we do it is driven for the most part by those who pay. If it is industry, it is usually directed and short term. General engineering industries, like carmakers, electronics, communications, finance, medicine and pharmacy want answers to problems now. They want specifically trained graduates.

Governments must have the vision to prepare for the future. This future is to be prepared for emerging industries and general improvements in areas such as health and the environment. Education must be geared towards this end. Dr. Barry Jones, a former Educational Minister in the Australian Government, when referring to Science, said that we must work 'Smarter not Harder' if we are to keep up with the rest of the world. Mathematics is the key to achieving that goal. The preparation of students must or will be required to respond to this desire for relevance.

5. ATTRIBUTES OF STUDENTS

In addition to the underlying basic knowledge other skills will be required of people entering the employment market, attributes which are relevant to the position. They will include the ability to self-teach, communication skills, new knowledge acquisition skills, written and oral presentation skills, and the ability to work as part of a team (Coxford 1998a; 1998b; 1999).[5]

Professor Kwyro Lee of KAIST, in an interview, 'Computing-Everywhere, Every-

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[5] See also Fadness, J.: *Contemporary Mathematics in Context, Summer Workshop* (2003), http://www.nden.k12.wi.us/nwacad/cpr.htm.

where-Computers,' stresses that

> 'People fail to recognise the importance of collaboration (group work)' and 'Although knowledge is important the ability to communicate is just as important. Students should therefore try to improve their communications skills.'

Self-teaching and the skill to acquire new knowledge can be introduced early in a student's education and teachers of mathematics are in an ideal position to assist and guide students in this pursuit.

Student must become aware of how powerful a tool the computer is in solving problems in mathematics. The curriculum for gifted students refers to the 'fullest use of computers in teaching and learning mathematics' (Shin & Han 2000).

6. BACKGROUND KNOWLEDGE OF MOTIVATORS

Professional Development Programmes should be available to enable teachers to add to their skills, those needed to motivate students to acquire these attributes. The programme should broaden their knowledge and experience by containing examples of mathematics relevant to our real world and the way in which it could be introduced to create opportunities for self-teaching and acquisition by students.

Examples could be drawn from areas such as mathematics in art, mathematics in music, mathematics in nature. A wide range should be provided to appeal to different teachers. In this context, Chick (1994) has drawn examples from fractals in nature and population growth while Donaldson (1994) describes patterns and continuity in nature.

In addition the teachers could be given appropriate computer training and part of the programmes could be devoted to specialist educational training such as gaining information on how to recognise and work with talented students and also with students who are less mathematically oriented.

7. WHAT DOES MATHEMATICS DO?

Before looking at a specific example it is important to know what mathematics attempts to do in a real world situation.

Barrow (1992) in relationship to science and consequentially to mathematics expresses the motivation as

> 'It seems to be a powerful human drive to demonstrate one's insight, wisdom and inside knowledge about the hidden causes of everything.'

Firstly we recognise a pattern such as the daily rising of the sun. Mathematics as the language of science sets out to describe the pattern in mathematical terms. This usually provides a deeper understanding of the pattern. Deeper inquiry, usually by finding equations involving changes of the patterns, leads to further information. The next step is usually to introduce external influences to control the pattern to advantage as suggested by the mathematical description.

8. THREE ELEMENTARY CHALLENGES

Ask your students if they can count to a million. They will all say 'Of course'. Then ask 'How long it will take?'

Take a piece of paper and repeatedly fold it, six times is enough. Suggest to the students that if they folded the paper once a week for a year and turned it on its side, they can walk along the edge to the moon. They should be able to show this using simple arithmetic and patience. On the practical side they will soon discover that folding the paper more than seven times is extremely difficult but then you can add another sheet of paper and so on. Again ask how long it will take and how much paper is needed.

Show your students how Pascal's triangle can be predicted by an experiment involving the repeated tossing of several coins at one time or vice-versa.

9. AN EXAMPLE IN NATURE

This example promotes self-teaching and new knowledge acquisition. Students should be encouraged to work in groups on different parts of the general problem and bring their findings to the attention of the whole class. The teacher should guide but the students should use their natural inquisitiveness.

Present the students with a photograph of different types of flowers including a daisy and a sunflower (see Figures 1, 2 and 3). They could bring real ones to the class. Ask what they observe. The replies will usually be a pretty flower or lots of colours. Ask them to look more closely and they will observe petals on the outside and little florets in the center. A more close observation should reveal the two sets of spirals formed by the florets. The next step is to count the number of petals and the number of spirals.

The flowers often have 5, 8 and 13 petals and 13, 21, 34, 55 and 89 spirals. Putting them together we have the Fibonacci[6] sequence and students will be able to discover how

[6] Websites for Fibonacci numbers and the golden ratio include
www.mcs.surrey.ac.uk/personal/R.Knott/Fibonacci/fib.html
www.branta.connectfree.co.uk/fibonacci.htm.

the sequence is constructed with a little help.

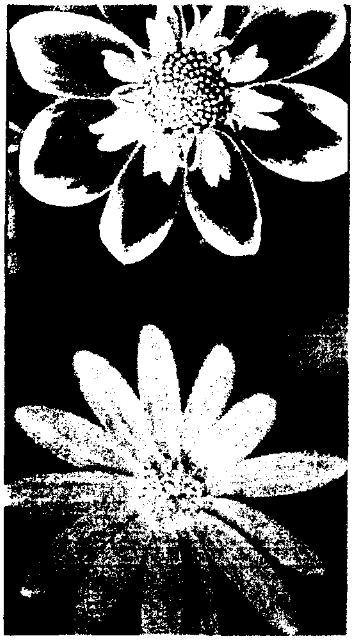

*Figure*1. Flowers: Colours and Petals: 8 and 13

Figure 2. Spirals of Daisies 13 and 21

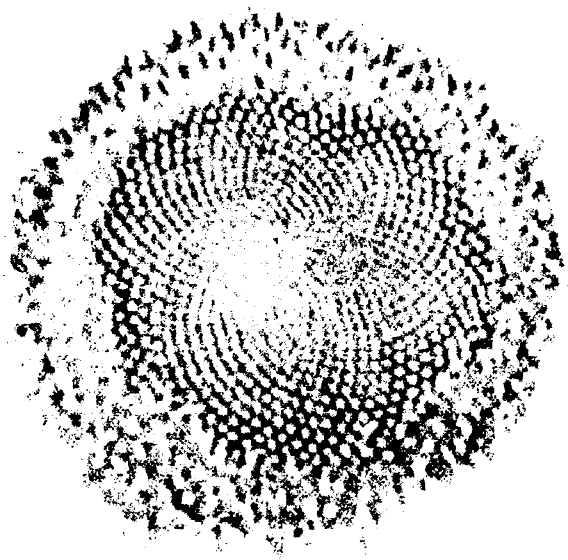

Figure 3. Spirals of a Sunflower 34 and 55

Now present the students with photographs of some architectural masterpieces such as the great Pyramid of Giza and the Parthenon in Athens, a violin and an ancient vase (see Figure 4). They will need help, but they will be able to observe that there is a consistency in the ratios of some of the lengths in the objects.

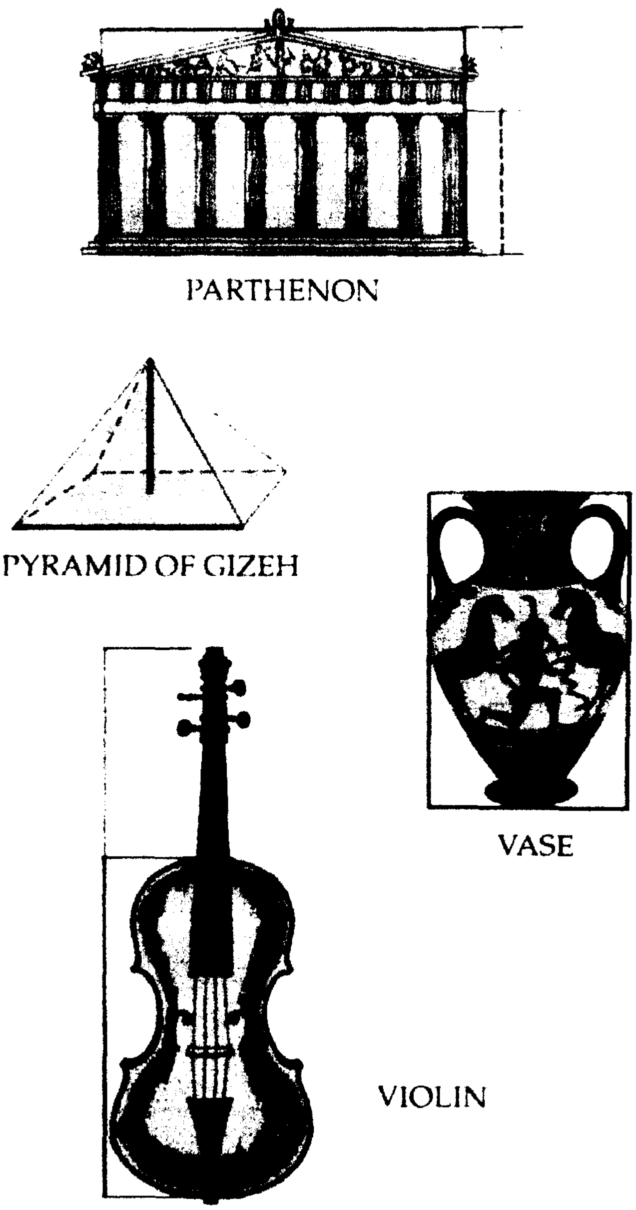

Figure 4. Golden Ratio: Man Made

It has been agreed that the flowers in nature are pretty while the man-made objects are also pleasing to the eye. Is there a connection?

Take the ratio of consecutive members of the Fibonacci sequence,

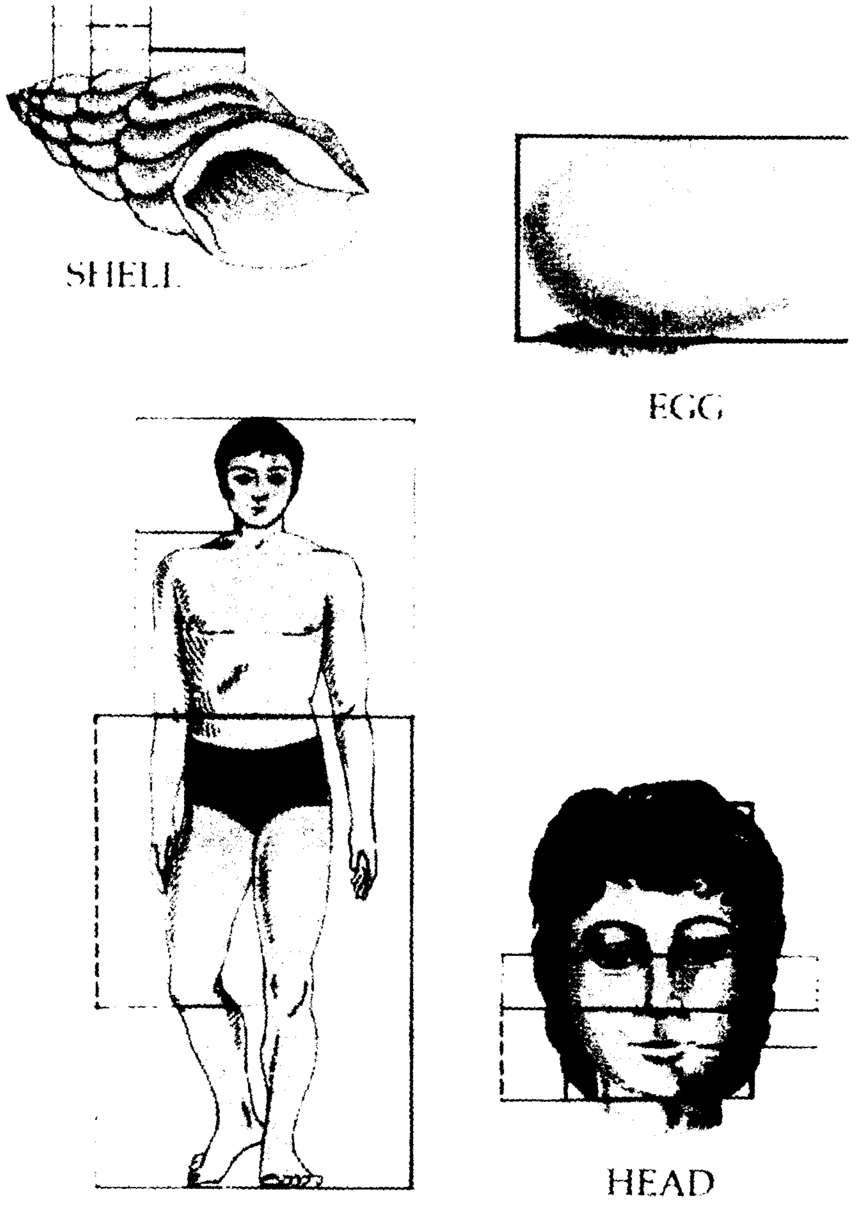

Figure 5. Golden Ratio: Nature

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

We have 1, 2, 1.5, 1.666..., 1.6, 1.625, 1.615..., 1.619..., which seems to be stabilising about 1.62... $(= (1+\sqrt{3})/2)$.

This is equal to the ratio previously observed. It is called the golden ratio.

Students should now be encouraged to look for other examples. Members of the sequence can be found in apple cores and on the surface of pineapples. The golden ratio can be found in seashells, branches in trees, eggs and the human head and body, see Figure 5. Obtain a copy of Leonardo da Vinci's 'Mona Lisa' and investigate some of the measurements in the painting. It also occurs in a regular pentagon (Figure 6(c)).

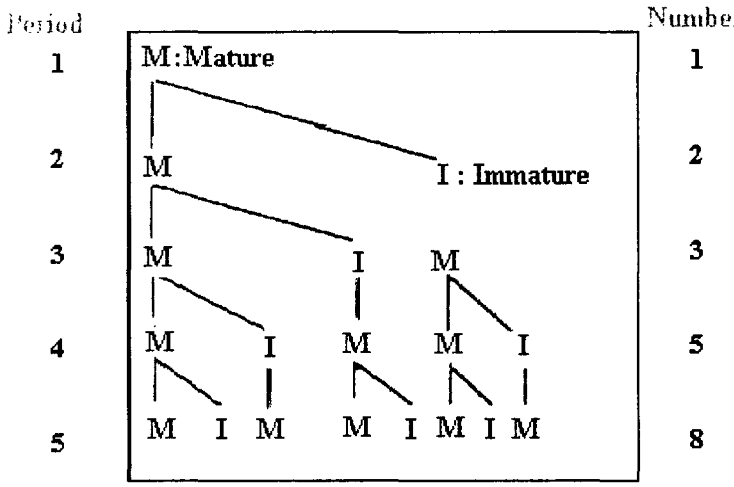

Figure 6(a): Population Growth - Fibonacci sequence

Golden Ratio

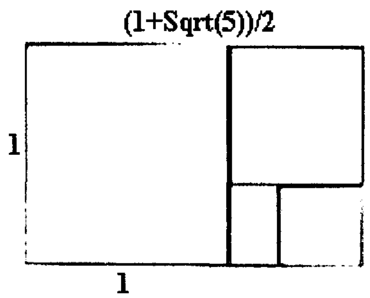

Figure 6(b): Subdivision of the golden rectangles into similar rectangles

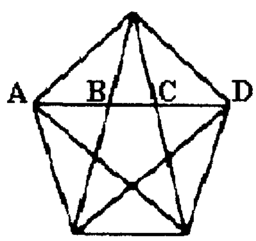

Figure 6(c): Pentagram BC/AB=AB/AC=AC/AD =(1+Sqrt(5))/2

Why do these numbers appear so often? This is too difficult a question, but it is suggested that it is an optimal growth process, in some cases the available area for growth is optimized and in others it is the area exposed to sunlight (Stewart 1995; 1997).[7]

10. GROWTH OF A POPULATION

Consider a population of rabbits starting from a single rabbit (see Figure 6(a)). A mature rabbit is responsible for the birth of an immature rabbit at the end of each gestation period. The immature rabbit takes one gestation period to become mature. What is the number of rabbits after 6 or 10 gestation periods? What limitations are suggested?

Hopefully the teacher will be motivated to study this problem a little further. The model can be made more realistic and the study uncovers some interesting examples which serve to introduce the fascinating mathematical theory of Chaos (*cf.* Gliek 1997, pp. 59–77; Stewart 1997 and the references to the WWW).

11. CONCLUSION

Mathematics is confronted by challenges on many fronts, not the least of which is its apparent lack of relevance to people in government and industry. Fortunately, the diversity of the discipline empowers it with the ability to respond successfully.

In this talk we have examined what general skills are expected from our students and shown how mathematics can be used to attain them.

However to acquire these skills we must provide them with teachers who have the knowledge and as a result the confidence to enthuse and motivate our students to realise the integral and essential part that mathematics will play in their future.

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[7] See also the WWW sites listed in the footnotes 2and 6.

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