

Conceptual and Procedural Learning in Mathematics

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Işık & Kılıç (1998: Mathematics Education and its Appraising in the Primary School Teacher Certificate) found that many prospective mathematics teachers for primary schools who attended at newly established certificate programs made significant misconception on mathematics education because they were not graduates of education faculties. The levels of conceptual knowledge and procedural knowledge of students from a secondary school in Erzurum, Turkey were investigated in order to reveal how serious misconception the teachers have been made so far. The conceptual knowledge is very important to students, however in this research, it was found that procedural knowledge was much more important than conceptual knowledge.

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ZDM classification: C79, D40

MSC2000 classification: 97C70, 97D40

INTRODUCTION

Many researches recently have done in mathematics education in showing that there are big differences between conceptual knowledge and procedural knowledge (*cf.* Ma 1999). Prospective mathematics teachers for primary schools attended at newly established certificate programs and, made significant mistakes on mathematics education (Işık & Kılıç 1998). Because they were not graduates of education faculties, they did not have enough chances to learn both conceptual knowledge and procedural knowledge in balance.

A concept is defined as knowledge structure of common characteristics of different substances and events captured by human brain. A triangle, a quadrangle and a pentagon might be examples of concepts. A triangle, a quadrangle and pentagon have different shapes, edges and sizes. All these objects are known as different objects with different angles; however, they can also be defined as a “concept of shape” due to their common characteristics (Ulgen 1996, pp. 34–84).

Concepts can also be defined as perceptive characteristics of events and subjects derived from life-experiences in the real world. Therefore, characteristics of concepts are continuously investigated and concepts are re-defined by time. For different people, the perceptive characteristics of subjects and events may not be the same. In order to fully understand, one needs to know the meanings of words that are related with concepts. The concepts should be taught during the conceptual activities.

The levels of conceptual knowledge and procedural knowledge of students from a secondary school in Erzurum, Turkey were investigated in order to reveal how big teachers have made misconceptions so far. The conceptual knowledge is very important to students, however in this research, it was found that procedural knowledge was much more important than conceptual knowledge.

Educational programs need to be prepared based upon consecutive and uninterrupted concepts. Teaching all the characteristics of a concept should be equally distributed through out the educational periods. Some information in mind can be built by learning of concept resulting in distinguishing some categories of stimulants. This information needs to be integrated with the behavior of human being for efficient education. The learning of concept, especially in primary schools and secondary schools, will help to learn the new things that are being used in life (Ulgen 1996, pp. 34–84).

CONCEPTUAL AND PROCEDURAL KNOWLEDGE

The concepts should be effectively used at right place and at right time. Thompson & Van de Walle (1985) and Van de Walle (1987, 1988, 1991) defined students’ ability in understanding mathematical knowledge as the follows (*cf.* Baykul 1999, pp. 34–45):

1. Students need to understand conceptual knowledge of mathematics.
2. Students need to understand procedural knowledge of mathematics.
3. Students need to understand relationship between conceptual and procedural knowledge.

Three statements made above are known as connected (relevant) understanding. Connected understanding can be explained as understanding of concepts in terms of their

elements in mathematics, explaining with symbols and formulas from these simple concepts. Thus, it can be learned through the meanings of words, understanding techniques of procedures in mathematics and explaining them with symbols and concepts. Followings are a few benefits of this kind of learning (Baykul 1999):

- Learning becomes joyful and students enjoy learning.
- Students can remember topics they learn with more ease and learning becomes permanent.
- New concepts are learned more easily, students can be self-learners and they need less help.
- Students' ability to solve problems improve and they will be more successful.
- Mathematical anxiety decreases, and positive manner improves.

About learning psychology, Skemp (1971) firstly searched mathematics knowledge. Skemp mentioned two kinds of knowledge. The first one is to recognize a set of symbols, which is mechanical knowledge that does not include conceptual understanding, but includes the ability to make procedures. The second one is the knowledge that can symbolize mathematical concepts; relate each other, and the knowledge that based upon abilities of making procedures with mathematical concepts. At the first glance from Skemp's explanation even if these types of knowledge seem to be independent from each other, however, conceptual knowledge and procedural knowledge complete and dependent on each other. The first one is the language of mathematics that consists of symbols and demonstrations. The other is to know procedures, rules, and algorithms used to solve mathematic problems (Baki 1998).

In mathematics, procedural knowledge defines symbols, rules and knowledge used in solving mathematical problems. On the other hand, conceptual knowledge is described as mathematical concepts and relationship to each other (Baykul 1999). However, it is not possible to separate conceptual knowledge and procedural knowledge precisely. If a person has conceptual knowledge which constitutes procedural knowledge, one can make strong connections between basic concepts, can reach to solutions by using data given, find mathematical construction wanted, and can easily explain mathematical construction relating with rules and symbols which one knew by one's conceptual knowledge. In mathematics, permanent and functional learning can be possible only by balancing procedural and conceptual knowledge (Noss & Baki 1998).

In the current educational system, successful mathematics students can carry out procedures and use algorithms, formulas, and mathematical rules, which they learned and memorized. In mathematics education, which is characterized by O. S. S. Exam¹, it is not

¹ It is the student selection examination for the university entrance in Turkey.

regarded as important students just concentrate on only procedural knowledge. Hence students only care about finding correct answers and filling blanks right. To be able to do that, students choose to memorize mathematical rules and formulas without thinking about reasons (*cf.* Hacısalıhoğlu 1998). This strikes university education mainly.

This type of education is also against general and special goals of high school mathematics education stated in Turkish National Education System. The difference between conceptual knowledge and procedural knowledge is not clear. However, to search whether there is a balancing point between conceptual and procedural knowledge or not, two classes were chosen randomly from the 10th grade classes of the Ibrahim Hakki Science High School² in Erzurum, Turkey.

We executed procedural knowledge test (see Test 1 of Appendix) and, after one month, we did the conceptual knowledge test (see Test 2 of Appendix). After the procedural and conceptual knowledge tests, we interviewed 5 students from each class, in the same level. We selected 10 students who gave right answers to the questions of the procedural knowledge test and gave wrong answers to the questions of the conceptual knowledge test. We asked the interviewees why they couldn't give right answers to the questions in conceptual knowledge test. Their answers were very interesting. They said that they didn't learn proving or giving solutions to these kinds of questions for their university entrance examination.

METHODOLOGY

Objective

The objective of this research is to compare of the equality between the levels of conceptual knowledge and procedural knowledge of the students attending to the same school.

Problem

The comparison of conceptual knowledge with procedural knowledge on mathematical education has been investigated in this research. Some of the problems handled in this research are given as follows:

1. Is there any remarkable difference between the levels of procedural knowledge of the students from two different classes at one school?

² The characteristics of the schools: Science High School students are chosen with entrance exam in Turkey. One percent of students became successful in the entrance exam. The first aim of the Science High Schools is to prepare students to get science grade in university. These schools take only 96 students every year and they are divided into four classes.

2. Is there any remarkable difference between the levels of conceptual knowledge of the students from different classes at one school?
3. What are the differences between conceptual knowledge and procedural knowledge levels in mathematic education of the two classes at one school?

Limitation

We investigated only the 10th grade students from the selected high school in Erzurum, Turkey.

Assumptions

Assumptions made for this research are as follows:

- 1) Students have the same social and economic conditions.
- 2) Students' attendance to mathematics classes is the same.
- 3) Students show real level of their knowledge in this research.

Universe and Sample

In this research, study area was the center of Erzurum, Turkey. The First test was applied to 42 students and the second test was applied to 40 students in the 10th grade of the Science High School in Erzurum, Turkey.

Data Collection and Analysis

Conceptual and procedural knowledge tests were given to 82 students who attend to the 10th grade of the Science High School in Erzurum, Turkey. The data was selected from the test results on students' procedural and conceptual knowledge.

Procedural knowledge tests have been given to the students who attend to the 10th grades in different classes at the same school. Conceptual knowledge tests have been given to same students 30 days after they took the procedural knowledge test. Procedural and conceptual knowledge test results were compared with each other. Five students randomly selected from different classes of one school were interviewed about procedural and conceptual knowledge. To analyze data that was selected from procedural and conceptual knowledge tests, *t*-test was used.

FINDING AND DISCUSSIONS

In Table 1, *p* value for levels of procedural knowledge of the student was found to be

0.228 for different classes at one school. The p value, $p = 0.228$, does not show that much difference between two classes.

Table 1: The levels of procedural knowledge in the Science High School in Erzurum.

Class Name	N	Average	Standard Deviation	t	df	p
10-A	22	90.95	18.32	-1.225	40	0.028
10-C	20	96.65	10.31			

In Table 2, p value was found to be 0.573 that does not make remarkable differences between two different classes. Based on t test, significant value of p was found to be 0.05.

Table 2: Conceptual knowledge level in Science High School in Erzurum.

Class Name	N	Average	Standard Deviation	t	df	P
10-A	17	19.47	20.55	-0.568	38	0.573
10-C	23	23.00	18.55			

Procedural and conceptual knowledge levels are given in Table 3. p for procedural knowledge and conceptual knowledge is 0.001, which shows a little differences.

Table 3: Procedural and conceptual knowledge level in the class 10-A

Class Name	N	Average	Standard Deviation	t	df	p
Conceptual	17	19.47	20.55	-11.462	37	0.001
Procedural	22	90.95	18.32			

In Table 4, conceptual and procedural levels of 10-C class are listed. The value p for procedural knowledge and conceptual knowledge is 0.001, which shows meaningful difference between them.

Table 4: Procedural and conceptual knowledge level in the class 10-C

Class Name	N	Average	Standard Deviation	t	df	p
Conceptual	23	23.00	18.55	16.357	35.263	0.001
Procedural	20	96.95	10.31			

Difference between procedural knowledge and conceptual knowledge levels is shown in Table 5.

Table 5: Procedural and conceptual knowledge level in total

Class Name	N	Average	Standard Deviation	<i>t</i>	<i>df</i>	<i>p</i>
Conceptual	40	21.50	19.25	18.808	74.064	0.001
Procedural	42	93.67	15.15			

CONCLUSION AND SUGGESTIONS

Results show that there is difference between procedural learning and conceptual leaning in mathematics education. This is especially meaningful because this research was done in the Science High Schools, where the purpose of school is to educate students who will major in science or mathematics in universities. Even though students in the Science High Schools were distinguished in elementary and middle schools and were selected by test³, it is understood that procedural learning takes in the first place. If we examine reasons for that, for students who want to enroll university after high school, mathematics education is no more than getting the correct answer. Because some topics of high school mathematics curriculum (especially limit, derivative, integral, sequence, and series) are not included in the test of O. S. S. Exam, senior students are not interested in high school mathematics, but private math courses. It is going to be boring for students to listen to concepts, drawings, and proofs. Thus, high school mathematics curriculum helps students improve thinking abilities (Hacısalıhoğlu 1998).

Some of the students, who will take O. S. S. Exam, are not attending classes after March. To solve these problems, we need following remedies:

- It is necessary to include all topics in high school mathematics curriculum while rearranging mathematics questionnaires in O. S. S. Exam.
- To increase the importance of school education, the role of the Graduate Point Average (GPA) should be increased so that students can be interested in conceptual learning as well as procedural learning.
- Mathematics should be related to art and music, and their esthetic aspects should be explained to the students (Pesen & Odabaş 2000, pp. 15–38). The examples indicating these aspects should be given to students so that they can have positive perspectives in mathematics.

³ Test for Entrance to Science High Schools.

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Appendix

Test 1 (Procedural Knowledge Test)

Q1: Which of the following are roots of the equation $ax^2 + bx + c = 0$ for $a, b, c \in \mathbf{R}$?

a) $x_{1,2} = \frac{b \mp \sqrt{b^2 - ac}}{2a}$

b) $x_{1,2} = \frac{-b \mp \sqrt{b^2 - ac}}{a}$

c) $x_{1,2} = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$

d) $x_{1,2} = \frac{-b \mp \sqrt{4ac}}{2a}$

Q2: Which of the following is equal to $\sin(x + y)$?

a) $\sin x \cos y + \cos x \sin y$

b) $\sin x \cos y - \cos x \sin y$

c) $\sin x \cos x + \sin y \cos y$

d) $\cos x \cos y - \sin x \cos y$

Q3: If $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$ and $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$, then $z_1 z_2$ equal to?

a) $(r_1 + r_2)[\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$

b) $(r_1 r_2)[\cos(\theta_1 + \theta_2) + i \sin(\theta_1 \theta_2)]$

c) $(r_1 r_2)[\cos(\theta_1 - \theta_2) + i \sin(\theta_1 + \theta_2)]$

d) $(r_1 r_2)[\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$

Test 2 (Conceptual Knowledge Test)

Q1: Show that the roots of the equation $ax^2 + bx + c = 0$ are $x_{1,2} = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$ for $a, b, c \in \mathbf{R}$.

Q2: Show that $\sin(x + y) = \sin x \cos y + \cos x \sin y$.

Q3: If $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$ and $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$, then show that $z_1 z_2 = (r_1 r_2)[\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$.