The Improvement of Convergence Characteristic using the New RLS Algorithm in Recycling Buffer Structures

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요 약

적응 횡단선 필터에서 수렴 속도의 개선을 위해 기존의 최소 평균 자승 알고리즘을 확장한 반복적 최소 자승 알고리즘의 탭 가중치 갱신 메커니즘에 재순환 데이터 버퍼를 이용함으로서 수렴특성을 개선시키는 효율적인 기법을 제시하였다.

본 논문은 기존의 적응 횡단선 필터에 데이터 재순환 버퍼 구조를 제안하여 새로운 *RLS* 탭 가중치 갱신 알고리즘을 유도하여 조화 평균 학습 곡선의 평균 자승 에러 값에 대한 반복수에 대해서 데이터 재순환 버퍼를 사용한 학습 곡선의 수렴 속도가 버퍼가 없는 경우의 재순환 버퍼 *RLS* 알고리즘의 수렴 속도보다 비례하여 빠르게 수렴한다는 것을 수학적인 연산을 통해 증명하였다. 채널 진폭의 왜곡의 정도와 재순환 데이터 버퍼 수에 따른 평균 자승 에러에 대한 삼차원 시뮬레이션 결과로부터 고유치 확산이 증가함에 따라 특정 값에 수렴하기 위한 요구된 샘플의 반복수가 비례하여 증가하였으며, 재순환 데이터 버퍼 수 *B*가 증가함에 따라 요구된 샘플의 반복수가 *B*배만큼 감소함으로서 제안된 구조에서 *RLS* 가중치 갱신 알고리즘의 수렴특성이 개선됨을 입증하였다.

ABSTRACT

We extend the sue of the method of least square to develop a recursive algorithm for the design of adaptive transversal filters such that, given the least-square estimate of this vector of the filter at iteration n-1, we may compute the updated estimate of this vector at iteration n upon the arrival of new data. We begin the development of the RLS algorithm by reviewing some basic relations that pertain to the method of least squares. Then, by exploiting a relation in matrix algebra known as the matrix inversion lemma, we develop the RLS algorithm. An important feature of the RLS algorithm is that it utilizes information contained in the input data, extending back to the instant of time when the algorithm is initiated.

In this paper, we propose new tap weight updated RLS algorithm in adaptive transversal filter with data-recycling buffer structure. We prove that convergence speed of learning curve of RLS algorithm with data-recycling buffer is faster than it of exiting RLS algorithm to mean square error versus iteration number. Also the resulting rate of convergence is typically an order of magnitude faster than the simple LMS algorithm. We show that the number of desired sample is portion to increase to converge the specified value from the three dimension simulation result of mean square error according to the degree of channel amplitude distortion and data-recycle buffer number. This improvement of convergence character in performance, is achieved at the B times of convergence speed of mean square error increase in data recycle buffer number with new proposed RLS algorithm

키워드

RLS Algorithm, Convergence Characteristic, Transversal filter, Recycling buffer, Tap weight

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Ⅰ. Instruction

Adaptive filter are essential for meeting the need to acquire more improved performance in digital signal processing and communication systems. Also, adaptive filter algorithms which have rapid convergence speed, lower mean squared error(MSE) and practicality for hardware implementation

We begin the development of the RLS algorithm by reviewing some basic relations that pertain to the method of least squares. Then, by exploiting a relation in matrix algebra known as the matrix inversion lemma, we develop the RLS algorithm. An important feature of the RLS algorithm is that it utilizes information contained in the input data, extending back to the instant of time when the algorithm is initiated.

In this paper, a simple and efficient technique for rapid convergence speed of a transversal filter which uses RLS algorithms is introduced. The basic idea of the technique is the use the discarded data samples to update the tap weight vector in a sample period. This can increase convergence speed by (B) times without increasing the computational complexity substantially, where B is the number of recycled data.

Ⅱ. Adaptive transversal filter

In Recursive implementations of the method of least square, we start the computation with known initial conditions and use the information contained in new data samples to update the old estimates. We therefore find that the length of observable data is valuable. Accordingly, we express the cost function to be minimized as $\xi(n)$, where $n$is the variable length of the observable data. Also, it is customary to introduce a weighting factor into the definition of the cost function $\xi(n)$. We thus write

$$\xi(n)=\sum_{i=1}^{n}\beta(n,i)|e(i)|^2 \qquad (1)$$

where $e(i)$ is the difference between the desired response $d(i)$ and the output $y(i)$ produced by a transversal filter whose tap input equal $u(i), u(n-1), \ldots, u(i-M+1)$, as in Fig. 1.

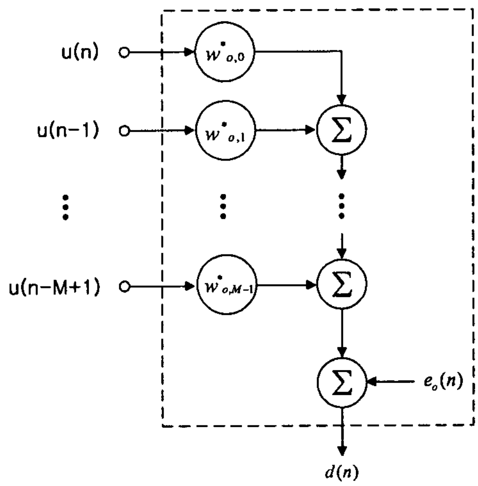


Fig. 1 Adaptive transversal filter

That is, $e(i)$ is defined by

$$e(i) = d(i)-y(i)=d(i)-w^H(n)u(i) \qquad (2)$$

where $u(i)$ is the tap-input vector at time i, defined by

$$u(i)=[u(i), u(i-1), \ldots, u(i-M+1)]^T \qquad (3)$$

and $w(n)$ is the tap-weight vector at time n, defined by

$$w(n)=[w_0(n), w_1(n), \ldots, w_{M-1}(n)]^T \qquad (4)$$

Note that the tap weights of the transversal filter remain fixed during the observation interval $1\le i\le n$ for which the cost function $\xi(n)$is defined

Ⅲ. Proposed data recycling filter structure

We proposed the new data recycling filter structure to improve convergency characteristic of recycling buffer RLS algorithm. Instead of using a single $u(n)$ to update the tap weight vector, we can use the discarded input data vector $u(n)-1), u(n-2), u(n-3), \ldots$ which are stored in some finite buffers in multi linear feedback adaptive transversal filter. The proposed structure with the buffers used for recycling data and the coefficient updating process are depicted in Fig 2.

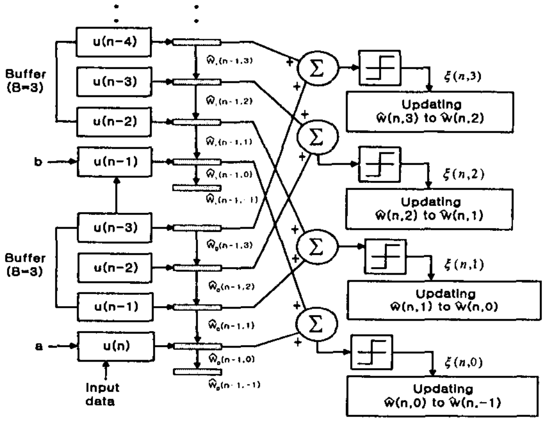


Fig. 2 Adaptive transversal filter structure with data-recycling buffer

For Simplicity, the proposed structure including just two taps a and b is described. Tap a and b have three recycling data(B=3) in their each buffers. Firstly, the recycling data $u(n-3)$ in the buffers of tap a and $u(n-4)$ in the buffer of tap b are used to update $\hat{w}(n,3)=[\hat{w}_0(n-1,3), \hat{w}_1(n-1,3)]^T$ to $\hat{w}(n,2)=[\hat{w}_0(n-1,2), \hat{w}_1(n-1,2)]^T$ using error $\xi(n,3)$.

Secondly, $\hat{w}(n,2)=[\hat{w}_0(n-1,2), \hat{w}_1(n-1,2)]^T$ is update to $\hat{w}(n,1)=[\hat{w}_0(n-1,1), \hat{w}_1(n-1,1)]^T$ using $u(n-2)=[u(n-2), u(n-3)]^T$ and $\xi(n,2)$. Finally, the data vector $u(n-0)=[u(n-0), u(n-1)]^T$ and coefficient vector $\hat{w}(n,0)=[\hat{w}_0(n-1,0), \hat{w}_1(n-1,0)]^T$ produce $\xi(n,0)$. Now $\hat{w}(n,-1)$ is copied to the TDL filter coefficient vector $u(n+1)$, which will be used to produce filter output.

Actually output estimation $\hat{d}(n)$ of adaptive transversal filter with recycling data buffer denoted in Fig. 2, product tap input and tap weight vector, as follows

$$\hat{d}(n) = \sum_{j=1}^{M+1} w_j(n-1)u(n) = \hat{w}^H(n-1)u(n) \quad (5)$$

A priori estimation error $\xi(n)$ defined by difference desired response $d(n)$ and actually adaptive transversal filter output $\hat{d}(n)$. We get the desired recursive equation with data recycling buffer RLS algorithm for updating the tap weight vector

$$\hat{w}(n) = \hat{w}(n-1) + k(n)\sum_{i=1}^{B}\xi^*(n,i) \quad (6)$$

we may express the a priori estimation error $\xi(n)$ as

$$\sum_{i=1}^{B}\xi(n) = e_0(n) - \sum_{i=1}^{B}[\hat{w}(n-1,i) - w_0]^H u(n-i)$$
$$= e_0(n) - \sum_{i=1}^{B}\varepsilon^H(n-1,i)u(n-i) \quad \cdots \quad (7)$$

where $\varepsilon(n-1)$ is the weight error vector at times $n-1$

IV. Tap weight updating with RLS algorithm in data recycling structures

As an index of statistical performance for the RLS algorithm, it is convenient to use the a priori estimation error $\xi(n)$ to define the mean-squared error

$$J(n) = \sum_{i=1}^{B} E[|\xi(n)|^2] \quad (8)$$

The Prime in the symbol $J(n)$ is intended to distinguish the mean-square value of $\xi(n)$ from that of $e(n)$. Substituting Eq.(7) in Eq.(8), and then expanding terms, we get

$$J(n) = E[|e_0(n)^2|] + \sum_{i=1}^{B}(E[u^H(n-i)\varepsilon(n-1,i)\varepsilon^H(n-1,i)u(n-i)] - E[\varepsilon^H(n-1,i)u(n-i)e_0^*(n)] - E[e_0(n)u^H(n-i)\varepsilon(n-1,i)]) \quad \cdots \quad (9)$$

With the measurement $e_0(n)$ assumed to be of zero mean, the first expectation on the right-hand side of Eq.(9). is simply the variance of $e_0(n)$, which is denoted by σ^2. The second expectation on the right-hand side of Eq.(9), is the estimate $\widehat{w}(n-1, i)$ and therefore the weight-error vector $\varepsilon(n-1,i)$, is independent of the tap-input vector $u(n)$; the latter is assumed to be drawn from a wide-sense stationary process of zero mean. Hence, we may use this statistical independence together with well-known results from matrix algebra to express the second expectation on the right-hand side of Eq.(9) as follows:

$$\sum_{i=1}^{B} E[(u^H(n-i)\varepsilon(n-1,i)\varepsilon^H(n-1,i)u(n-i)]$$
$$= \sum_{i=1}^{B} E[tr\{u^H(n-i)\varepsilon(n-1,i)\varepsilon^H(n-1,i)u(n-i)\}]$$
$$= \sum_{i=1}^{B} E[tr\{u(n-i)u^H(n-i)\varepsilon(n-1,i)\varepsilon^H(n-1,i)\}]$$
$$= \sum_{i=1}^{B} tr\{E[u(n-i)u^H(n-i)\varepsilon(n-1,i)\varepsilon^H(n-1,i)]\}$$
$$= \sum_{i=1}^{B} tr\{E[u(n-i)u^H(n-i)]E[\varepsilon(n-1,i)\varepsilon^H(n-1,i)]\}$$
$$= \sum_{i=1}^{B} tr\{RK(n-1,i)\} \qquad (10)$$

where, in the last line, we have made use of the definitions of the ensemble-averaged correlation matrix R and weight-error correlation matrix $K(n-1)$.

The measurement error $e_0(n)$ depends on the tap-input vector $u(n)$. The weight-error vector $\varepsilon(n-1)$ is therefore independent of both $u(n)$ and $e_0(n)$. Accordingly, we may show that the third expectation on the right-hand side of Eq.(9) is zero by first reformulating is as follows:

$$\sum_{i=1}^{B} E[\varepsilon^H(n-1,i)u(n-i)e_0^*(n)] = \sum_{i=1}^{B} E[\varepsilon^H(n-1,i)]E[u(n-i)e_0^*(n)] \qquad (11)$$

We now recognize form the principle of orthogonal that all the elements of the tap-input vector $u(n)$ orthogonal to the measurement error $e_0(n)$. We therefore have

$$\sum_{i=1}^{B} E[\varepsilon^H(n-1,i)u(n-i)e_0^*(n)] = 0 \qquad (12)$$

The forth and final expectation on the right-hand side of Eq.(9) has the same mathematical form as that just considered in point 2, except for a trivial complex conjugation. We may therefore set this expectation equal to zero, too;

$$\sum_{i=1}^{B} E[e_0(n)u^H(n-i)\varepsilon(n-1,i)] = 0 \qquad (13)$$

Thus, recognizing that $e_0(n)^2$ is equal to σ^2, and using the results of Eqs.(10) to (13) in (9), we get the following simple formula for the mean-squared error in the RLS algorithm.

$$J(n) = \sigma^2 + \sum_{i=1}^{B} tr[RK(n-1,i)] \qquad (14)$$

We may express the weight-error correlation matrix $K(n)$ as

$$K(n) = \frac{\sigma^2}{n-M-1}R^{-1}, \qquad n > M+1 \qquad (15)$$

Next, substituting Eq.(15) in Eq.(14), we get for $\lambda = 1$

$$J(n) = \sigma^2 + B\frac{M\sigma^2}{n-M-1}, \qquad n > M+1 \qquad (16)$$

Based on this result, we may make the following deduction. The ensemble - averaged learning curve of the recycling buffer RLS algorithm converges more rapidly B times, if proposed structure has the B number of data recycling buffers. M, is denoted in

Eq.(16), means the number of taps in the transversal filter. This means that the rate of convergence of the RLS algorithm is typically an order magnitude faster than that of the LMS algorithm. As the number of iterations, n, approaches infinity, the mean-squared error $J'(n)$ approaches a final value equal to the variance σ^2 of the measurement error $e_0(n)$. In other words the RLS algorithm, in theory, produce zero excess mean-squared error when operating in a stationary environment Convergence of the RLS algorithm in the mean square is independent of the eigenvalue of the ensemble-averaged correlation matrix R of the input vector $u(n)$.

V. Computer simulation result and analysis

For our computer simulation, we use recycling buffer RLS algorithm to reject interference of channel which produces in the adaptive equalization of a linear dispersive communication channel. Two independent random-number generators are used. one, denoted by x_n, for probing the channel, and the other, denoted by $v(n)$, for simulating the effect of additive white noise at the receiver input. The sequency x_n is a Bernoulli sequence with $x_n = \pm 1$; the random variable x_n has zero mean and unit variance. The second sequency $v(n)$ has no zero mean; its variance σ^2 is determined by the desired signal to noise ratio. The equlizer has 11 tap. The impulse response of the channel is defined by

$$h_n = \begin{cases} \frac{1}{2}\left[1+\cos\left(\frac{2\pi}{W}(n-2)\right)\right], & n=1,2,3 \\ 0, \quad \text{otherwise} & \cdots \quad (17) \end{cases}$$

where W control the amount of amplitude distortion produced by the channel.

Equivalently, the parameter W controls the eigenvalue spread $\chi(R)$ of the correlation matrix of the tap inputs in the equalizer, with the eigenvlaue spread increasingly with W. The Sequence $v(n)$, produced by the second random generator, has the zero mean and variance $\sigma^2 = 0.001$.

The first tap input of the equalizer at time n equals

$$u(n) = \sum_{k=1}^{n} h_k a(n-k) + v(n) \qquad (18)$$

where all the parameter are real valued. Hence the correlation matrix R of the tap inputs of the equalizer, $u(n), u(n-1), \ldots, u(n-10)$, is a symmetric 11 by 11 matrix. Also, Since the impulse response h_n has non-zero values only for n=1,2,3 and the noise process $v(n)$ is white with zero mean and variance σ_v^2 , the correlator matrix R is quantdiagonal. That is the only non-zero elements of R are on the main diagonal and the four diagonals directly above and below it, two on either sided, as shown by the special structure.

$$R = \begin{bmatrix} r(0) & r(1) & r(2) & 0 & \cdots & 0 \\ r(1) & r(0) & r(1) & r(2) & \cdots & 0 \\ r(2) & r(1) & r(0) & r(1) & \cdots & 0 \\ 0 & r(2) & r(1) & r(0) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & r(0) \end{bmatrix} \qquad (19)$$

where $r(0) = h_1^2 + h_2^2 + h_3^2 + \sigma_v^2$, $r(1) = h_1 h_2 + h_2 h_3$, $r(2) = h_1 h_3$. The variance σ_v^2 = 0.001, hence h_1, h_2, h_3 are determined by the value assigned to parameter W in Eq.(17). We have obtained values of the autocorrelation function $r(l)$ with Eq.(17) and (19) for $l = 0, 1, 2$ and the smallest eigenvalue, $\lambda_{\min}$, the largest eigenvalue, $\lambda_{\max}$, and the eigenvalue spread $\chi(R) = \lambda_{\max}/\lambda_{\min}$.

Computer simulation with recycling buffer RLS algorithm can use time variance of channel that inputs in adaptive transversal filter. The impulse response of channel is used in Eq.(18). The adaptive transvesal filter with proposed recycling data buffer RLS algorithm has 11 taps, and additive white Gaussian noise variance, σ_v^2, is 0.0001.

The result of the experiment for a fixed eigenvalue spread $\chi(R)$=878.1699 and varying the number of buffer were presented in Fig. 3. The four parts of that figure correspond to the parameter B=0, 1, 4, 9 respectively.

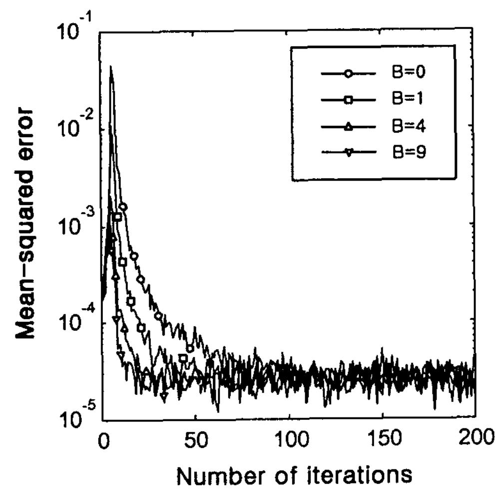
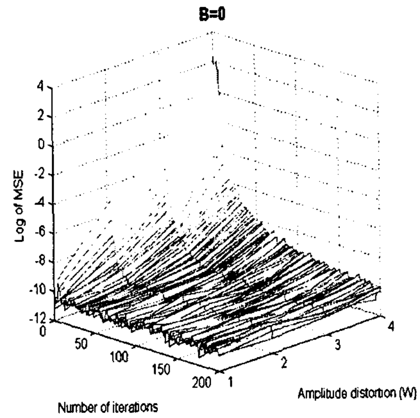


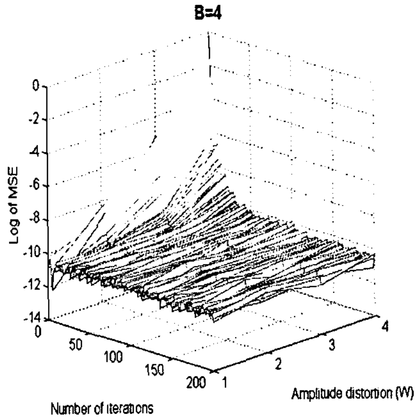
Fig. 3. MSE learning curves of RLS Algorithm with number of taps M=11, standard deviation parameter σ= 0.01, eigenvalue spread $\chi(R)$=878.1699 and recycling data Buffer B=0, 1, 4, 9

The result of the experiment for a fixed eigenvalue spread $\chi(R)$=878.1699 and varying the number of buffer were presented in Fig. 3. The four parts of that figure correspond to the parameter B=0,1,4,9 respectively. The result presented in Fig. 4 clearly show that the superior rate of convergence of the recycling buffer RLS algorithm with buffer over RLS algorithm without buffer. The recycling buffer RLS algorithm converges much faster B times than the RLS algorithm. Convergence of the recycling buffer RLS algorithm is attained in about 134, 67, 23, 15 iterations, correspondence to the four different values of the buffer, that is ,B=0,1,4,9 respectively.

Finally, the new algorithm, recycling buffer RLS algorithm, in proposed structure is prove to be efficient to control of channel interference from the computer simulation above.

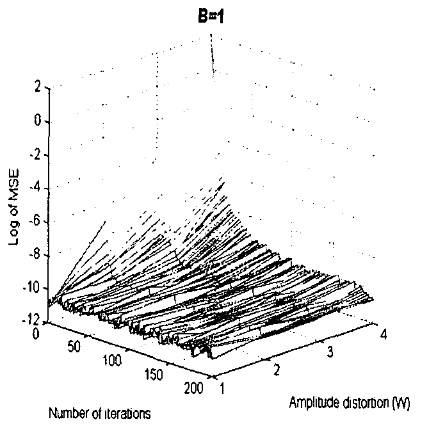


(a) B=0

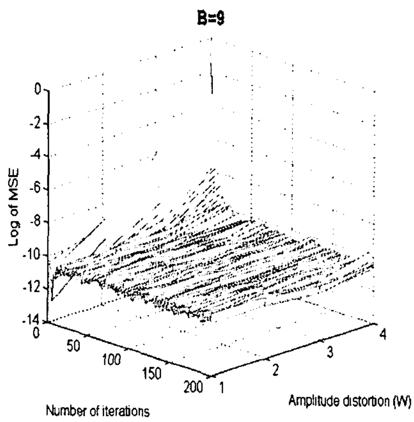


(b) B=1

Three dimension simulation denoted in Fig. 4 (a) was typically result that was applied to the case of bufferless in adaptive transversal filter with recycling data buffer structure. This result is obtained from the state of convergence of the mean square error is correspondence of distortion of channel which controlled by the tap weight vector with error, the difference actually estimation value and desired response. x axis and y axis presented Fig. 4 means the number of iteration of sample and degree of amplitude distortion respectively.



(b) B=4



(c) B=9

Fig. 4. Three dimensional simulation of MSE learning curves of RLS algorithm with number of taps M=11, standard deviation parameter σ= 0.01, varying amplitude distortion w and recycling data buffer B = 0, 1, 4, 9

For the computer simulation, the number of iteration is set to 300, and amplitude distortion is established from 1 to 4. z axis in Fig. 4 means log value of mean square error. The simulation from Fig. 4(b) to Fig. 4(d) are presented convergence of MSE varying eigenvalue spread $\chi(R)$ correspondence amplitude distortion, W. From the simulation result, we show that value of MSE increase proportional to increasing amplitude distortion. Also, simulation results Fig. 4(b), Fig. 4(c) and Fig. 4(d) are presented to the case of the number of data recycling buffer, B=4, 9, equivalent above parameters, number of taps, white Gaussian noise variance σ^2 and eigenvalue spread $\chi(R)$.

The result of the computer simulation demonstrate that the simulation results converges B times of convergence speed in accordance with recycling data buffer B, for the controlling tap weight using recycling buffer RLS algorithm in proposed structure.

Ⅵ. Conclusion

We have presented efficient method for rapidly adjusting the tap weight control with the recycling data buffer in proposed structure. We begin the development of the RLS algorithm by reviewing some basic relations that pertain to the method of least squares. Then, by exploiting a relation in matrix algebra known as the matrix inversion lemma, we develop the RLS algorithm. An important feature of the RLS algorithm is that it utilizes information contained in the input data, extending back to the instant of time when the algorithm is initiated.

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