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# An Adaptive Algorithm Applied to a Design of Robust Observer

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Primary goal of adaptive observers would be to estimate the true states of a plant. Identification of unknown parameters is of secondary interest and is achieved frequently with the persistent excitation condition of some regressors. Nevertheless, two problems are linked to each other in the classical approaches to adaptive observers; as a result, we get a good state estimate once after a good parameter estimate is obtained. This paper focuses on the state estimation without parameter identification so that the state is estimated regardless of persistent excitation. In this direction of research, Besancon (2000) recently summarized that most of adaptive observers in the literature share one common canonical form, in which unknown parameters do not affect the unmeasured states. We enlarge the class of linear systems from the canonical form of (Besancon, 2000) by proposing an adaptive observer (with additional dynamics) that allows unknown parameters to affect those unmeasured states. A recursive algorithm is presented to design the proposed dynamic observer systematically. An example confirms the design procedure with a simulation result.

**Key Words :** Adaptive Observer, Linear System, Unknown Parameter, Persistent Excitation, Passivity

## 1. Introduction

The design of adaptive observers[1] has received considerable attention in the last several years. The first contribution to adaptive observer design was made in (Carroll and Lindorfe, 1973) for linear time-invariant systems with unknown parameters. Since then, many interesting results have been reported in the literature. Based on a new canonical form for a linear system, a significantly simplified observer structure was suggested by Luders and Narendra (1973; 1974). The construction of adaptive observers with arbitrarily high rates of convergence was considered in (Kreisselmeier, 1977). Several years later, an adaptive observer for nonlinear systems was proposed in (Bastin and Gevers, 1988) by extending the result of (Luders and Narendra, 1974).

However, most of adaptive observers in the literature require the condition of persistent excitation for the regressor in order to have the state

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1) Some authors refer to 'adaptive observer' as an observer that yields both the state and the parameter estimates. In this paper, it just means an observer which has a parameter update law regardless of its convergence.

estimate. In case of aforementioned linear adaptive observers, the observer includes a parameter identifier so that they first estimate unknown parameters (which is usually achieved under the persistent excitation condition) and then the standard Luenberger observer technique is applied to estimate the state. On the other hand, the adaptive observer proposed in (Marino, 1990; Marino and Tomei, 1992) is designed for the 'adaptive observer canonical form' and the persistent excitation is not required as long as the system has the canonical form. If the system is not in the canonical form, some parameter-dependent coordinate change needs to be applied to transform the system into the canonical form, so that parameter estimation becomes necessary to have the estimate of states in the original coordinates.

In this paper, we consider a system given by

$$\begin{aligned} \dot{x} &= Ax + Bu + G\theta \\ y &= Cx \end{aligned} \tag{1}$$

where $x$ is the state in $R^n$, $u$ the input in $R^m$, $y$ the output in $R^p$, and $\theta$ is a vector of unknown constant parameters in $R^q$. Since $G$ is a constant matrix, it is not likely to be persistently excited for several unknown parameters.

When we do not have persistently excited regressors like (1), the class of systems admitting an adaptive observer is quite restricted. In (Besancon, 2000), Besancon presented a unified framework for many existing adaptive observers that do not require parameter estimation[2]. According to (Besancon, 2000), almost all adaptive observers in the literature, that can estimate the state $x$ without first estimating $\theta$, have been designed for the following particular class of systems:

$$\begin{aligned} \dot{y} &= A_{ya}y + A_{yb}z + B_y u + G_y\theta \\ \dot{z} &= A_{za}y + A_{zb}z + B_z u \end{aligned} \tag{2}$$

where $y$ is the output of the system and $A_{zb}$ is Hurwitz. In this form, $\theta$ does not affect the unmeasured state $z$.

We present in this paper a new adaptive observer for different classes of systems from (2), in which uncertain parameters enter the unmeasured states. While the design requires no hypothesis of persistent excitation, what is assumed in this paper is the following:

**Assumption 1**. *Let us define*

$$H_k := \begin{bmatrix} C \\ CA \\ \vdots \\ CA^k \end{bmatrix}$$

*For the system (1), there exists an integer $r$ ($1 \le r$) such that*

*1. $H_{r-1}G = 0$, and*

*2. there are some matrices $L$, $P$, $\Gamma$ of appropriate dimensions satisfying*

$$P(A - LH_r) + (A - LH_r)^T P < 0 \tag{3a}$$

$$PG = H_r^T \Gamma \tag{3b}$$

$$P > 0 \tag{3c}$$

**Remark 1**. It is presupposed in this assumption that $r \ge 1$. However, when the conditions (3a)-(3c) hold with $r = 0$, the system (1) is the very case considered in (Besancon, 2000), and the standard technique can be applied to obtain an adaptive observer. (This technique also appears as the initial step of the proposed recursion in this paper.) If $(C, A)$ is an observable pair, then the conditions (3a)-(3c) become easy to hold as $r$ in creases, since the column and the row spaces of $H_r$ are enlarged; on the other hand, the condition $H_{r-1}G = 0$ gets more restrictive. Therefore, each $r$ characterizes its own class of systems.

As an example, consider the system

$$\begin{aligned} y = x_1, \ \dot{x}_1 &= x_2, \\ \dot{x}_2 &= z + g_2\theta, \\ \dot{z} &= A_z z \end{aligned} \tag{4}$$

where $A_z$ is Hurwitz. Clearly, it is not in the adaptive observer form proposed in (Besancon, 2000), but can be shown to satisfy Assumption 1 with $r = 1$. An interesting way to show this is to apply the technique of (Besancon, 2000), assuming that the output $y$ is $H_1 x = [x_1, x_2]^T$ so

2) In (Besancon, 2000), parameter estimation is a bonus when the regressors are persistently excited. Also, note that only linear systems are dealt with in this paper while nonlinear systems are considered in (Besancon, 2000).

that $x_2$ is also measurable. Then, the system (4) becomes in the adaptive observer form of (2), and thus it admits the error Lyapunov function suggested in (Besancon, 2000), which has the positive definite matrix $P=\text{diag}\{I_{2\times 2}, P_z\}$ where $P_z$ is such that $P_zA_z+A_z^TP_z<0$. Finally, it is easy to show that the matrix $P$ satisfies Assumption 1.

In the next section, we present a dynamic adaptive observer for (1) only under Assumption 1, followed by the recursive algorithm to design the gains of the proposed observer in a systematic manner. Section 3 illustrates a design example with a simulation result. Conclusions are found in Section 4.

## 2. Main Results

For the system (1) that satisfies Assumption 1, we propose a dynamic adaptive observer of the form:

$$\begin{aligned}\dot{\hat{\theta}}&=\Phi_a(C\hat{x}-y)+\Phi_b\lambda\\ \dot{\hat{x}}&=A\hat{x}+Bu+G\hat{\theta}+N_a(C\hat{x}-y)+N_b\lambda\\ \dot{\lambda}&=\Psi_a(C\hat{x}-y)+\Psi_b\lambda\end{aligned} \tag{5}$$

where $\hat{x}$ is the estimate of the true state $x$, and $\hat{\theta}\in R^q$ and $\lambda=R^{rp}$ are the internal states of additional dynamics (thus, we know their values). Then, the observer problem is solved if we design all $\Phi=[\Phi_a, \Phi_b]$, $N=[N_a, N_b]$ and $\Psi=[\Psi_a, \Psi_b]$ matrices so that, by defining $\tilde{\theta}:=\hat{\theta}-\theta$ and $e:=\hat{x}-x$, the following error dynamics

$$\begin{aligned}\dot{\tilde{\theta}}&=\Phi_aCe+\Phi_b\lambda\\ \dot{e}&=Ae+G\tilde{\theta}+N_aCe+N_b\lambda\\ \dot{\lambda}&=\Psi_aCe+\Psi_b\lambda\end{aligned} \tag{6}$$

guarantees that $e(t)\to 0$ and $\lambda(t)\to 0$ as $t\to\infty$, in the sense that there exist positive definite matrices $P$ and Q satisfying

$$\frac{d}{dt}\left(\begin{bmatrix}\tilde{\theta}\\ e\\ \lambda\end{bmatrix}^T P\begin{bmatrix}\tilde{\theta}\\ e\\ \lambda\end{bmatrix}\right)=-\begin{bmatrix}e\\ \lambda\end{bmatrix}^T Q\begin{bmatrix}e\\ \lambda\end{bmatrix}$$

(Indeed, LaSalle-Yoshizawa theorem, (Krstic et al., 1995, p. 24), proves the convergence. Note that the dimension of $Q$ is lower than $P$ because we are not interested in the convergence of $\tilde{\theta}$.)

In the subsequent part of the paper, we will show the design of the matrices $\Phi$, $N$ and $\Psi$ by a recursive algorithm. Therefore, the main contribution of the paper is summarized as

**Theorem 1.** *For the system (1) satisfying Assumption 1, there exists a dynamic adaptive observer (5) with additional $\lambda$-dynamics of order $(r\times p)$, so that $\hat{x}(t)\to x(t)$ as $t\to\infty$*

The idea of the construction of (5) is to assume, in the beginning, that $H_re=(H_r\hat{x}-H_rx)$ is available for measurement although it is not true since $H_rx$ is not all measurable. Then, the standard technique of adaptive observer yields an adaptive observer with an error Lyapunov matrix pair $P$ and $Q$ of appropriate sizes. Now we change our virtual assumption so that $H_{r-1}e$ is available for measurement but $CA^re$ is not. Then, the designed observer in the previous step is not implementable since it depends on the signal $CA^re$. Thus we extract the signal $CA_re$ from the observer structure and design additional dynamics with which the use of $CA_re$ is eliminated. In the next step, we proceed by assuming that $H_{r-2}e$ is measurable but $CA_{r-1}e$ is not. The recursion goes to the end if we get a dynamic observer that requires only the measurement of $H_0e=Ce$ but not others.

The recursion begins by the following initial step.

### 2.1 Initial step

Assuming that $H_re$ is measurable, we choose our initial error system $S_r$ as follows (compare this with (6)):

$$S_r:\begin{cases}\dot{\tilde{\theta}}=-\Gamma^TH_re &=D_{1a}H_{r-1}e+D_{1b}(CA^re)\\ \dot{e}=G\tilde{\theta}+Ae-LH_re=G\tilde{\theta}+A_e-D_{2a}H_{r-1}e-D_{2b}(CA^re)\end{cases}$$

where $\Gamma$ and $L$ are given in Assumption 1 and $-\Gamma^T=[D_{1a}, D_{1b}]$ and $L=[D_{2a}, D_{2b}]$. Clearly, this error system is obtained from the observer

$$\begin{aligned}\dot{\hat{\theta}}&=-\Gamma^T(H_r\hat{x}-H_rx)\\ \dot{\hat{x}}&=G\hat{\theta}+A\hat{x}+Bu-L(H_r\hat{x}-H_rx).\end{aligned} \tag{7}$$

Error convergence easily follows since, with

$\gamma=[\tilde{\theta}^T, e^T]^T$, the function $V(\gamma)=\frac{1}{2}\gamma^T P\gamma=\frac{1}{2}\tilde{\theta}^T\tilde{\theta}+\frac{1}{2}e^T Pe$ satisfies that

$$\begin{aligned}\dot{V}&=\tilde{\theta}^T(-\Gamma^T H_r e)+e^T PG\tilde{\theta}+e^T P(A-LH_r)e\\&=-e^T Qe\end{aligned}$$

where $Q=-\text{sym}\{P(A-LH_r)\}$, in which sym $\{A\}$ denotes $\frac{1}{2}(A+A^T)$, is positive definite by Assumption 1.

### 2.2 Recursive design

Suppose that a system $S_k$ ($k$ is an index between 0 and $r$ and the recursion begins when $k=r$ and ends with $k=0$) given by

$$S_k:\begin{cases}\dot{\tilde{\theta}}=D_{12}H_k e+D_{13}\lambda=D_{1a}H_{k-1}e+D_{13}\lambda+D_{1b}(CA^k e)\\ \dot{e}=G\tilde{\theta}+Ae+D_{22}H_k e+D_{23}\lambda=G\tilde{\theta}+Ae+D_{2a}H_{k-1}e+D_{23}\lambda+D_{2b}(CA^k e)\\ \dot{\lambda}=D_{32}H_k e+D_{33}\lambda=D_{3a}H_{k-1}e+D_{33}\lambda+D_{3b}(CA^k e)\end{cases}$$

where $\tilde{\theta}\in R^q$, $e\in R^n$ and $\lambda\in R^{p(r-k)}$. The matrices $G$, $A$ and $H_k$ (from $A$ and $C$) are given in (1), and all $D$ matrices have appropriate dimensions (for example, $D_{12}=[D_{1a}, D_{1b}]$). Note that $\lambda$ is null when $k=r$, but increases its dimension as the recursion proceeds.

The system $S_k$ will be concisely denoted by

$$\dot{\gamma}=F\gamma+D_b v+w \tag{8}$$

were $\gamma=[\tilde{\theta}^T, e^T, \lambda^T]^T$, $D_b=[D_{1b}^T, D_{2b}^T, D_{3b}^T]$ and

$$F=\begin{bmatrix}0 & D_{1a}H_{k-1} & D_{13}\\ G & A+D_{2a}H_{k-1} & D_{23}\\ 0 & D_{3a}H_{k-1} & D_{33}\end{bmatrix} \tag{9}$$

if $v$ and $w$ are taken as

$$v=CA^k e \text{ and } w=0 \tag{10}$$

By introducing $v$ (and the zero input $w$) the system $S_k$ is now decomposed into the term including $CA^k e$ and the rest. However, since $CA^k e$ is not available for measurement (when $k\geq 1$), we will propose an alternative design of $v$ and $w$ which depend only on $H_{k-1}e$ and the state of added dynamics that is known to observer. Before presenting the alternative design of $v$ and $w$, we confirm the following claim holds for $S_k$ at this stage.

**Claim 1**. *There exist positive definite matrices* $P=R^{(q+n+p(r-k))^2}$ *and* $Q\in R^{(n+p(r-k))^2}$ *such that, with* $V(\gamma)=\frac{1}{2}\gamma^T P\gamma$,

$$\begin{aligned}\dot{V}&=\gamma^T P(F\gamma+D_b(CA^k\gamma_2))\\&=-\begin{bmatrix}\gamma_2\\ \gamma_3\end{bmatrix}^T Q\begin{bmatrix}\gamma_2\\ \gamma_3\end{bmatrix}\end{aligned} \tag{11}$$

*where* $\gamma_2=e$ *and* $\gamma_3=\lambda$.

The equation (11) implies that, if $v$ and $w$ are taken as (10), then the states $e$ and $\lambda$ of the system $S_k$ converge to zero. This claim holds true from the initial step when $k=r$ and will be justified by Corollary 1 as the recursion proceeds.

Now we assume that $H_{k-1}e$ is available for measurement but $CA^k e$ is not. Then, the following theorem shows that, by attaching additional dynamics, we can design an alternative $v$ and $w$, instead of (10), that does not depend on the unmeasurable quantity $CA^k e$.

**Theorem 2**. *Suppose that the system (8) satisfies Claim 1 when v and w are taken as (10). If the following dynamic system is appended to (8)*

$$\dot{\eta}=-v-CA^{k-1}D_{2b}v-CA^{k-1}D_{2a}H_{k-1}e-CA^{k-1}D_{23}\lambda \tag{12a}$$

$$\bar{y}=\eta+CA^{k-1}e \tag{12b}$$

*then the system (8) and (12) guarantees that the states* $e$, $\lambda$ *and* $\eta$ *converge to zero by redesigning*

$$v=V_k\bar{y},\ w=(FD_b+D_b CA^k D_{2b})\bar{y}=:W_k\bar{y} \tag{13}$$

*in which* $\bar{y}$ *is measurable if* $H_{k-1}e$ *is assumed to be measurable. The matrix gain* $V_k$ *is chosen so that*

$$Q_*:=\begin{bmatrix}Q & \begin{bmatrix}-\frac{1}{2}(CA^k)^T\\ 0\end{bmatrix}\\ \begin{bmatrix}-\frac{1}{2}CA^k & 0\end{bmatrix} & \text{sym}\{V_k+CA^k D_{2b}\}\end{bmatrix}>0 \tag{14}$$

**Remark 2**. Note that the linear matrix inequality (14) always has a solution that is $V_k=\phi I$ with sufficiently large $\phi>0$.

**Corollary 1**. *Under the assumptions of Theorem 2, the augmented system (8), (12) and (13) can be written as a single system :*

$$\frac{d}{dt}\begin{bmatrix}\tilde{\theta}\\ e\\ \lambda\\ \eta\end{bmatrix}=\begin{bmatrix}0 & D_{2a}H_{k-1}+(D_{1b}V_{k+W_{k,1}})CA^{k-1} & D_{13} & D_{1b}V_k+W_{k,1}\\ G & \begin{bmatrix}A+D_{2a}H_{k-1}\\ +(D_{2b}V_{k+W_{k,2}})CA^{k-1}\end{bmatrix} & D_{23} & D_{2b}V_k+W_{k,2}\\ 0 & D_{3a}H_{k-1}+(D_{3b}V_{k+W_{k,3}})CA^{k-1} & D_{33} & D_{3b}V_k+W_{k,3}\\ 0 & \begin{bmatrix}-CA^{k-1}D_{2a}H_{k-1}\\ -(I+CA^{k-1}D_{2b})V_kCA^{k-1}\end{bmatrix} & -CA^{k-1}D_{23} & -(I+CA^{k-1}D_{2b})V_k\end{bmatrix}\begin{bmatrix}\tilde{\theta}\\ e\\ \lambda\\ \eta\end{bmatrix} \quad (15)$$

*where* $W_k^T=[W_{k,1}^T, W_{k,2}^T, W_{k,3}^T]$. *For this system, it follows that*

$$\frac{d}{dt}\left(\begin{bmatrix}\tilde{\theta}\\ e\\ \lambda\\ \eta\end{bmatrix}^T \bar{P}\begin{bmatrix}\tilde{\theta}\\ e\\ \lambda\\ \eta\end{bmatrix}\right)=-\begin{bmatrix}e\\ \lambda\\ \eta\end{bmatrix}^T \bar{Q}\begin{bmatrix}e\\ \lambda\\ \eta\end{bmatrix}$$

*where*

$$\bar{P}=T^T\begin{bmatrix}P & 0\\ 0 & I\end{bmatrix}T>0 \quad (16)$$

$$\bar{Q}=T_*^TQ_*T_*>0 \quad (17)$$

$$T=\begin{bmatrix}I+D_b[0 \;\; CA^{k-1} \;\; 0 \;\; D_b]\\ [0 \;\; CA^{k-1} \;\; 0]] & I\end{bmatrix} \quad (18)$$

*in which,* $T_*$ *is the lower-right block of* $T$ *with the size of* $(n+p(r-k+1))$ *by* $(n+p(r-k+1))$, *i.e.,*

$$T_*=\begin{bmatrix}I+\begin{bmatrix}D_{2b}CA^{k-1} & 0\\ D_{3b}CA^{k-1} & 0\end{bmatrix} & \begin{bmatrix}D_{2b}\\ D_{3b}\end{bmatrix}\\ [CA^{k-1} \;\; 0] & I\end{bmatrix}$$

Finally the recursion procedure is quite obvious. By the initial step, Claim 1 holds for $S_r$ and Corollary 1 presents the system $S_{r-1}$ by the equation (15). Indeed the new $D_{ij}$ matrices are identified by redefining $[\lambda^T, \eta^T]^T$ as the new $\lambda$ and by extracting $CA^{r-1}e$ term. Then Claim 1 again holds for $S_{r-1}$, which enables to apply Corollary 1 to the system $S_{r-1}$ and the system $S_{r-2}$ is obtained. This recursion will end with $S_1$, because Theorem 2 will yield an implementable adaptive observer (i.e., the system $S_0$). As a result, the system (15) of Corollary 1 will be the same as (6), and all matrices $\Phi$, $N$ and $\Psi$ of (5) are derived straightforwardly.

*Proof of Theorem 2 and Corollary 1.*

First of all, note that

$$\begin{aligned}\dot{\bar{y}}&=\dot{\eta}+CA^{k-1}\dot{e}\\ &=(-v-CA^{k-1}D_{2b}v-CA^{k-1}D_{2a}H_{k-1}e-CA^{k-1}D_{23}\lambda)\\ &\quad+CA^{k-1}(G\tilde{\theta}+Ae+D_{2a}H_{k-1}e+D_{23}\lambda+D_{2b}v)\\ &=-v+CA^ke\end{aligned}$$

where the assumption that $H_{r-1}G=0$ (Assumption 1) is used.

We now define

$$\xi=\gamma+D_b\bar{y} \quad (19)$$

Then

$$\begin{aligned}\dot{\xi}&=F\gamma+D_bv+w-D_bv+D_bCA^k\gamma_2\\ &=F\xi-FD_b\bar{y}+D_bCA^k\xi_2-D_bCA^kD_{2b}\bar{y}+w\end{aligned} \quad (20)$$

Also, let

$$V(\xi, \bar{y})=\frac{1}{2}\xi^TP\xi+\frac{1}{2}\bar{y}^T\bar{y} \quad (21)$$

be a Lyapunov function candidate for the augmented system $S_k$ and (12). Then, the derivative of $V$ becomes

$$\begin{aligned}\dot{V}&=\xi^TP(F\xi+D_bCA^k\xi_2)-\xi^TP(FD_b+D_bCA^kD_{2b})\bar{y}+\xi^TPw\\ &\quad-\bar{y}^Tv+\bar{y}^TCA^k\xi_2-\bar{y}^TCA^kD_{2b}\bar{y} \qquad \text{by (20)}\\ &=-[\xi_2^T, \xi_3^T]Q[\xi_2^T, \xi_3^T]^T-\bar{y}^TV_k\bar{y}^TCA^k\xi_2-\bar{y}^TCA^kD_{2b}\bar{y} \qquad \text{by (11) and (13).}\end{aligned}$$

The last equality can be rewritten using (14) as

$$\dot{V}(\xi, \bar{y})=-\begin{bmatrix}\xi_2\\ \xi_3\\ \bar{y}\end{bmatrix}^T Q_*\begin{bmatrix}\xi_2\\ \xi_3\\ \bar{y}\end{bmatrix} \quad (22)$$

Therefore, it is concluded, by LaSalle-Yoshizawa theorem, that $\xi_2$, $\xi_3$ and $\bar{y}$ converge to zero, which in turn implies the states $e$, $\xi$ and $\eta$ converge to zero by (19) and (12).

To prove Corollary 1, we would simply need to express in the original coordinates the augmented system (1), the function $V(\xi, \bar{y})$ of (21) and $Q_*$ of (22). In fact, through (19) and (12b), the states $\gamma=[\tilde{\theta}, e, \xi]$ and $\eta$ are transformed to $\xi$ and $\bar{y}$. This can be written concisely by

$$\begin{bmatrix}\gamma\\ \eta\end{bmatrix}=T\begin{bmatrix}\xi\\ \bar{y}\end{bmatrix}$$

where $T$ is already given by (18).

## 3. Design Example

The mechanical system shown in Fig. 1 is composed of a mass-spring-damper system and an actuator that generates the force $F$. We assume

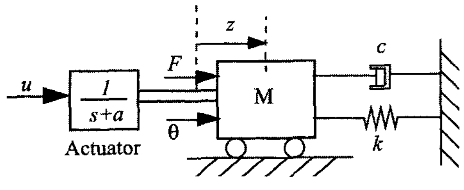


**Fig. 1** Mechanical system example

that the actuator has dynamics as follows:

$$\dot{F}=-aF+u \tag{23}$$

where $a$ is the time constant and $u$ is the control input. Suppose that not only the force $F$ generated by the actuator, but also an unknown constant force $\theta$ drives the system. The equation of motion is given by

$$M\ddot{z}+c\dot{z}+kz=F+\theta \tag{24}$$

where $M$, $c$, $k$ are the mass, the damping coefficient, and the spring constant of the system, respecti vely. By choosing the state variables $x=[x_1, x_2, x_3]^T=[z, \dot{z}, F]^T$ and assuming that the displacement $z$ is measured, we have the state space model:

$$\dot{x}=\begin{bmatrix} 0 & 1 & 0 \\ -\frac{k}{M} & -\frac{c}{M} & \frac{1}{M} \\ 0 & 0 & -a \end{bmatrix} x+\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u+\begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \end{bmatrix} \theta =: Ax+Bu+G\theta \tag{25}$$
$$y=[1\ 0\ 0]x=:Cx$$

When the system parameters are given that $M=1$, $k=0.5$, c$=0.3$, and $a=1.5$, it can be shown that Assumption 1 holds with $r=1$ and with

$$L=\begin{bmatrix} 1 & 2 \\ -0.5 & -0.3 \\ 0 & 1 \end{bmatrix}, Q=\begin{bmatrix} 20.0000 & 9.6667 & -11.3333 \\ 9.6667 & 6.0000 & -3.3333 \\ -11.3333 & -3.3333 & 20.0000 \end{bmatrix},$$
$$P=\begin{bmatrix} 10.0000 & 3.0000 & -3.3333 \\ 3.0000 & 6.6667 & 0 \\ -3.3333 & 0 & 6.6667 \end{bmatrix}, \text{ and } \gamma=\begin{bmatrix} 3 \\ 6.6667 \end{bmatrix}$$

With $r=1$, the system $S_1$ in Section 2.2 satisfies Claim 1. Indeed, the initial step of Section 2.1 guarantees the claim with the updated $P$ and $Q$, that is, diag{1, $P$} and $Q$.

However, since $CAe$ is not available for measurement, we proceed one step further using Theorem 2. Hence, the following adaptive observer is obtained:

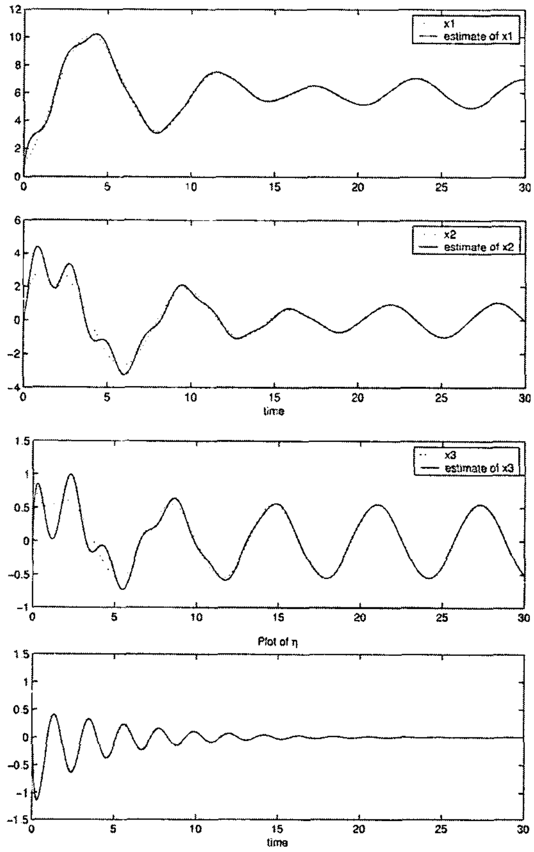


**Fig. 2** Simulation results

$$\dot{\hat{\theta}}=D_{1a}(C\hat{x}-y)+D_{2b}v+\omega_1$$
$$\dot{\hat{x}}=A\hat{x}+G\hat{\theta}+D_{2a}(C\hat{x}-y)+Bu+D_{2b}v+\omega_2$$
$$\dot{\eta}=-v-CD_{2b}v-CD_{2a}(C\hat{x}-y)$$

where

$$D_{1a}=-3,\ D_{1b}=-6.6667,\ D_{2a}=[-1\ 0.5\ 0]^T,\ D_{2b}=[-2\ 0.3\ -1]^T$$
$$\omega_1=(D_{1a}H_0D_{2b}+D_{1b}CAD_{2b})\bar{y}$$
$$\omega_2=[GD_{1b}+(A+D_{2a}C)D_{2b}+D_{2b}CAD_{2b}]\bar{y}$$
$$v=10\bar{y}$$
$$\bar{y}=\eta+C\hat{x}-y$$

The simulation results are given in Fig. 2 where all the initial conditions of the system are set to 1 while all the initial conditions in the observer are set to 0. The results show that the estimates converge to the true states.

## 4. Conclusion

In this paper, a recursive algorithm to design the adaptive observer for the linear systems that do not have persistently excited regressors. By Assumption 1, the class of systems that admits the observer is different from that of (Besancon, 2000), and the index $r$ characterizes the class. The larger index $r$ implies the unknown parameter has larger relative degree from the output $y$ when the parameter is regarded as an input. The recursive design indicates the higher order dynamics is necessary when the index $r$ increases.

From the proposed recursion algorithm, it seems easy to develop an automated design package on a PC.

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