

Frequency-stabilized Femtosecond Mode-locked Laser for Optical Frequency Metrology

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We demonstrated an optical frequency synthesizer based on a femtosecond (fs) mode-locked Ti:sapphire (Ti:s) laser by simultaneously stabilizing the carrier-offset frequency, f_{ceo} , and repetition rate, f_{rep} , referenced to the Cs atomic frequency standard. By using two wide-band digital phase-detectors we realized a phase-coherent link between f_{rep} and f_{ceo} with the relation $f_{ceo} = f_{AOM} - 5/6f_{rep} \equiv 0$, where $f_{AOM} = 5/6f_{rep}$ is the phase-locked driving frequency of an acousto-optic modulator (AOM) in a self-referencing interferometer and $f_{rep} = 100$ MHz. As a result, we could stabilize all components of the fs laser comb at once with an equal frequency separation $f_{rep} = 100$ MHz with $f_{ceo} = 0$. In our optical frequency synthesizer, the frequency of the n th component (f_n) is given exactly by the simple relation $f_n = nf_{rep}$, enabling us to use the fs laser comb as a frequency ruler in the optical frequency metrology.

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I. INTRODUCTION

Recently, a revolutionary advance has been made in the precision optical frequency metrology with the help of an octave-spanning optical frequency comb generated by nonlinear self-phase modulation of fs mode-locked pulses in a photonic crystal fiber [1]. Such an ultra-short fs mode-locked laser has been successfully developed as a precision frequency ruler for the absolute optical frequency measurement in a single step [2-5]. In this new optical frequency synthesizer the optical frequency corresponding to the n th component of the comb can be described with two parameters, i.e., the spacing between the modes given by the repetition rate, f_{rep} , and the carrier-envelope offset frequency, f_{ceo} , that arises from the difference between phase and group velocities of the pulses in the laser cavity. The carrier-envelope offset frequency in the frequency domain is related to the pulse to pulse phase slip $\Delta\Phi$ (not absolute phase) in the time domain by the relation $\Delta\Phi = 2\pi f_{ceo}/f_{rep}$ [2,6]. Therefore, simultaneous frequency stabilizations of f_{rep} and f_{ceo} in the frequency domain directly result in the pulse-to-pulse time separation and carrier-envelope offset-phase stabilizations in the time domain with unprecedented stability.

Since the frequency of the n th mode of the comb is

precisely given by the relation $f_n = nf_{rep} + f_{ceo}$ with two low-frequency components f_{rep} and f_{ceo} [1], we can easily stabilize the optical frequency f_n by simply stabilizing f_{rep} and f_{ceo} referenced to the precisely known frequency standard. Therefore, in the new optical frequency synthesizer, it is essential to phase-lock both f_{rep} and f_{ceo} to the most accurate atomic frequency standards, i.e., the Cs atomic clock, to be assured the accurate division of optical frequency f_n by the integer n .

Furthermore, if we can make $f_{ceo} = 0$, i.e., $\Delta\Phi = 0$, we can develop an ultimate optical frequency synthesizer, where the frequency of the n th component is exactly given by the simple relation $f_n = nf_{rep}$. To realize the above idea, we have experimented in our previous paper the phase (frequency) stabilization of the repetition rate by using a wide-band digital phase detector referenced to the KRISS Cs atomic clock [7]. In addition, we have introduced an orthogonalization scheme with a 3-axis piezo-electric transducer (PZT) to control independently both degrees of freedom of the fs laser in [7]. In this paper, we report the experimental results for the complete control of the two frequencies by employing two wide-band digital phase detectors and the orthogonalization scheme in [7]. For the phase (frequency) stabilization of f_{rep} and f_{ceo} at the same time, we realized a phase-coherent frequency

relationship between f_{rep} and f_{ceo} by the relation $f_{ceo} = f_{AOM} - 5/6 f_{rep} \equiv 0$, where $f_{AOM} = 5/6 f_{rep}$ is the phase-locked driving frequency of an acousto-optic modulator (AOM) in a self-referencing interferometer and $f_{rep} = 100$ MHz, by employing a phase locking technique. As a result, we could stabilize all components of the fs laser comb at once with an equal frequency separation of $f_{rep} = 100$ MHz with $f_{ceo} = 0$, leading to the fs laser comb as a frequency ruler in the optical frequency metrology.

II. EXPERIMENTS AND RESULT

The experimental scheme to realize our optical frequency synthesizer is shown in Fig. 1. The details of the performance characteristics of the fs Ti:s is described in [7], thus we describe here the system briefly. The Ti:s laser generates a 100 MHz pulse train with pulse width as short as 15 fs via Kerr lens mode-locking. The output power of the Ti:s laser operating in the self mode-locked regime was about 300 mW at the pumping power of 3.3 W. The output pulse spectrum was centered at 820 nm with a bandwidth of 50 nm. To reduce passively the cavity length drift within the tuning range of a PZT ($5 \mu\text{m}$ dynamic range at 150 V) which was used for the active control of both f_{rep} and f_{ceo} , temperature of the whole parts of the fs laser including base plate was controlled under 10 mK at slightly higher value than room temperature, and the laser itself was encased in a pressure-sealed box to reduce the airflow-induced frequency fluctuations.

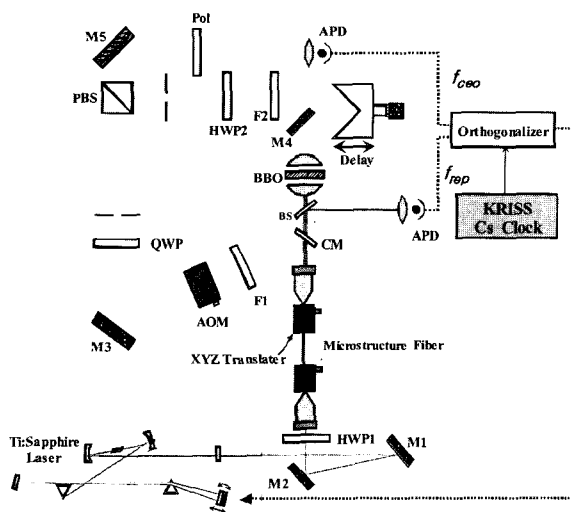


FIG. 1. Experimental setup to realize an optical frequency synthesizer. M1~M5: Total Mirror, Pol: Linear Polarizer, APD: Avalanche Photo Diode, HWP: $\lambda/2$ Plate, QWP: $\lambda/4$ Plate, F1, F2: Interference Filter (532 nm), BS: Beam Splitter, BBO: Second Harmonic Crystal, CM: Cold Mirror, AOM: Acousto-optic Modulator.

In reference [7], we have demonstrated a phase-locking of f_{rep} at 100 MHz referenced to the KRISS Cs atomic clock with a fractional stability below 5×10^{-11} at a 1 s average time without stabilizing f_{ceo} . To realize an optical frequency synthesizer using an orthogonal control scheme for f_{rep} and f_{ceo} , however, one need to detect f_{ceo} with high signal-to-noise (S/N) ratio. The detection of f_{ceo} , however, is not so simple as the detection of f_{rep} , rather it requires an octave-spanning spectrum of the fs comb to implement the self-referencing technique [8]. A tapered fiber or a nonlinear microstructure fiber (MS) can generate an extremely broadband continuum spectrum from the visible to the near infrared spectrum by use of low-energy fs pulses [3]. Generally, the properties of broadened spectrum generated by a MS fiber strongly depend on the peak power and polarization state of the input pulses, and also on the MS fiber length. To maintain the broadened comb spectrum with high stability, we controlled the input polarization state by a broadband half-wave plate placed in front of the fiber input (see Fig. 1). Also, we have tested the efficiency of the super-continuum generation depending on the length of the MS fiber and finally employed a 7 cm long MS fiber with $\sim 50\%$ transmission efficiency.

The carrier-envelope offset frequency was measured on a fast avalanche photo-diode (APD) by employing the f-to-2f self-referencing interferometer [2] as shown in Fig. 1. We used a 1 mm thick β -barium-borate (BBO) crystal for the second harmonic generation of the infrared part (around 1064 nm) of the comb spectrum. Fig. 2 shows the typical RF spectrum detected at the output of the self-referencing interferometer. With the detected two RF beat signals corresponding to f_{rep} and f_{ceo} , we implemented an orthogonal control scheme with a 3-axis PZT as described below.

For the orthogonal control of the two frequency

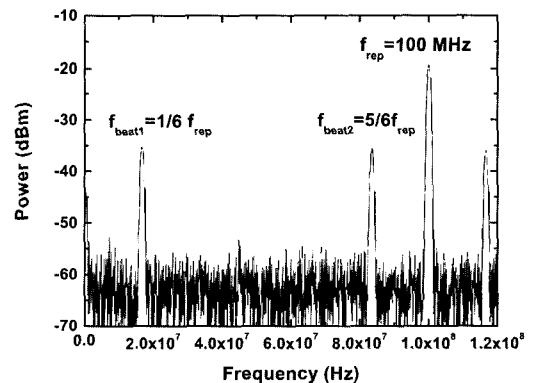


FIG. 2. RF power spectrum showing both repetition rate and heterodyne beat note from f-to-2f self-referencing interferometer. f_{beat1} and f_{beat2} are the beat frequencies corresponding to $1/6 f_{rep}$ and $5/6 f_{rep}$, respectively.

parameters, i.e., f_{rep} and f_{ceo} , we have coherently generated a 12th subharmonic of 100 MHz external clock signal from a precision frequency synthesizer which was phase-locked to the Cs clock, i.e., $100 \text{ MHz}/12 = 8.33 \dots \text{ MHz}$, for the reference frequency of a wide-band digital phase detector used for the phase stabilization of f_{ceo} . In this way, we could phase-lock f_{rep} and f_{ceo} at the same time with one reference external signal at 100 MHz from a precision frequency synthesizer of which frequency was phase-locked to the Cs atomic clock. In addition, the coherently generated $5/6$ th subharmonic of f_{rep} , i.e., $5/6 f_{rep} = 83.33 \dots \text{ MHz}$, was amplified up to 30 dBm in order to drive the AOM in Fig. 1 which shifts the high-frequency parts of the comb spectrum with an equal amount of $f_{AOM} = 5/6 f_{rep}$ in the self-referencing interferometer allowing the phase-locking of f_{ceo} to the value of zero [7]. Consequently, the beat frequency divided by an arbitrary integer N detected at the output of the self-referencing interferometer could be expressed by the relation

$$\frac{f_{ceo} + (f_{rep} - f_{AOM})}{N} = \frac{f_{rep}}{12}, \quad (1)$$

where N is an integer. We made an electronic circuit to set selectively $f_{ceo} = (N - 2)/12 f_{rep}$, $N = 0, 1, \dots, 11$ to realize Eq. (1) experimentally. It is obvious that for $N = 2$, the value of f_{ceo} is to be zero, resulting in the every comb frequency being precisely the integer multiple of f_{rep} , thereby the relation $f_n = n f_{rep}$ holds strictly.

The reason of using AOM in the self-referencing interferometer was to stabilize the value of f_{ceo} to zero as mentioned above and described in detail in our previous paper [7]. The resulting beat signal including AOM is expressed as

$$f_{beat} = f_{ceo} + (f_{rep} - f_{AOM}) \quad \text{or} \quad (2) \\ = f_{AOM} - f_{ceo}.$$

Clearly, one can see that $f_{beat} \neq 0$ for $f_{ceo} = 0$ since f_{rep} and $f_{AOM} \neq 0$. In this experiment we set $f_{AOM} = (5/6) f_{rep}$, thus, when $f_{ceo} = 0$, f_{beat1} and f_{beat2} become $f_{beat1} = (1/6) f_{rep} \simeq 16.7 \text{ MHz}$ and $f_{beat2} = (5/6) f_{rep} \simeq 83.3 \text{ MHz}$, respectively.

To stabilize both f_{rep} and f_{ceo} independently with phase error signals from the two phase detectors as described above, we employed an electronic orthogonalization scheme of Cundiff *et al.* [8] with a three-axis PZT [7]. Fig. 2 shows the RF spectrum of the repetition rate and the beat signals detected at the output port of the self-referencing interferometer in Fig. 1 (resolution bandwidth of 820 kHz). In this figure, the spectrum having RF power of about -20 dBm correspond to the repetition rate at $f_{rep} = 100 \text{ MHz}$ and the spectra having RF power below -20 dBm are due

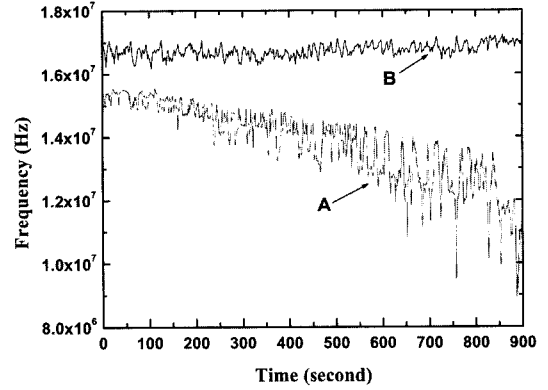


FIG. 3. Frequency fluctuation of the beat frequency around 16.7 MHz, i.e., $f_{ceo} \sim 0$, before (A) and after (B) the phase stabilization of the repetition rate at 100 MHz.

to the beat frequencies $f_{beat1} = 1/6 f_{rep}$ and $f_{beat2} = 5/6 f_{rep}$, respectively. In this experiment we used the beat frequency $f_{beat1} = 1/6 f_{rep} = 16.7 \dots \text{ MHz}$ as the control signal to stabilize the carrier-envelope offset frequency to zero. The beat signal at $\sim 16.7 \text{ MHz}$ was strongly filtered out by using a passive Chebyshev low-pass filter.

Fig. 3 shows the frequency fluctuation of $f_{beat1} = 1/6 f_{rep} \sim 16.7 \text{ MHz}$ before and after the stabilization of the repetition rate. As can be seen in Fig. 3, f_{beat1} , i.e., f_{ceo} , was stabilized as the result of the phase-locking of f_{rep} . This result can be understood from the analysis of Cundiff *et al.* [8]. Since f_{rep} depends on the cavity length and the swivel angle, while f_{ceo} depends mainly on the swivel angle, but weakly on the cavity length, the active stabilization of f_{rep} by adjusting the cavity length resulted in the weak stabilization of f_{ceo} . The standard deviation of f_{beat1} after stabilizing f_{rep} in 900 s was measured to be 194 kHz, and the maximum frequency jitter was about 1 MHz, which corresponds to the f_{ceo} phase fluctuation of 63 mrad, as estimated from the relation $\Delta\Phi = 2\pi f_{ceo}/f_{rep}$. To reduce the frequency noise further, we used a frequency-to-voltage (F/V) converter (AD652) for the pre-stabilization of f_{ceo} under the locking range of the digital phase-detector at $1/6 f_{rep}$. Due to the limited dynamic range of the F/V converter up to 2 MHz, we divided f_{beat1} by 16 and obtained a stable output signal that is linearly proportional to $f_{beat1}/16$. By adjusting the reference voltage of the frequency discriminator, we were able to lock f_{beat1} to any desired value around $1/6 f_{rep} \sim 16.7 \text{ MHz}$.

The two error signals, i.e., ε_1 from the F/V converter for f_{ceo} stabilization and ε_2 from the f_{rep} stabilization phase-detector, were processed through an orthogonalization circuit introduced in [7] and independently controlled the two degrees of freedom through the three-axis PZT in Fig. 1 with the servo bandwidth

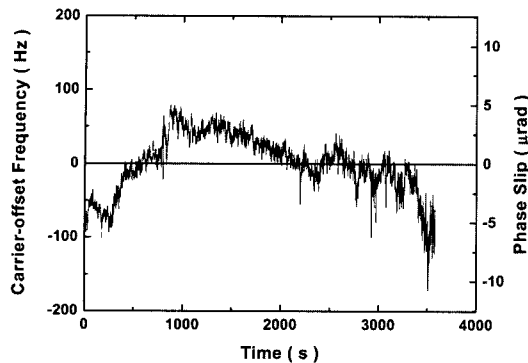


FIG. 4. Frequency jitter of the carrier-envelope offset frequency when the repetition rate and carrier-envelope offset frequency were stabilized simultaneously to the Cs atomic clock, leading to the unprecedented fluctuation of pulse-to-pulse phase slip of under $\pm 10\mu\text{rad}$.

of around 600 Hz. Fig. 4 shows the frequency jitter of f_{ceo} around the setting value of zero, when the repetition rate and the carrier-envelope offset frequencies were stabilized simultaneously to the Cs atomic clock. The standard deviation of the f_{ceo} was about 40 Hz, which corresponds to the phase fluctuation of $10\mu\text{rad}$. As one can see in Fig. 4, f_{ceo} was not phase-locked fully, but frequency-locked with the F/V converter as described above. We are now finalizing the phase-locking servo of f_{ceo} to make the frequency stability below 1 Hz and realize the relation $f_{ceo} = 0$. With this phase-coherent implementation of the optical and electrical circuits, we could stabilize all the frequencies of the fs comb components to the Cs atomic clock at once, which enables us to use the frequency comb as a frequency ruler in the optical frequency metrology. We are now finalizing our optical frequency synthesizer by increasing the phase-locking frequency for f_{rep} up to 5 GHz and improving the frequency stability of f_{ceo} under 1 Hz. The absolute optical frequency measurement should be now straightforward with our new optical frequency synthesizer and the first measurement results will be reported in the subsequent publications.

III. CONCLUSION

We have demonstrated a novel optical frequency synthesizer based on a frequency-stabilized fs mode-locked Ti:Sapphire laser with zero carrier-envelope offset frequency. We used a well-established digital phase-locking technique which phase coherently stabilized the two degrees of freedom of the fs laser, i.e., f_{rep} and f_{ceo} , to the Cs atomic frequency standard. As

a result, we have established a phase coherent link between the microwave frequency standard and the n th mode of the optical frequency comb with the simple relation $f_n = n f_{rep}$. The absolute optical frequency measurement should be straightforward now with our new optical frequency synthesizer.

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