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On Testing Exponentiality Against NBURFR Class Of Life Distributions

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Abstract. A non-parametric test based on U-statistic for testing exponentiality against the new better than used renewal failure rate (NBURFR) alternatives is introduced and the percentiles of this test statistic are tabulated for sample size 5(1)50. Its properties are also discussed including the Pitman asymptotic efficiency relative to the tests of the new better than used and new better than used failure rate (Ahmed (1994) and Hendi (2000)). The powers of this test are also calculated for some used life distributions. An example from blood cancer patients demonstrates a practical application of our test in the medical sciences is presented. Finally the problem when right-censored data is available is handled.

Key Words : *NBURFR, U-statistic, hypotheses testing, life testing, exponential distribution.*

1. INTRODUCTION

For over three decades, studies have led to declaring several families of life distributions to characterize aging. Many classes of life distributions have been introduced in reliability, See for example Barlow et al. (1963), Bryson and Siddiqui (1969), Barlow and Proschan (1981), Zachks (1992), Abouammoh and Ahmed (1992). All of these statisticians and reliability analysts have shown a growing interest in modeling survival data using classification of life distributions based on some aspects of aging. Among the most well known families are the classes of increasing failure rate (IFR), increasing failure rate in average (IFRA), new better than used (NBU), new better than used in expectation (NBUE), new better than used failure rate (NBUFR), new better than average failure rate (NBAFR) and their dual classes, decreasing failure rate (DFR), decreasing failure rate in average (DFRA), new worth than used (NWU), new worth than used in expectation (NWUE), new worth than failure rate (NWUFR), new worth than

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average failure rate (NWAFR). The implications among these classes of life distributions are as follows:

IFR → IFRA → NBU → NBUA → NBUE → NBUFR → NBAFR
↓
NBURFR

Testing exponentiality against various classes of life distributions has got a good deal of attention. With respect to testing against IFR, see Proschan and Pyke (1967), Barlow (1968), and Ahmed (1975), (1976) among others. For testing against IFRA, See Deshpande (1986), Linmk (1989), Aly (1989), and Ahmed (1994). For testing a gainst NBU, see Hollander and Proschane (1972), Koul (1977), Kumazawa (1983) and Ahmed (1994). Finally, for testing against NBUE, NBUFR, NBAFR classes, we refer to Klefsjo (1981 and 1982), Deshpande et al. (1986), Aboammoh and Ahmed (1988), Loh (1984) and Hendi et al. (2000). Now let T be a non negative random variable with life distribution $F(t)$, where $F(t)=0$ for t<0 and F(0) may not be zero. The corresponding survival function of new system and its pdf are given by $\overline{F}(t)$ and $f(t)$ respectively. Failure rate at time t is defined by $r_F(t)=f(t)/\overline{F}(t), t\ge 0$. In the long run, if a device is replaced by sequence of mutually and identically distributed, the remaining life distribution of the system under operation at time t is given by stationary renewal distribution as follows,

$$W_F(t)=\mu_F^{-1}\int_0^t\overline{F}(u)du, \quad \text{for } 0\le t<\infty$$

where

$$\mu_F=\int_0^\infty\overline{F}(u)du<\infty.$$

The corresponding renewal survival function is given by

$$\overline{W}_F(t)=1-W_F(t)=1-\mu_F^{-1}\int_0^t\overline{F}(u)du=\mu_F^{-1}\int_t^\infty\overline{F}(u)du.$$

The density function of the renewal distribution $W_F(t)$ is given by

$$w_F(t)=\frac{d}{dt}W_F(t)=\frac{d}{dt}\left(\mu_F^{-1}\int_0^t\overline{F}(u)du\right)=\mu_F^{-1}\overline{F}(t)=-\frac{d}{dt}\overline{W}_F(t) \text{ for } 0\le t<\infty.$$

The failure rate of the renewal distribution $W_F(t)$ is given by

$$r_w(t)=\frac{w_F(t)}{\overline{W}_F(t)}=\frac{\overline{F}(t)}{\int_t^\infty\overline{F}(u)du}=(\mu_F(t))^{-1} \text{ for } 0\le t<\infty,$$

where $\mu_F(t)$ is the mean remaining life distribution of a used unit at time t.

Definition 1.1. F is new better than used renewal failure rate (NBURFR) if

$$t \geq 0 \qquad (1.1)$$

i.e the failure rate of a new system is less than the renewal failure rate of a used system.

Definition 1.2. F is new worth than used renewal failure rate (NWURFR) if

$$r_F(0) \geq r_w(t), t \geq 0 \qquad (1.2)$$

i.e the failure rate of a new system is greater than the renewal failure rate of a used system. For convenience, we note that (1.1) and (1.2) are equivalent to

$$\int_t^{\infty} \overline{F}(u)du \leq \overline{F}(t)/r_F(0), t > 0 \qquad (1.3)$$

$$\int_t^{\infty} \overline{F}(u)du \geq \overline{F}(t)/r_F(0), t > 0, \text{ see (Abouammoh and Ahmed (1992))} \qquad (1.4)$$

The main theme of this article is to test H_0 :F is exponential against H_1 :F∈ NBURFR and not exponential. A non-parametric test based on U-statistic for testing exponentiality a against the new better than used renewal failure rate (NBURFR) alternatives is introduced and the percentiles of this test statistic are tabulated for sample size 5(1)50. Its properties are also discussed including the Pitman asymptotic efficiency relative to the tests of the new better than used and new better than used failure rate (Ahmed (1994) and Hendi et al (2000)). The powers of this test are also calculated for some used life distributions. An example from blood cancer patients demonstrates a practical application of our test in the medical sciences is presented. Finally the problem when right-censored data is available is handled.

2. TESTING AGAINST NBURFR CLASS ALTERNATIVES IN THE NONCENSORED CASE

In this section we derive a non parametric U-statistic test for testing

H_0 : F is exponential against H_1 : F∈ NBURFR and not exponential.

For more details about U-statistics, see Lee (1989). Here, the problem is to test H_0 :F is standard exponential against H_1 :F∈ NBURFR and not exponential based on sample $X_1, X_2, X_3, \ldots X_n$ from F. Since F is NBURFR, means that $\int_t^{\infty} \overline{F}(u)du \leq \overline{F}(t)/r_F(0)$ for all t, we use the following measure of departure from H_0.

$$\delta_F = E\left(\overline{F}(t) - f(0)w(t)\right) \geq 0$$

$$= \frac{1}{2} - f(0)\int_0^{\infty} w(t)dF(t),$$

where $w(t)=\int_t^\infty \bar{F}(u)du$. Note that $\delta_F=0$ under H_0 and $\delta_F>0$ under H_1. To estimate δ_F let $X_1,X_2,X_3,...X_n$ be a random sample from F, then F(t), $w(t)$ and f(0) will be empirically estimated by $\hat{F}_n(t)$, $\hat{w}_n(t)=\frac{1}{n}\sum_{i=1}^{n}(X_i-t)I(X_i>t)$, and $\hat{f}_n(0)=\frac{1}{na_n}\sum_{j=1}^{n}K(-X_j/a_n)$, where K(.) be a known pdf, symmetric and bounded with 0 mean and variance $\sigma_k^2>0$. For more details about K and the sequence $\{a_n\}$ See Hardle (1991). So the empirical form of δ_F is as follows:

$$\hat{\delta}_{F_n}=\frac{1}{2}-\hat{f}_n(0)\frac{1}{n^2}\sum_{j=1}^{n}\sum_{i=1}^{n}\left((X_i-X_j)I(X_i\geq X_j)\right)$$

$$=\frac{1}{n^3}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\left[\frac{1}{2}-\frac{1}{a_n}K\left(\frac{-X_k}{a_n}\right)\left((X_i-X_j)I(X_i\geq X_j)\right)\right].$$

If we define $\phi(X_1,X_2,X_3)=\frac{1}{2}-\frac{1}{a_n}K\left(\frac{-X_3}{a_n}\right)(X_2-X_1)I(X_2\geq X_1)$, and defining the symmetric kernel

$$\psi(X_1,X_2,X_3)=\frac{1}{3!}\sum_R\phi(X_{i1},X_{i2},X_{i3}),$$

where, the summation over all arrangements of X_{i1},X_{i2},X_{i3}, then $\hat{\delta}_{F_n}$ is equivalent to U-statistic

$$U_n=\frac{1}{\binom{n}{3}}\sum_R\psi(X_i,X_j,X_k).$$

The following theorems summarize the large sample properties of $\hat{\delta}_{F_n}$.

Theorem 2.1. (i) As $n\to\infty$, $\sqrt{n}\left(U_n-\delta_{F_n}\right)\sim N(0,\sigma^2)$

where,

$$\sigma^2=Var\left\{\frac{3}{2}-f(0)\left(\begin{array}{c}\int_X^\infty udF(u)-\int_0^X udF(u)-X\bar{F}(X)+XF(X)+\\ \int_0^\infty u\bar{F}(u)dF(u)-\int_0^\infty\int_0^X udF(u)dF(v)\end{array}\right)\right\}$$

(ii) Under H_0, $\sigma^2=1/3$

(iii) If F is continuous NBURFR, then the test is consistent.

Proof. (i) and (ii) follow from the standard theory of U-statistic cf- Lee(1989) and by direct calculations respectively.

(iii) δ_F can be written in the form

$$\delta_F = \int_0^\infty \left(\frac{1}{2} - f(0)w(t) \right) dF(t).$$

Let D(t) $= \frac{1}{2} - f(0)w(t)$. Since F is NBURFR and continuous, then D(t)>0 and since F is not exponential then D(t)>0 for at least one t, call it t_0. Set $t_1 =$. $\inf\{t \mid t \le t_0 \text{ and } \bar{F}(t) = \bar{F}(t_0)\}$ Thus, $D(t_1) = \frac{1}{2} - f(0)w(t_1) \ge \frac{1}{2} - f(0)w(t_0)$ $= D(t_0) > 0$ and $F(t_1 + \delta) - F(t_1) > 0$, and since t_1 is the point of increase of F, thus δ_F >0.

To conduct the test, calculate $\sqrt{3n}\hat{\delta}_{F_n}$ and reject H_0 if this value exceeds Z_α, the standard normal variate at level α.

2.1 Monte Carlo Null Distribution Critical Points

A FORTRAN program is used to compute $\hat{\delta}_{F_n}$ based on 5000 simulated samples from the standard exponential distributions of order 5(1)50. Table 2.1 gives lower and upper percentiles points of the statistic of $\hat{\delta}_{F_n}$.

Table 2.1. Critical values of $\hat{\delta}_{F_n}$.

n	.01	.05	.10	90	95	98	99
5	.0199	.0932	.1242	.2475	.2573	.2676	.2745
6	.0033	.0788	.1112	.2303	.2410	.2519	.2577
7	.0196	.0808	.1077	.2191	.2283	.2380	.2461
8	.0172	.0738	.0984	.2089	.2190	.2271	.2327
9	.0234	.0767	.0983	.2014	.2104	.2197	.2245
10	.0255	.0729	.0953	.1959	.2054	.2146	.2202
11	.0230	.0695	.0932	.1895	.1983	.2065	.2135
12	.0249	.0688	.0902	.1856	.1942	.2029	.2091
13	.0260	.0706	.0879	.1812	.1897	.1998	.2044
14	.0283	.0686	.0877	.1779	.1863	.1940	.1990
15	.0333	.0700	.0871	.1739	.1819	.1904	.1960
16	.0302	.0696	.0875	.1722	.1802	.1881	.1937
17	.0318	.0688	.0851	.1677	.1757	.1839	.1888

18	.0402	.0680	.0857	.1664	.1746	.1826	.1883
19	.0301	.0683	.0862	.1631	.1706	.1788	.1837
20	.0388	.0702	.0847	.1622	.1696	.1770	.1823
21	.0418	.0724	.0876	.1612	.1684	.1759	.1810
22	.0417	.0709	.0849	.1586	.1655	.1718	.1765
23	.0436	.0715	.0858	.1582	.1653	.1728	.1773
24	.0424	.0708	.0852	.1564	.1632	.1706	.1749
25	.0463	.0739	.0853	.1548	.1625	.1704	.1750
26	.0446	.0719	.0845	.1537	.1605	.1668	.1718
27	.0478	.0706	.0865	.1525	.1592	.1670	.1713
28	.0468	.0716	.0854	.1515	.1578	.1648	.1696
29	.0467	.0716	.0854	.1496	.1561	.1634	.1689
30	.0505	.0747	.0858	.1491	.1558	.1629	.1664
31	.0516	.0745	.0851	.1488	.1556	.1623	.1668
32	.0521	.0734	.0865	.1475	.1543	.1617	.1659
33	.0503	.0734	.0865	.1462	.1526	.1599	.1640
34	.0513	.0748	.0860	.1453	.1518	.1590	.1637
35	.0532	.0748	.0852	.1454	.1518	.1582	.1619
36	.0543	.0755	.0862	.1445	.1511	.1584	.1613
37	.0512	.0741	.0854	.1440	.1505	.1572	.1606
38	.0535	.0758	.0860	.1436	.1496	.1570	.1611
39	.0563	.0751	.0863	.1432	.1497	.1551	.1590
40	.0574	.0774	.0865	.1425	.1488	.1553	.1593
41	.0537	.0769	.0859	.1417	.1478	.1544	.1580
42	.0534	.0754	.0863	.1411	.1470	.1533	.1575
43	.0570	.0763	.0850	.1405	.1464	.1521	.1561
44	.0575	.0770	.0862	.1401	.1463	.1518	.1559
45	.0606	.0774	.0861	.1395	.1455	.1511	.1542
46	.0589	.0779	.0873	.1393	.1458	.1519	.1554
47	.0581	.0774	.0866	.1391	.1451	.1510	.1548
48	.0593	.0774	.0859	.1383	.1444	.1493	.1527
49	.0601	.0783	.0871	.1385	.1437	.1493	.1525
50	.0615	.0777	.0859	.1369	.1433	.1491	.1528

3. ASYMPTOTIC RELATIVE EFFICIENCY AND POWERS

Since the above test is new and no other tests known for NBURFR we compare our test to the smaller classes and choose NBU and NBUFR classes which are studied by Ahmed (1994) and Hendi (2000). The comparisons are achieved by using Pitman asymptotic relative efficiency, which is defind as follows:

Let T_{1n}, T_{2n} be two test statistics for testing $H_0 : F_\theta \in \{F_{\theta_n}\}, \theta_n = \theta + cn^{-1/2}$ with c is an arbitrary constant, then the asymptotic relative efficiency of T_{1n} relative to T_{2n} is defined by $e(T_{1n}, T_{2n}) = [\mu_1'(\theta_0)/\sigma_1(\theta_0)]/[\mu_2'(\theta_0)/\sigma_2(\theta_0)]$, where

$\mu_i'(\theta_0) = \lim_{n\to\infty}\left(\frac{\partial}{\partial\theta}E(T_{in})\right)_{\theta\to\theta_0}$ and $\sigma_i^2(\theta_0) = \lim_{n\to\infty} Var_0(T_{in})$, i=1,2 is the null variance.

We choose the following alternatives:

(i) Linear failure rate family : $\overline{F}(t) = \exp(-t - \theta t^2/2)$, $t > 0, \theta \ge 0$

(ii) Makeham family $\overline{F}(t) = e^{-t-\theta(t+e^{-t}-1)}$ (iii) Pareto family $\overline{F}(t) = (1-\theta t)^{1/\theta}$

Note that H_0 is attained at $\theta = 0$ in (i) and (ii), while as $\theta \to 0$ in (iii). Direct calculations give the asymptotic efficiencies and the relative efficiencies as follows in Table 3.1.

Table 3.1. Asymptotic relative efficiency of $\hat{\delta}_{F_n}$, $\hat{\delta}_{F_n^{(1)}}$ Ahmed (1994), $\hat{\delta}_{F_n^{(2)}}$ Hendi (2000).

Efficiency	F_1 LFR	F_2 Makeham	F_3 Pareto
$\hat{\delta}_{F_n^{(1)}}$ Ahmed (1994)	0.8056	0.2854	....
$\hat{\delta}_{F_n^{(2)}}$ Hendi (2000)	0.433	0.289	0.1880
$\hat{\delta}_{F_n}$	1.2990	0.5774	1.2990
$e(\delta_{F_n}, \delta_{F_n^{(1)}})$	1.6100	2.0300	....
$e(\delta_{F_n}, \delta_{F_n^{(2)}})$	3.0000	2.0000	6.9100

It is clear from the Table 3.1 that our test performs well for all previous alternatives.

For the previous alternatives, a Fortran program is used to tabulate the powers for the proposed test in Table 3.2 by using simulated number of sample 5000 for sample sizes 10.20 and 30 and θ values 2.3 and 4.

Table 3.2. Powers for NBURFR class test.

	N		Powers		Powers		Powers
LFR	10 20 30	$\theta=2$	1.000 1.000 1.000	$\theta=3$	1.000 1.000 1.000	$\theta=4$	1.000 1.000 1.000
Pareto	N 10 20 30	$\theta=2$	1.000 1.000 1.000	$\theta=3$	1.000 1.000 1.000	$\theta=4$	1.000 1.000 1.000
Weibull	N 10 20 30	$\theta=2$	1.000 1.000 1.000	$\theta=3$	1.000 1.000 1.000	$\theta=4$	1.000 1.000 1.000
Gamma	N 10 20 30	$\theta=2$	0.987 0.996 0.996	$\theta=3$	0.999 1.000 1.000	$\theta=4$	1.000 1.000 1.000

It is clear from this table that our test has a good powers.

4. AN APPLICATION OF $\hat{\delta}_{F_n}$ TEST

Consider the data in Abouamoh et al.(1994), these data represent set of 40 patients suffering from blood cancer (Leukemia) from one of Ministry of Health Hospitals in Saudi Arabia and the ordered values in days are :

115, 181, 255, 418, 441, 461, 516, 739, 743, 789, 807, 865, 924, 983, 1024, 1062, 1063, 1165, 1191, 1222, 1222, 1251, 1277, 1290, 1357, 1369, 1408, 1455, 1478, 1549, 1578, 1599, 1603, 1605, 1696, 1735, 1799, 1815, 1852, 1899.

It was found that the test statistic for the data set, by formula (2.1) is $\hat{\delta}_{F_n} = 0.2565625$ which is greater than the critical value of the Table 2.1. Then we reject the null hypothesis of exponentiality agreeing with the conclusion in Hendi et al (2000).

5. ON TESTING EXPONENTIALITY AGAINST NBURFR ALTERNATIVES IN THE CENSORED CASE

In this Section, a test statistic is proposed to test H_0 versus H_1 with randomly right censored samples. In the censoring model, instead of dealing with $X_1, X_2, X_3, \ldots X_n$, we observe the pair (Z_i, δ_i), i=1,2,3,…,n, where $Z_i = \min(X_i, Y_i)$ and $\delta_i = 1$ if $Z_i = X_i$, $\delta_i = 0$ if $Z_i = Y_i$, where $X_1, X_2, X_3, \ldots X_n$ denote their true life time from a distribution F and $Y_1, Y_2, Y_3, \ldots Y_n$ be i.i.d according to censored distribution G. Also we assume X's and Y's are independent. Let $Z_{(0)} = 0 < Z_{(1)} < Z_{(2)} < Z_{(3)} \ldots < Z_{(n)}$ denote the ordered Z's and $\delta_{(i)}$ is the δ_i corresponding to $Z_{(i)}$, respectively. Using the censored data (Z_i, δ_i), i=1,2,3,…,n by replacing the product limit estimator of Kaplan and Meier (1958)

$$\hat{\bar{F}}_n(X) = 1 - \hat{F}_n(X) = \prod_{(i<Z_{(i)} \le X)} \left[\frac{n-i}{n-i+1}\right]^{\delta_{(i)}}, \mathrm{X} \in \left[0, \mathrm{Z}_{(n)}\right].$$

Tanner (1983) proposed the following kernel estimator of the hazard rate in the censored case

$$\hat{r}_n(t) = \frac{1}{2R_k} \sum_{i=1}^{n} \left[\frac{\delta_{(i)}}{n-i+1} K\left(\frac{t - Z_{(i)}}{2R_k}\right)\right],$$

where

R_k distance between point t and its k th nearest failure point

K(.) a function of bounded variation with compact support on the interval [-1,1].

The proposed test statistic is

$$\hat{\delta}^c_{F_n} = \int_0^\infty \bar{F}_n(t) d\bar{F}_n(t) - \int_0^\infty \hat{r}(0)\hat{\omega}(t) d\bar{F}_n(t) \tag{3.1}$$

For computation use, $\hat{\delta}^c_{F_n}$ in (3.1) can be written in the following form

$$\hat{\delta}^c_{F_n} = \sum_{i=1}^{n} \prod_{j=1}^{i-1} \left(C_{(j)}\right)^{\delta_{(i)}} \left\{ \prod_{m=1}^{i-2} \left(C_{(m)}\right)^{\delta_{(m)}} - \prod_{mm=1}^{i-1} \left(C_{(mm)}\right)^{\delta_{(mm)}} \right\} -$$

$$\sum_{i=1}^{n} \hat{r}(0) \sum_{j=i}^{n} \prod_{k=1}^{j-1} \left(C_{(k)}\right)^{\delta_{(k)}} \left(Z_{(j)} - Z_{(j-1)}\right) \left\{ \prod_{m=1}^{i-2} \left(C_{(m)}\right)^{\delta_{(m)}} - \prod_{mm=1}^{i-1} \left(C_{(mm)}\right)^{\delta_{(mm)}} \right\}$$

where $C_k = (n-k)/(n-k+1)$ and $dF_n(Z_j) = \bar{F}_n(Z_{j-1}) - \bar{F}_n(Z_j)$, Table 3.3 gives the critical values of $\hat{\delta}^c_{F_n}$ test for sample sizes 5(1),50, 60, 70, 80, 81.

Table 3.3. Critical values of $\hat{\delta}^c_{F_n}$.

n	.01	.05	.10	90	95	98	99
7	-.4754	-.0051	.0912	.3671	.3777	.3875	.3912

8	-.4314	.0000	.1409	.3748	.3864	.3973	.4028
9	-.5349	.0000	.1264	.3846	.3951	.4066	.4124
10	-.3977	.0538	.1545	.3860	.3999	.4132	.4185
11	-.3684	.0415	.1538	.3898	.4066	.4198	.4256
12	-.3925	.0518	.1684	.3883	.4064	.4226	.4297
13	-.4035	.0614	.1565	.3917	.4112	.4264	.4348
14	-.3730	.0669	.1709	.3889	.4099	.4284	.4380
15	-.3720	.0626	.1624	.3888	.4098	.4313	.4388
16	-.3027	.0708	.1720	.3831	.4046	.4299	.4406
17	-.3010	.0671	.1645	.3866	.4081	.4342	.4438
18	-.2612	.0881	.1808	.3833	.4057	.4346	.4476
19	-.3376	.0766	.1694	.3846	.4067	.4349	.4478
20	-.2491	.0907	.1797	.3782	.3998	.4296	.4467
21	-.2967	.0852	.1707	.3812	.4035	.4353	.4510
22	-.2747	.0820	.1774	.3767	.3997	.4354	.4526
23	-.2725	.0810	.1697	.3757	.4009	.4310	.4517
24	-.2431	.0942	.1825	.3727	.3964	.4290	.4521
25	-.2729	.0735	.1691	.3747	.3986	.4282	.4544
26	-.4626	.0256	.1291	.3745	.3988	.4257	.4565
27	-.4263	.0145	.1261	.3777	.4029	.4514	.4613
28	-.5101	-.0039	.1120	.3778	.4046	.4530	.4615
29	-.5909	-.0204	.1074	.3774	.4084	.4543	.4613
30	-.5648	-.0261	.1028	.3801	.4073	.4550	.4631
31	-.5255	-.0248	.0970	.3811	.4092	.4596	.4651
32	-.5723	-.0334	.0965	.3801	.4104	.4610	.4658
33	-.5517	-.0279	.0930	.3795	.4067	.4566	.4658
34	-.5882	-.0235	.0953	.3809	.4087	.4590	.4667
35	-.6389	-.0225	.0931	.3821	.4098	.4628	.4680
36	-.5998	-.0299	.0937	.3815	.4098	.4632	.4684
37	-.6014	-.0300	.0902	.3815	.4120	.4644	.4691
38	-.5485	-.0299	.0935	.3819	.4117	.4646	.4694
39	-.5813	-.0499	.0911	.3821	.4071	.4649	.4706
40	-.6366	-.0543	.0910	.3799	.4063	.4651	.4704
41	-.5691	-.0245	.0995	.3809	.4073	.4645	.4705
42	-.5686	-.0195	.1069	.3798	.4060	.4570	.4705
43	-.4874	-.0091	.1111	.3809	.4070	.4635	.4711
44	-.4940	.0009	.1118	.3782	.4044	.4646	.4721
45	-.5676	.0177	.1209	.3787	.4051	.4635	.4715
46	-.4995	.0184	.1230	.3764	.4050	.4604	.4723
47	-.2602	.0511	.1321	.3779	.4034	.4533	.4720

48	-.2882	.0488	.1274	.3749	.3996	.4527	.4722
49	-.2770	.0490	.1294	.3720	.3975	.4534	.4726
50	-.2698	.0415	.1218	.3725	.3962	.4355	.4721
60	-.8927	-.1304	.0455	.3762	.4269	.4759	.4777
70	-1.0341	-.1501	.0454	.3837	.4439	.4784	.4802
80	-.4298	.0501	.1372	.3657	.4035	.4782	.4809
81	-.5127	.0342	.1362	.3640	.4051	.4793	.4816

6. A APPLICATION OF $\hat{\delta}^c_{F_n}$

Consider the data in susarla and vanryzin (1978). These data represent 81 survival times of patients of melanoma. Of them 46 represent whole life times (non-censored data) and the ordered values are :

13,14,19,19,20,21,23,23,25,26,26,27,27,31,32,34,34,37,38,38,40,46,50,53,54,57,58,59,60, 65,65,66,70,85,90,98,102,103,110,118,124,130,136,138,141,234.

The ordered censored observations are:

16,21,44,50,55,67,73,76,80,81,86,93,100,108,114,120,124,125,129,130,132,134,140,147, 148,151,152,152,158,181,190,193,194,213,215.

A simple computer program is written to calculate $\hat{\delta}^c_{F_n}$ for these censored data and the value we get leads to a $\hat{\delta}^c_{F_n}$ =0.3124690 which is less than the critical value in Table 3.3 at 95% upper percentile. Then we reject H_1 which states that the set of data have NBURFR property.

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