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EXTENSION OF CR-FUNCTIONS DEFINED ON WEDGE-LIKE DOMAINS IN CR-MANIFOLDS

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ABSTRACT. We give results describing behavior of regions of holomorphic extension of CR functions near boundary points of their domain of definition.

1. Introduction

A remarkable phenomenon in several complex variables is an automatic analytic extension of holomorphic functions defined in an open subset $\Omega \subset \mathbb{C}^N$ to a larger open subset Ω'. Given such a subset Ω, it is a nontrivial problem is to determine geometric shapes of a corresponding subset Ω'. An effective way of attacking this problem is its localization near a boundary point, leading to the study of the local holomorphic extension of boundary values of holomorphic functions (or, more generally, CR-functions).

Our goal in this paper is to study holomorphic extension of CR-functions (always assumed continuous in this paper) defined on an open piece V of a smooth hypersurface $M \subset \mathbb{C}^N$, in particular, to understand the behavior of the region of extension as one approaches the boundary of V in M. Whereas such local extension near interior points of V has been intensively studied by many authors, much less is known about extension near boundary points. The main difficulty here is to give a *quantitave* estimate for the size of the region of extension rather than just a *qualitative* description. Of course, one can obtain certain quantitative estimates by analysing the proofs of various results giving qualitative pictures. However, in most cases, such estimates are very rough.

To illustrate the "roughness" mentioned above, consider the case when the Levi form $L(w, w)$ of M in the direction of some complex tangent vector w is nontrivial. Then the classical extension results of

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Kneser [11] and Lewy [12] yield holomorphic extension of CR-functions on V to the one-sided neighborhood in the direction of $L(w,w)$. Suppose now that V is a relatively compact open subset with smooth boundary in another real hypersurface M and p is its boundary point. Then the domain of extension of CR-functions on V may a priori shrink arbitrarily fast as one goes to the boundary. If the Levi form of M is nonzero on a tangent vector w at p, the classical argument gives holomorphic extension to a region in the direction of $L(w,w)$ whose size near a point $x \in M$ behaves like $\text{dist}(x, \partial V)^2$, where "dist" stands for the euclidean distance. If, however, the boundary ∂V is generic in $\mathbb{C}^N$ near p, i.e. if $T_p\partial V$ spans $T_p\mathbb{C}^N$ over $\mathbb{C}$, then it follows from our results in [17] that the above estimate can be improved to $\text{dist}(x, \partial V)$. In particular, the extension takes place to a wedge W with edge on the boundary ∂V such that W is bounded by V itself and another hypersurface through ∂V showing into the "Levi-positive" side. Here we have to mention that our results have been inspired by recent results of Tumanov [13, 14] proving in the above (and also more general cases) holomorphic extension to a wedge with edge ∂V in some direction, possibly not related to the Levi form of M at p.

From the mentioned general results of Tumanov it follows that holomorphic extension to wedges occurs also for "corners" of V. More precisely, if M is minimal at p and V is a wedge-like domain in M, whose edge E is generic in $\mathbb{C}^N$, it is shown in [14] then CR-functions on V extend holomorphically to a wedge W with edge E. Here no information is given about the direction of W. In view of the above mentioned fact for smooth boundaries, one may conjecture that any direction of the Levi form at p still gives a direction of extension for some wedge in $\mathbb{C}^N$ (with edge E at p) in case V is a wedge with generic edge E as in Tumanov's result. This, however, turns out to be false in general as was recently observed by Eastwood-Graham [9, 10] (see [9, Example 1.2] and Example 2.2 below).

In [17] the authors proposed an invariant geometric way of selecting those Levi form directions that are responsible for the extension. The main idea was to consider Levi form directions $L(w,w)$ only for those w, for which the complex line $\mathbb{C}w$ contains a sector of angle $> \pi/2$. Each such directions leads to an extension to wedges in that direction (see [17]). This gives a rather general description of the additional directions of extension in terms of the Levi form of M at a boundary point $p \in \partial V$. (For a different use of sectors for the description of directions of extendibility at the interior points of V, based on a different method, the reader is referred to Baouendi-Treves [5] and Baouendi-Rothschild [3].)

On the other hand, Tumanov's results [14] mentioned above still give extension even if the Levi form of M completely vanishes at p. This suggests that the above description should be also possible to generalize to this and other analogous situations. In this paper we give such a generalization also treating the case when M is of infinite type at p, where no information about extension can be extracted from the Taylor series of the defining equation of M.

Finally, we say few words about the methods used here to show the extension. One of the ideas of Tumanov was to obtain the required extension through analytic discs that are partially attached to the boundary of ∂V. The discs can be still chosen to be smooth ($C^{1,\beta}$) and hence one can discuss their directions. Once a sufficiently small disc is found, whose direction at a boundary point $p \in \partial V$ is transversal to M, the required wedge is obtain by deforming the disc. A problem arises here if one want to control the direction of the disc rather than just say that it is transversal. This was done by Baracco and the second author in [6] for directions of the Levi form evaluated at vectors tangent to the edge E. Unless the Levi form at p vanishes in all directions, this approach always gives some new directions. However, in view of Example 2.4 below, in this way one may not obtain all directions of extension that can be obtained from the mentioned results in [17]. The "missing" directions are no more tangent to E and, as a consequence, one cannot attach smooth analytic discs to $E \cup V$ in those directions.

The method of this paper is based on attaching *nonsmooth analytic discs* for which the derivative at the reference point does not exists. These discs can be written as a sum of a nontrivial factor of $(1-\tau)^\alpha$ with $\alpha < 1$ and a smooth term. The replacement for the "direction" of such a disc is the coefficient of the first factor. However, one can see that this coefficient (as a vector) is never transversal to M and hence one cannot "fill" a wedge by deforming such a disc and applying the implicit function theorem as it is done for smooth discs. Furthermore, the information about the coefficient of $(1-\tau)^\alpha$ does not suffice for a description of the region filled by those discs. What now comes into play is the derivative of the remaining term. The first main step in the proof is to get control of that derivative. Then, by deforming such a disc, one does not fill a full wedge but rather a smaller region called here "α-wedge". Since, after normalizing, the defining equation of M begins from terms of order 2, such an α-wedge with $\alpha > 1/2$ dominates the terms of M. This is the main advantage of having an α-wedge and is crucial for the second step, where an α-wedge V' is used to obtain a family of smooth analytic discs attached to V' and filling a wedge in the usual sense.

2. Main results and examples

Let M be a smooth real hypersurface in $\mathbb{C}^N$ through 0 given by

(2.1)
$$M = \{(x+iy, w) \in \mathbb{C} \times \mathbb{C}^{N-1} : y = h(x,w)\}, \quad h(0) = 0, \quad h'(0) = 0.$$

By a wedge $V \subset M$ with edge E at $p \in M$, we understand a diffeomorphic image of a neighborhood of 0 in $\mathbb{R}^{\dim E} \times \Gamma$, where $\Gamma \subset \mathbb{R}^{\dim M - \dim E}$ is a convex cone such that E is the image of a neighborhood of 0 in $\mathbb{R}^{\dim E} \times \{0\}$. Denote by C_pV the closed (Whitney) tangent cone to V at p. By a sector with angle γ in a complex vector space $\mathbb{C}^n$, we mean the image of an open sector of this angle in $\mathbb{C}$ under a nontrivial complex-linear map. We have the following result.

THEOREM 2.1. *Let V be a wedge with edge E at 0 in a C^4-smooth hypersurface $M \subset \mathbb{C}^N$ given by (2.1), where E is a C^2-smooth generic submanifold of $\mathbb{C}^N$. Suppose that for some $\varepsilon > 0$ and sector $S \subset T_0^cM \cap C_0V$ with angle $> \pi/2$, the following holds:*

$$(2.2) \qquad h(x,w) > 0 \quad \text{for} \quad w \in S, \quad |x| < \varepsilon|w|, \quad |w| < \varepsilon.$$

Then all continuous CR-functions on V extend holomorphically to a wedge V' in $\mathbb{C}^N$ with edge E at 0 such that

(i) *$\overline{V'}$ contains a neighborhood of 0 in V;*
(ii) *V' is contained in the positive side $\{y > h(x,w)\}$.*

Theorem 2.1 generalizes the corresponding result from [17]. Indeed, in [17] the same conclusion was obtained unter the assumption that there exists a sector S as above such that the Levi form of M at 0 is positive if evaluated at $w \in S$. Then one can easily choose coordinates $(x+iy, w)$ in (2.1) such that (2.2) holds for $\varepsilon > 0$ sufficiently small. One the other hand, it is clear that the assumption (2.1) may still hold even when the Levi form of M at 0 vanishes in all directions.

The angle $\pi/2$ in Theorem 2.1 is optimal and cannot be replaced by any smaller angle. We give here the corresponding example from [17].

EXAMPLE 2.2. Consider the hypersurface

$$M := \{z = (x+iy, w_1, w_2) \in \mathbb{C}^3 : y = (\operatorname{Im} w_2)^2 - (\operatorname{Im} w_1)^2\}$$

and the wedge $V := \{z \in M : \operatorname{Im} w_1 > |\operatorname{Im} w_2|\}$ with edge $E := \mathbb{R}^3$ at 0. Then, for

$$\xi := (0, 1, 1 - \delta - i\sqrt{2\delta}) \subset T_0^c M,$$

the Levi form of M at 0 in the direction ξ shows up. Hence, after a change of coordinates, (2.2) holds for any sector $S \subset \mathbb{C}\xi \cap C_0 V$. Furthermore, if $\delta > 0$ arbitrarily small, the angle of S can be taken arbitrarily close to $\pi/2$. However, any domain of holomorphic extension of CR-functions on V must be contained in the half-space $y < 0$. Hence the conclusion of Theorem 2.1 cannot hold in this case.

The assumption that the edge E is *generic* in Theorem 2.1 is also important and cannot be dropped as it is shown by another example from [17]:

EXAMPLE 2.3. Consider the hypersurface $M := \{z = (x + iy, w) \in \mathbb{C}^2 : y = |w|^2\}$ and, for a real number $1/2 < \alpha < 1$, the wedge $V := \{z \in M : y < \operatorname{Im} w^{1/\alpha}\}$ with edge $E := \{z \in M : w = 0\}$ at 0. Then (2.2) holds for the sector $S := T_0^c M \cap C_0 V$, whose angle is $\pi\alpha > \pi/2$. However, the CR-function $f(z) := (x + iy - w^{1/\alpha})^{-1}$ on V does not extend holomorphically to any wedge V' as in Theorem 2.1.

Finally, the following example (also from [17]) shows the optimality of the angle $\pi/2$ in Theorem 2.1 "from the other side" in the sense that a replacement $\pi/2$ by any larger angle in the theorem would lead to a strictly weaker statement.

EXAMPLE 2.4. Consider the hypersurface

$$M := \{z = (x + iy, w_1, w_2) \in \mathbb{C}^3 : y = -|w_1|^2 - |w_2|^2 + (2 + \delta)\operatorname{Im} w_1\overline{w}_2\}$$

for a small $\delta > 0$. Then the Levi form is nonnegative only for vectors $\xi \in T_0^c M$ whose direction is sufficiently close to that of $\xi_0 = (0, -1+i, 1+i)$. We choose $V := \{z \in M : \operatorname{Im} w_1 > 0, \operatorname{Im} w_2 > 0\}$ so that the sector $S_0 := \mathbb{C}\xi_0 \cap C_0 V$ has angle $\pi/2$. Then one can find sectors S arbitrarily close to S_0 with angle $> \pi/2$ for which Theorem 2.1 yields extension to a wedge on the positive side of M. On the other hand, any sector satisfying (2.2), even for a different choice of coordinates, must be close to S_0 and hence, its angle must be close to $\pi/2$.

3. Nonsmooth analytic discs attached to $V \cup E$

The previously known techniques for proving wedge-extendibility of CR-functions on V in the setting of Theorem 2.1 were based on attaching smooth ($C^{1,\beta}$) analytic discs to $E \cup V$ (see e.g. [13, 14]) and then on filling a wedge by these discs. However, in the direction of a sector S as in Theorem 2.1 such discs may not exist and hence other methods are required. Our approach here consists of two main steps. Using tools developed in [16], we first attach nonsmooth analytic discs to V that will not fill a wedge anymore but rather a smaller region V' that we call an "α-wedge", where $\alpha > 1/2$ is a suitable number. Such an α-wedge is given by inequality with some coordinates bounded by αth powers of some others. (Here α is chosen such that the angle of S is $\alpha\pi$.) Then we attach analytic discs to properly chosen submanifolds $\widetilde{M}$ approximating M such that, over a certain region, $\widetilde{M}$ is contained in V'. For this step, the property $\alpha > 1/2$ is crucial and guarantees that the directions of $(1/\alpha)$th powers are not affected by the Taylor expansion of (the defining functions of) M.

We discuss now how to attach analytic discs to a generic submanifold M of $\mathbb{C}^N$. The reader is referred to [2, 7, 15] for general details and to [16, 17] for attaching nonsmooth discs. By an *analytic disc* in $\mathbb{C}^N$ we mean a holomorphic mapping A from Δ, the unit disc in $\mathbb{C}$, into $\mathbb{C}^N$ which extends at least continuously to $\overline{\Delta}$; we still denote by A the image $A(\Delta)$. We say that A is *attached* to M if $A(\partial\Delta) \subset M$. Usually one considers smooth ($C^{k,\beta}$ with $k \geq 1$) discs. (The reason of the fractional regularity is due to the continuity of the Hilbert transform in the corresponding classes of functions.) One of the main technical tools developed in [16] was attaching analytic discs that are C^1 except at $\tau = 1$, where they are only Hölder-continuous. As in [16], for $\frac{1}{2} < \alpha < 1$, we put

$$\beta := 2\alpha - 1.$$

For any real δ, consider the principal branch of $(1-\tau)^\delta$ on Δ that is real on the interval $(0,1)$; note that $(1-\tau)^{2\alpha} \in C^{1,\beta}$. As in [16], define $\mathcal{P}^\alpha(\partial\Delta) \subset C^\alpha(\partial\Delta)$ to be the subspace of all linear combinations of $(1-\tau)^\alpha$ and functions in $C^{1,\beta}(\partial\Delta)$ and $\mathcal{P}^\alpha(\bar{\Delta}) \subset C^\alpha(\overline{\Delta})$ to be the subspace of linear combinations of $(1-\tau)^\alpha$, $(1-\bar{\tau})^\alpha$ and functions in $C^{1,\beta}(\overline{\Delta})$. These subspaces naturally become Banach spaces (see [16]). It is easy to see that the Hilbert transform is a bounded operator on $\mathcal{P}^\alpha(\overline{\Delta})$.

We now choose coordinates $(z,w) \in \mathbb{C} \times \mathbb{C}^n = \mathbb{C}^N$, $z = x + iy$, where M is defined by (2.1). It is well-known (cf. [7]) that, given a sufficiently small holomorphic function $w(\tau)$ in Δ that is C^α in $\overline{\Delta}$ and a sufficiently small vector $x \in \mathbb{R}^l$, there exists a (unique) small analytic disc $A(\tau) = (z(\tau), w(\tau))$ attached to M in the same class with prescribed small value $x(1) = x$. It is further shown in [16] that the same statement also holds in the class $\mathcal{P}^\alpha$ defined above and that the obtained disc A depends smoothly on $w(\cdot)$ and x in this class. More precisely, one has:

PROPOSITION 3.1 ([16]). *Let M be C^{k+2}-smooth ($k \geq 1$) and fix $\alpha > 1/2$. Then for every sufficiently small $x \in \mathbb{R}$ and holomorphic function $w(\cdot) \in \mathcal{P}^\alpha(\overline{\Delta})$ there is an unique sufficiently small analytic disc $A = (z(\cdot), w(\cdot)) \in \mathcal{P}^\alpha$ with $x(1) = x$ attached to M that depends in a C^k fashion on $w \in \mathcal{P}^\alpha$ and $x \in \mathbb{R}^l$.*

It is further shown in [16] that, if $A(1) = 0$ and if the coordinates (z, w) are chosen as above, the component $z(\tau)$ is in fact in $C^{1,\beta}$ and depends in a C^k fashion on the data also as an element in $C^{1,\beta}$.

4. Extension to an α-wedge

Our fist main step in proving Theorem 2.1 will be to obtain a weaker statement about the extension to a region smaller than a wedge. More precisely, given a hypersurface $M \subset \mathbb{C}^N$ as in (2.1) through 0 and an open subset $V \subset M$ with $0 \in \partial V$, define an *α-wedge in M' over V at 0 showing up* to be the open subset of $\mathbb{C}^N$ of the form

(4.1)
$$V' = \{z = (x+iy, w) \in \mathbb{C}^N : y > h(x,w),\ \operatorname{dist}(z, V) < c \operatorname{dist}(z, \partial V)^{1/\alpha}\}$$

for some positive constant c.

The following statement is the main technical tool for the first step in the proof of Theorem 2.1:

PROPOSITION 4.1. *Under the assumptions of Theorem 2.1, choose $1/2 < \alpha < 1$ such that the angle of S is greater than $\pi\alpha$. Then there exists a family of analytic discs of class $\mathcal{P}^\alpha$ in $\mathbb{C}^N$ attached to V that fill an α-wedge over V at 0 showing up.*

Since V is a wedge with generic edge E, the arguments of Baouendi-Treves [4] can be used to show that continuous CR-functions on a neighborhood of 0 in V can be uniformly approximated on compacta by holomorphic polynomials. Hence, replacing V by such a neighborhood, we obtain from Proposition 4.1:

COROLLARY 4.2. *Under the assumptions of Proposition 4.1, all continuous CR-functions on V admit a (unique) holomorphic extension to an α-wedge over V at 0 shown up.*

Indeed, given a sequence of holomorphic polynomials converging uniformly to a continuous CR-function on V on compacta, it also converges uniformly on the union of discs by the maximum principle. Hence, if the discs fill an α-wedge over V, the sequence of polynomials converges there to a holomorphic function.

REMARK 4.3. If the boundary ∂V does not contain a generic submanifold E, the classical arguments of [4] cannot be applied to show that any continuous CR-function on V is approximated by polynomials on compacta, even after replacing V by a smaller neighborhood of p in V. Nevertheless it is shown in [16] by a different construction that holomorphic extension of CR-functions to an α-wedge can be obtained also in this case.

Proof of Proposition 4.1. Let S be the sector satisfying (2.2). Then we can find $w_0 \in T_0^c M$ such that $(1-\tau)^{\alpha'} w_0 \in S$ for some $\alpha' > \alpha$ and all $\tau \in \Delta$. For a small real parameter $\eta > 0$ set

$$w(\tau) = w_\eta(\tau) := \eta(1-\tau)^\alpha w_0.$$

We attach discs $A(\tau) = A_\eta(\tau)$ to M with "w-components" $w(\tau)$ (the above choice of w_0 will guarantee that our discs with be automatically attached to V). We write

$$z(\tau) = u(\tau) + iv(\tau), \quad \tau = re^{i\theta} \in \overline{\Delta},$$

and solve the Bishop's equation

$$u(\tau) = -T_1 h(u(\tau), w(\tau)), \quad \tau = e^{i\theta} \in \partial\Delta,$$

where T_1 is the Hilbert transform on $\partial\Delta$ normalized by the condition $T_1(\cdot)|_{\tau=1} = 0$. By Proposition 3.1, for small η, this equation has solution $u(\cdot)$ in $\mathcal{P}^\alpha$ with $u(1) = 0$. It is proved in [16] that $u(\cdot)$ is in $C^{1,\beta}$ and moreover $\eta \mapsto v_\eta$, $\mathbb{R} \to C^{1,\beta}$ is of class C^k. Then also $v := T_1 u \in C^{1,\beta}$ and $\eta \mapsto v_\eta$ is of class C^k. Since $h(0) = 0$ and $h'(0) = 0$, we have $v|_{\eta=0} \equiv 0$ and $\dot{v}|_{\eta=0} \equiv 0$, where the dot stands for the derivative in η. Hence also $\dot{u}|_{\eta=0} \equiv 0$ because $\dot{u}$ is related to $\dot{v}$ by the Hilbert transform.

Let $\varepsilon > 0$ be chosen as in (2.2). Then $\dot{u}|_{\eta=0} \equiv 0$ implies that $|u(\tau)| < \varepsilon|w(\tau)|$ for $\tau \in \partial\Delta$ and η sufficiently small. Hence, in view of (2.2), η can be chosen such that

$$v(\tau) = h(u(\tau), w(\tau)) > 0, \quad \tau \in \partial\Delta \setminus \{1\}. \tag{4.2}$$

By the Hopf Lemma, the radial derivative of the harmonic extension of v at $\tau = 1$ is negative. We write A_0 for the corresponding analytic disc with the chosen small value of η.

Thus we have found an analytic disc A_0 attached to $V \cup E$ with singularity at $\tau = 1$ such that the normal component $v(\cdot)$ is smooth and has a nonvanishing derivative at $\tau = 1$ showing up. If the tangential component $w(\cdot)$ were also smooth, this would imply that the disc A_0 is transversal to M and we would be able to obtain a wedge in $\mathbb{C}^N$ over V just by deforming A_0. However, since $\eta > 0$, the component $w(\cdot)$ cannot be smooth. In fact, it is easy to see that the disc A_0 is tangent to M at 0 in the sense of Whitney tangent cones. Thus, we cannot hope to obtain a wedge directly from A_0. Instead, we shall obtain an α-wedge. More precisely, the above construction can be used to obtain a family of attached analytic discs A_q for $q \in \overline{V}$ near 0 having the same "w-component" and such that $A_q(1) = q$. The conclusion about filling an α-wedge over V follows now from a result of [16] that we restate here for the reader's convenience in the interesting for us case:

LEMMA 4.4 ([16]). *Let $V \subset \mathbb{R}^l \times \{0\} \subset \mathbb{R}^l \times \mathbb{R}$ be an open subset with Lipschitz boundary at $0 \in \partial V$. Let $A\colon [0,1] \times \overline{V} \to \mathbb{R}^{l+1}$ be a map of the form*

$$\varphi(t,q) = q + t^\alpha a(q) + b(t,q), \quad 0 < \alpha < 1,$$

with $a(\cdot)$ and $b(\cdot, q)$, $q \in \overline{V}$, being of class $C^{1,\gamma}$ for some $0 < \gamma \leq \frac{1}{\alpha} - 1$ such that $a(q) \in C_0 V \times \{0\}$, $b(0,q) \equiv 0$ and $\partial_t b(0,0) \in \mathbb{R}^l \times \mathbb{R}_{>0}$. Assume that the induced map $q \mapsto b(\cdot, q)$ between $\overline{V}$ and $C^{1,\gamma}$ is also of class $C^{1,\gamma}$. Then there exists $\varepsilon > 0$ and a neighborhood U of 0 in $\mathbb{R}^l \times \{0\}$ such that A defines a homeomorphism between $(0,\varepsilon) \times (V \cap U)$ and an α-wedge over V at 0 showing up.

□

5. The end of the proof

Once extension to an α-wedge is established, the second remaining step is a consequence of [17, Proposition 6.1] that we restate here in our case. The assumption that the edge E was not important in Proposition 4.1 above but is now crucial for this step.

PROPOSITION 5.1 ([17]). *Let V be a wedge in M with generic edge E at 0 and V' be an α-wedge over V at 0 showing up for some $1/2 < \alpha < 1$. Then the union of analytic discs attached to $V \cup V'$ contains a wedge V'' with edge E at 0 such that $\overline{V''}$ contains a neighborhood of 0 in V and V'' is contained in the positive side of M. In particular, all* *continuous functions on V admitting holomorphic extension to V' are also holomorphically extendible to V''.*

Proof of Theorem 2.1. In the setting of Theorem 2.1, Corollary 4.2 implies that all continuous CR-functions on V extend holomorphically to an α-wedge V' over V at 0 showing up, for some $1/2 < \alpha < 1$. Then Proposition 5.1 can be applied to V' and yield a holomorphic extension to a wedge V'' with edge E at 0 as required. The proof is complete. □

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