The Korean Communications
in Statistics Vol. 10 No. 2, 2003
pp. 361-370

Interval Estimation for Sum of Variance Components in a Simple Linear Regression Model with Unbalanced Nested Error Structure

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Abstract

Those who are interested in making inferences concerning linear combination of variance components in a simple linear regression model with unbalanced nested error structure can use the confidence intervals proposed in this paper. Two approximate confidence intervals for the sum of two variance components in the model are proposed. Simulation study is peformed to compare the methods. The methods are applied to a numerical example and recommendations are given for choosing a proper interval.

Keywords : variance components, mixed model, inference

1. Introduction

One might be interested in making inferences for a linear function of variance components in simple linear regression model with unbalanced nested error structure. This model includes two variance components; one in the primary level and the other in the secondary level of the model. Since this model written in (2.1) contains regression coefficients and a predictor variable which are fixed part and a random variable among primary levels which is random part, it is regarded as a mixed model. The two confidence intervals are proposed in Section 3. A simulation study is performed in Section 4 to compare the two proposed intervals. The proposed confidence intervals are applied to a numerical example in Section 5 and Section 6 includes concluding remarks.

2. A Simple Linear Regression Model With An Unbalanced Nested Error Structure

A simple linear regression model with an unbalanced nested error structure is written as

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$$Y_{ij} = \mu + \beta X_{ij} + A_i + E_{ij} \qquad (2.1)$$
$$i = 1, \ldots, I; \quad j = 1, \ldots, J_i$$

where Y_{ij} is the j th observation in the i th primary level, μ and β are unknown constants, X_{ij} is a fixed predictor variable, and A_i and E_{ij} are jointly independent normal random variables with zero means and variances σ_A^2 and σ_E^2, respectively, $I > 2$, $J_i \geq 1$, and $J_i > 1$ for at least one value of i. A_i is an error term associated with the first-stage sampling unit(primary level) and E_{ij} is an error term associated with the second-stage sampling unit. Model (2.1) is unbalanced since the number of observations in cells are not all equal.

In order to form confidence intervals on linear functions of the variance components, an appropriate set of sums of squares is needed. The model is written $\boldsymbol{y} = \boldsymbol{X}\underline{\boldsymbol{\alpha}} + \boldsymbol{Z}\boldsymbol{u} + \boldsymbol{e}$ where $\boldsymbol{y}$ is a $J_. \times 1$ vector of observations, $\boldsymbol{X}$ is a $J_. \times 2$ matrix of known values with a column of 1's in the first column and a column of X_{ij}'s in the second column, $\underline{\boldsymbol{\alpha}}$ is a 2×1 vector of parameters with μ and β as elements, $\boldsymbol{Z}$ is a $J_. \times I$ design matrix with 0's and 1's, i.e. $\boldsymbol{Z} = \oplus_{i=1}^{I} \boldsymbol{1}_{J_i \times 1}$, $\boldsymbol{u}$ is an $I \times 1$ vector of random effects, $\boldsymbol{e}$ is a $J_. \times 1$ vector of random error terms, and $J_. = \Sigma_{i=1}^{I} J_i$. By the assumptions in (2.1) the response variables have a multivariate normal distribution, $\boldsymbol{y} \sim N(\boldsymbol{X}\underline{\boldsymbol{\alpha}}, \sigma_A^2 \boldsymbol{Z}\boldsymbol{Z}' + \sigma_E^2 \boldsymbol{D}_{J_.})$ where $\boldsymbol{D}_{J_.}$ is a $J_. \times J_.$ identity matrix.

3. Confidence Intervals For Sum Of Two Variance Components

In this article we propose two approximate confidence intervals for sum of two variance components $\gamma = \sigma_A^2 + \sigma_E^2$ using the sum of squares proposed by Olsen et al.(1976). The sum of squares is applied to Ting et al.(1990) method and to generalized p-values approach proposed by Tsui and Weerahandi(1989) and Khuri(1998).

Olsen et al.(1976), Thomas and Hultquist(1978), and El-Bassiouni(1994) used spectral decomposition method to obtain following statistics. They proposed a statistic $SSM = \boldsymbol{U}'\boldsymbol{U}$ which is asymptotically chi-squared distributed. $E(SSM) = (I - 1)(\sigma_A^2 + \sigma_E^2/\lambda_H)$ where λ_H is the harmonic mean of positive eigenvalues λ_i of $\boldsymbol{C}$, $\lambda_H = \Sigma_i r_i / (\Sigma_{i=1}^{I} r_i / \lambda_i)$, and r_i is the multiplicity of positive eigenvalues λ_i, $\boldsymbol{C} = \boldsymbol{Z}'(\boldsymbol{I} - \boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}')\boldsymbol{Z}$. It was also shown that $SSM/(\sigma_A^2 + \sigma_E^2/\lambda_H)$ and R_T/σ_E^2 are independent. The error sum of squares R_T associated with within regression coefficient estimator is written as $R_T = S_{wyye} - \hat{\beta}_T^2 S_{wxxe} = \mathbf{y}'\mathbf{T}\mathbf{y}$ where $S_{wyye} = \Sigma_{i=1}^{I}\Sigma_{j=1}^{J_i}(Y_{ij} - \overline{Y}_{i.})^2$, $S_{wxxe} = \Sigma_{i=1}^{I}\Sigma_{j=1}^{J_i}(X_{ij} - \overline{X}_{i.})^2$, $\hat{\beta}_T = S_{wxye}/S_{wxxe}$, $\mathbf{T} = \mathbf{D}_{J.} - \mathbf{X}^*(\mathbf{X}^{*'}\mathbf{X}^*)^-\mathbf{X}^{*'}$, $(\mathbf{X}^{*'}\mathbf{X}^*)^-$ is a generalized inverse of $\mathbf{X}^{*'}\mathbf{X}^*$, and $\mathbf{X}^* = [\mathbf{X}\ \ \mathbf{Z}]$. In addition, R_T/σ_E^2 is shown to be a chi-squared random variable with $J_. - I - 1$ degrees of freedom. The expected mean squares are thus summarized using the distributional properties of error sums of squares

$$E(S_M^2) \doteq \sigma_A^2 + \frac{1}{\lambda_H}\sigma_E^2 = \theta_M \quad \text{and} \tag{3.1a}$$

$$E(S_T^2) = \sigma_E^2 = \theta_T \tag{3.1b}$$

where $S_M^2 = SSM/(I-1)$ and $S_T^2 = R_T/(J_. - I - 1)$. S_M^2 and S_T^2 are now used to construct a confidence interval for sum of two variance components γ represented by (3.1*a*) and (3.1*b*),

$$\gamma = \theta_M + \left(1 - \frac{1}{\lambda_H}\right)\theta_T. \tag{3.2}$$

An approximate confidence interval on γ can therefore be constructed using the method of Ting et al.(1990). In particular, the $100(1-2\alpha)$ two-sided confidence interval for γ is

$$\begin{aligned}&[S_M^2 + (1-\tfrac{1}{\lambda_H})S_T^2 - (L_1^2 S_M^4 + (1-\tfrac{1}{\lambda_H})^2 L_2^2 S_T^4 + (1-\tfrac{1}{\lambda_H})^2 L_{12} S_M^2 S_T^2)^{\frac{1}{2}};\\ &S_M^2 + (1-\tfrac{1}{\lambda_H})S_T^2 + (H_1^2 S_M^4 + (1-\tfrac{1}{\lambda_H})^2 H_2^2 S_T^4 + (1-\tfrac{1}{\lambda_H})^2 H_{12} S_M^2 S_T^2)^{\frac{1}{2}}]\end{aligned} \tag{3.3}$$

where $F_1 = F_{(\alpha:\, I-1,\, J_.-I-1)}$, $F_2 = F_{(1-\alpha:\, I-1,\, J_.-I-1)}$, $L_1 = 1 - 1/F_{(1-\alpha:\, I-1,\infty)}$, $L_2 = 1/F_{(\alpha:\, J_.-I-1,\infty)} - 1$, $L_{12} = [(F_2-1)^2 - L_1^2F_2^2 - L_2^2]/F_2$, $H_1 = 1/F_{(\alpha:\, I-1,\,\infty)} - 1$, $H_2 = 1 - 1/F_{(1-\alpha:\, J_.-I-1,\infty)}$, $H_{12} = [(1-F_1)^2 - H_1^2F_1^2 - H_2^2]/F_1$ and $F_{(\delta:\, n_1,n_2)}$ is the F-percentile with degrees of freedom of n_1 and n_2 degrees of freedom with δ area to the left. Since $\sigma_A^2 > 0$, any negative bound is defined to be zero. Interval (3.3) is referred to as TING method. This TING method is different from two confidence intervals, TINGW and TINGU, for σ_A^2 in that TING method (3.3) uses SSM proposed by Olsen and other researchers whereas TINGW and TINGU methods of confidence intervals on σ_A^2 use sums of squares appeared in ANOVA table of model (2.1)(refer Park et al.(2002)).

Tsui and Weerahandi(1989) and Khuri(1998) methods now apply to (3.2) to construct a confidence interval on γ. From $R_T / \sigma_E^2 \sim \chi^2_{(J_. - I - 1)}$, the estimates of σ_E^2 are obtained by $(J_. - I - 1)\ s_T^2\ /\ U_T^*$ where s_T^2 is an observed value of S_T^2. That is, if a specific value of $\theta_T = \sigma_E^2$ is given, then an observed value of $S_T^2 = \theta_T \cdot \chi^2_{(J_. - I - 1)}\ /\ (J_. - I - 1)$ is generated and random variables of U_T^* are generated by $\chi^2_{(J_. - I - 1)}$ distribution. Similarly, The estimates of σ_A^2 are obtained by $(I - 1)\ s_M^2\ /\ U_M^* - (J_. - I - 1)\ s_T^2 / (\lambda_H U_T^*)$ where s_M^2 is an observed value of S_M^2, . That is, if a specific value of $\theta_M = \sigma_A^2 + \sigma_E^2 / \lambda_H$ is given, an observed value of $S_M^2 = \theta_M \cdot \chi^2_{(I-1)} / (I - 1)$ is generated and random variables of U_M^* are generated by $\chi^2_{(I-1)}$ distribution. The estimates of γ are calculated by substituting the estimates of σ_A^2 and σ_E^2 into (3.2). That is, they are generated by

$$\gamma = \frac{(I - 1)\ s_M^2}{U_M^*} + \left(1 - \frac{1}{\lambda_H}\right) \cdot \frac{(J_. - I - 1) s_T^2}{U_T^2} \tag{3.4}$$

Accordingly, an approximate $100(1 - 2\alpha)$ two-sided confidence interval for γ is constructed by

$$[G_\alpha\ ;\ G_{1-\alpha}] \tag{3.5}$$

where G_α and $G_{1-\alpha}$ are, respectively, the α th and $1 - \alpha$ th percentile of the distribution G constructed by γ. Interval (3.5) is referred to as GEN method.

4. Simulation Study

The methods proposed in Section 3 are compared using simulation study. The criteria for analyzing the performance of the methods are; 1) their ability to maintain stated confidence coefficient, and 2) the average length of two-sided confidence intervals. Although shorter average interval lengths are preferable, it is necessary that the methods first maintain the stated confidence coefficient. Thomas and Hultquist(1978) derived an easily calculated statistics that is used to construct confidence intervals on variance components in unbalanced case of one-way random effects model. They used nine designs representing a wide spectrum of unbalancedness. The middle two unbalanced patterns in Table 1 below were selected from design 9 and 2 from Table 1 in Thomas and Hultquist(1978) and the other two patterns were, respectively, chosen for demonstrating a numerical example in Section 5 and for more common practical use.

TABLE 1. Unbalanced Patterns Used in Simulation

Pattern	I	J_i
1	3	3 5 10
2	3	10 20 40
3	6	5 10 15 20 25 30
4	10	1 1 1 5 5 5 5 10 10 10

Let $\rho = \sigma_A^2/(\sigma_A^2 + \sigma_E^2)$. Without loss of generality $\sigma_A^2 = 1 - \sigma_E^2$ so that $\rho = \sigma_A^2$ and $1 - \rho = \sigma_E^2$. A_i and E_{ij} are independently generated from normal populations with zero means and variance ρ and $1 - \rho$, respectively, using RANNOR routines of SAS. Any value of μ and β and any fixed values of X_{ij}'s are used. Then Y_{ij}'s are calculated according to model (2.1) and generated values for S_M^2 and S_T^2 are substituted into appropriate formula and confidence intervals of (3.3) and (3.5) are computed. Values of ρ are varied from 0.001 to 0.999 in increments of 0.1. Each value of ρ is simulated 2000 times for each pattern. Two-sided intervals are computed based on equal tailed F-values. Confidence coefficients are determined by counting the number of the intervals that contain γ. Using the normal approximation to the binomial, if the true coefficient is 0.90(0.95), there is less than a 2.5% chance that an estimated confidence coefficient based on 2000 replications will be less than 0.8866(0.9404). The average lengths of the two-sided confidence intervals are also calculated.

Tables 2, 3, 4, and 5 present the results of the simulation for stated 90% and 95% confidence intervals on γ. The numbers in the body of Tables 2, 3, 4, and 5 report range of simulated confidence coefficients, average interval lengths, and minimum and maximum values for the range as ρ ranges from 0.001 to 0.999. The simulation results for 99% confidence intervals on γ are not shown here because they have exactly same trend as ones for 90% and 95% confidence intervals on γ. Different combinations of μ and β do not affect simulation results. The values with * in Tables 2 and 3 represent simulated confidence coefficients less than 0.8866 for 90% confidence intervals on γ and 0.9404 for 95%. TING method is very conservative when $\rho < 0.5$ for pattern 1 because the simulated confidence coefficients are much bigger than 0.9 or 0.95. TING method is too liberal when $\rho < 0.4$ for

TABLE 2. 90% Range of Simulated Confidence Coefficients

Pattern	1		2		3		4	
ρ	TING	GEN	TING	GEN	TING	GEN	TING	GEN
0.001	0.9530	0.9105	0.9545	0.9165	0.9265	0.9005	**0.8470***	0.8995
0.1	0.9510	0.9110	0.9470	0.9025	0.9315	0.9110	**0.8635***	0.8960
0.2	0.9455	0.9070	0.9260	0.9025	0.9235	0.9005	**0.8815***	0.8970
0.3	0.9335	0.9140	0.9045	0.9110	0.9130	0.8985	**0.8780***	0.8990
0.4	0.9340	0.9120	0.9110	0.8975	0.8965	0.9000	0.8880	0.9045
0.5	0.9080	0.8995	0.9015	0.9020	0.8910	0.8895	0.9010	0.8890
0.6	0.9170	0.8960	0.8935	0.9010	0.9050	0.8985	0.9060	0.8940
0.7	0.9145	0.8935	0.8935	0.9105	0.8870	0.8900	0.8985	0.8970
0.8	0.9045	0.8935	0.9000	0.9070	0.9020	0.9000	0.8960	0.9045
0.9	0.8980	0.8970	0.8970	0.8985	0.8955	0.8925	0.9115	**0.8860***
0.999	0.9120	0.8970	0.9075	0.9125	0.8935	0.9040	0.9190	0.9000
MAX	0.9530	0.9140	0.9545	0.9165	0.9315	0.9110	0.9190	0.9045
MIN	0.8980	0.8935	0.8935	0.8975	0.8870	0.8895	**0.8470***	**0.8860***

TABLE 3. 95% Range of Simulated Confidence Coefficients

Pattern	1		2		3		4	
ρ	TING	GEN	TING	GEN	TING	GEN	TING	GEN
0.001	0.9800	0.9585	0.9835	0.9565	0.9645	0.9455	**0.9195***	0.9525
0.1	0.9785	0.9545	0.9780	0.9515	0.9670	0.9635	**0.9270***	0.9505
0.2	0.9800	0.9555	0.9625	0.9540	0.9690	0.9510	0.9425	0.9480
0.3	0.9690	0.9565	0.9620	0.9530	0.9580	0.9520	**0.9385***	0.9440
0.4	0.9595	0.9580	0.9425	0.9555	0.9485	0.9540	0.9455	0.9505
0.5	0.9650	0.9515	0.9495	0.9480	0.9565	0.9410	0.9490	0.9415
0.6	0.9625	0.9490	0.9505	0.9565	0.9405	0.9495	0.9495	0.9490
0.7	0.9420	0.9495	0.9485	0.9525	0.9530	0.9455	0.9600	0.9475
0.8	0.9555	0.9445	0.9570	0.9510	0.9520	0.9480	0.9465	0.9565
0.9	0.9600	0.9510	0.9525	0.9460	0.9435	0.9425	0.9525	0.9455
0.999	0.9490	0.9485	0.9460	0.9590	0.9580	0.9520	0.9525	0.9500
MAX	0.9800	0.9585	0.9835	0.9590	0.9690	0.9635	0.9600	0.9565
MIN	0.9420	0.9445	0.9425	0.9460	0.9405	0.9410	**0.9195***	0.9415

TABLE 4. 90% Range of Average Interval Lengths

Pattern	1		2		3		4	
ρ	TING	GEN	TING	GEN	TING	GEN	TING	GEN
0.001	4.8931	8.1532	1.5354	2.0846	0.6088	0.6560	1.0820	1.1891
0.1	6.3293	9.0126	3.2270	3.8384	0.8725	0.9198	1.1783	1.2550
0.2	7.9243	10.0143	5.0128	5.4512	1.1638	1.2067	1.2595	1.3549
0.3	9.2495	11.2606	6.7925	7.2664	1.4434	1.4986	1.3694	1.4430
0.4	10.3255	12.0546	8.3604	8.8032	1.8091	1.8029	1.4733	1.5422
0.5	11.8025	13.5760	10.4851	10.6218	2.1380	2.2105	1.6043	1.6601
0.6	13.7805	15.4741	11.8562	12.2068	2.5132	2.5292	1.6865	1.7602
0.7	15.0355	15.8402	14.3987	13.5947	2.8430	2.8663	1.8065	1.8265
0.8	16.0754	17.1787	15.6405	15.6265	3.1567	3.2101	1.9643	1.9696
0.9	17.5622	18.2204	17.1147	17.5500	3.5582	3.5504	2.0522	2.0432
0.999	18.7191	19.2593	19.1646	18.2839	3.8836	3.8506	2.1746	2.1635
MAX	18.7191	19.2593	19.1646	18.2839	3.8836	3.8506	2.1746	2.1635
MIN	4.8931	8.1532	1.5354	2.0846	0.6088	0.6560	1.0820	1.1891

TABLE 5. 95% Range of Average Interval Lengths

Pattern	1		2		3		4	
ρ	TING	GEN	TING	GEN	TING	GEN	TING	GEN
0.001	9.4783	16.3502	2.7574	4.0244	0.8042	0.8602	1.4593	1.5438
0.1	12.7415	18.2206	6.5280	7.6946	1.2006	1.2737	1.5528	1.6357
0.2	15.2230	20.2783	9.8545	11.0626	1.6527	1.7048	1.7044	1.7713
0.3	18.8790	22.9298	13.9374	14.8076	2.0771	2.1372	1.8312	1.8914
0.4	21.6431	24.5458	17.7547	17.9409	2.6192	2.5822	1.9800	2.0242
0.5	22.7583	27.7255	20.9164	21.7153	3.0433	3.1712	2.1327	2.1809
0.6	26.1632	31.5588	24.1532	24.9415	3.5217	3.6326	2.2626	2.3136
0.7	29.6617	32.3651	29.0134	27.8342	4.0513	4.1167	2.3344	2.4006
0.8	34.2588	35.1546	31.4451	31.9583	4.5374	4.6116	2.5462	2.5879
0.9	35.4866	37.3545	36.2419	35.9364	5.1425	5.1037	2.6958	2.6865
0.999	39.2238	39.3962	38.2388	37.4129	5.4181	5.5293	2.8916	2.8450
MAX	39.2238	39.3962	38.2388	37.4129	5.4181	5.5293	2.8916	2.8450
MIN	9.4783	16.3502	2.7574	4.0244	0.8042	0.8602	1.4593	1.5438

pattern 4 because it generates much smaller than the threshold values, 0.8866 or 0.9404. Except these values of ρ TING method generates the simulated confidence coefficients close to 0.9 or 0.95. GEN method generally maintains the stated confidence coefficients through four patterns although its average interval lengths are slightly wider than TING method.

5. An Example

The methods proposed in Section 3 are applied to a data set. Scheffe (1959, p. 216) wrote a data set of 94 observations for seven types of starch film. The dependent variable in the data set is the breaking strength in grams and the independent variable is the thickness in 10^{-4} inch from tests of starch film. The data set was constructed by selecting three types of starch, Potato, Canna, and Wheat. In order to conform to pattern 1 in Table 1 three observations are selected from Potato, five from Canna, and ten from Wheat. This data set is used to fit the simple linear regression model of the breaking strength on the thickness of starch film assuming an unbalanced nested error structure. The selected data set was listed in Table 6. A necessary SAS code was programmed and the resulting 90% and 95% confidence intervals on γ were given in Table 7.

TABLE 6. The Data Set Used for Example

Observation	Type					
	Potato		Canna		Wheat	
	Y	X	Y	X	Y	X
1	983.3	13.0	791.7	7.7	263.7	5.0
2	958.8	13.3	610.0	6.3	130.8	3.5
3	747.8	10.7	710.0	8.6	382.9	4.7
4			940.7	11.8	302.5	4.3
5			990.0	12.4	213.3	3.8
6					132.1	3.0
7					292.0	4.2
8					315.5	4.5
9					262.4	4.3
10					314.4	4.1

From SAS output and $E(S^2_{WB}) = c_1\sigma^2_A + \sigma^2_E$ in Park(2002, (4.1a)) the estimator $\hat{\sigma}_A{}^2$ is computed as 12517.76 and from $E(S^2_T) = \sigma^2_E$ the estimator $\hat{\sigma}_E{}^2$ is computed as 3063.89. Therefore, the estimate of the ratio of variance in the primary unit to total variance $\hat{\rho}$ is approximately 0.8. In this case TING method should be used because it keeps the stated confidence level and generates shorter average interval length than GEN method for $\rho = 0.8$ in pattern 1. The calculated 90% and 95% confidence intervals in Table 7 are consistent with the simulation study in Section 4.

TABLE 7. Confidence Intervals on γ

	90% confidence interval			95% confidence interval		
Methods	Lower Bound	Upper Bound	Length	Lower Bound	Upper Bound	Length
TING	4,382.5	208,119.3	203,736.8	3,659.5	420,776.3	417,116.8
GEN	12,262.6	948,612.8	936,350.4	9,491.3	1,891,571.3	1,882,079.9

6. Conclusion

One might often confront with making inferences on variance components. This note proposes two approximate confidence intervals for sum of two variance components in a simple regression model with unbalanced nested error structure. The two approximate confidence intervals use the sum of squares proposed by Olsen et al.(1976), Thomas and Hultquist(1978), and El-Bassiouni(1994). The sum of squares is applied to Ting et al.(1990) method and to generalized p-values approach proposed by Tsui and Weerahandi(1989) and Khuri et al.(1998).

The two methods finding confidence interval for sum of two variance components, $\gamma = \sigma_A^2 + \sigma_E^2$, using the sum of squares proposed by researchers mentioned above in general yield simulated confidence coefficients close to stated confidence levels unlike methods using the sums of squares appeared in ANOVA table to find σ_A^2 in the model as Park et al.(2002) show. TING method generates somewhat conservative simulated confidence coefficients when $\rho < 0.5$ for pattern 1 in Table 1 whereas it is liberal for $\rho < 0.4$ for pattern 4. GEN method generally maintains the stated confidence coefficients across all patterns in Table 1 except one case in pattern 4. The GEN method gives slightly wider average interval lengths than TING method. Therefore, except $\rho < 0.4$ in pattern 4 TING or GEN method should be selected depending on shorter average interval length for constructing confidence interval for γ in the model as Tables 3 and 4 suggest.

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[2003년 3월 접수, 2003년 5월 채택]