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Computation of Stratified Flows using Finite Difference Lattice Boltzmann Method

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Abstract

A stratified flow is simulated using the finite difference lattice Boltzmann method (FDLBM). The effect of body force (gravity) in a simple one-dimensional model with the lattice BGK 9 velocity is examined. The effect of body force in the compressible fluid is greatly different from that of the incompressible fluid. In a compressible fluid under gravitational force, the density stratification is not sufficient and the entropy stratification is essential. The numerical simulation of a line sink compressible stratified flow in two-dimensional channel is also carried out. The results show that selective withdrawal is established when the entropy of the upper part increases, and the simulated results using FDLB method are satisfactory compared with the theoretical one.

1. Introduction

The effect of gravity in the compressible fluid is greatly different from that of the incompressible fluid. In the incompressible fluid, the fluid is stable when density increases downward, whereas the fluid pattern becomes unstable when it is reversed[1],[2]. However, the density changes due to pressure when the fluid is compressible, and it is insufficient in the selective withdrawal criteria of stratified fluids[3]. In the compressible fluid, even though there are many cases in which the concepts such as potential temperature are used, it is convenient to introduce an entropy stratification as a general concept[4].

In this report, we examine the effect of body force (gravity) in finite difference

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lattice Boltzmann method(FDLBM) using a simple one-dimensional model. Then the selective withdrawal phenomenon[4],[5], which is a typical phenomenon in the stratified flow, is simulated, using the lattice BGK compressible fluid model, to confirm the effectiveness of this model.

2. Lattice BGK Model

In this section we present a brief description of the FDLB method[6],[7] which was developed from lattice Boltzmann method(LBM)[8]-[12]. The basic approach of the FDLB method is to construct a lattice on which one solves for the evolution of a particle distribution function that obeys a lattice Boltzmann equation. The following lattice Boltzmann equation with BGK collision term describes the evolution of the distribution $f_i(x,t)$

$$f_i(x+\mathbf{c}_i\tau, t+\tau) = f_i(x,t) + \Omega_i \quad (1)$$

Here, the real number $f_i(x,t)$ is the mass of fluid at each lattice node x, and time step t, moving in direction i.

The discrete lattice BGK equation, a simplified version of the discrete lattice Boltzmann equation,

$$\frac{\partial f_i(x,t)}{\partial t} + c_{i\alpha}\frac{\partial f_i(x,t)}{\partial x_\alpha} = -\frac{1}{\phi}[f_i(x,t) - f_i^{(0)}(x,t)] \quad (2)$$

is used. The microscopic dynamics associated with Eq. (2) can be viewed as a two-step process of movement and collision. In the collision step, the distribution functions at each site relax toward a state of local equilibrium. The collision operator Ω_i in Eq. (1) conserves local mass, momentum and kinetic energy, while the parameter ϕ in Eq. (2) controls the rate at which the system relaxes to the local equilibrium of $f_i^{(0)}(x,t)$.

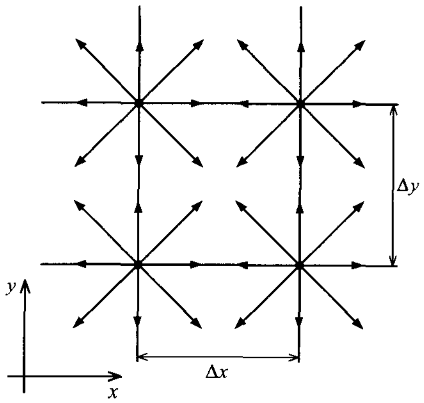


Fig. 1 Two-dimentional space lattice

The local equilibrium distribution function in Eq. (2) is expressed as[13],

$$f_i^{(0)} = F_i\rho[1 - 2Bc_{i\alpha}u_\alpha + 2B^2(c_{i\alpha}u_\alpha) + Bu^2 - \frac{4}{3}B^3(c_{i\alpha}u_\alpha)^3 - 2B^2c_{i\alpha}u_\alpha u^2] \quad (3)$$

The moving particles are allowed to move with five kinds of speed, c, $2c$, $3c$, $\sqrt{2}c$ and $2\sqrt{2}c$, and the space lattice in two-dimentional is shown in Fig. 1.

3. The Effect of Body Force

In LB method, the body force is added at the collision stage of the particle as a change of the discrete momentum(flow velocity). Hence, it is necessary to take an average of the distribution function after the collision process and after the movement process. However, the collision and translation process in FDLB method is simultaneously evaluated in order to perform an Eulerian calculation.

For simplicity, we just consider the case of a one-dimensional model as shown in Fig. 2, where the gravity works downward in the direction perpendicular to x-direction. When the gravity and buoyancy due to the pressure gradient match, it is considered that the fluid has reached a steady state at the macroscopic viewpoint.

Using the advance Euler method for the time term and the first order up-wind scheme for the convective term, the difference equation for the collision term of BGK(Eq.(2)) model becomes as follows:

$$f_1^{n+1}(y) = f_1^n(y) + \triangle t\left[-\left\{c_1\frac{f_1^n(y) - f_1^n(y-\triangle y)}{\triangle y}\right\} - \frac{1}{\phi}\{f_1^n(y) - f_1^{(0)}(y)\}\right] \quad (4)$$

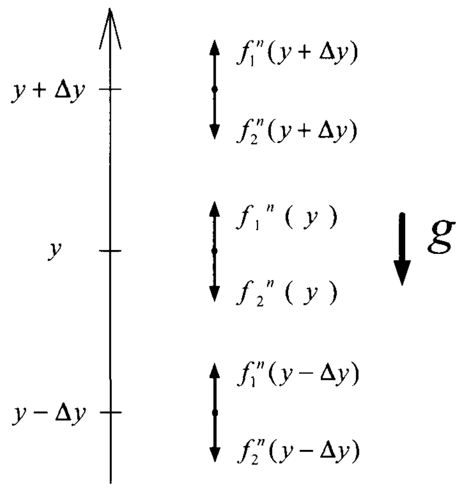


Fig. 2 One-dimensional lattice under the effect of gravity in FDLBM

$$f_2^{n+1}(y) = f_2^n(y) + \triangle t\left[-\left\{c_2\frac{f_2^n(y) - f_2^n(y-\triangle y)}{\triangle y}\right\} - \frac{1}{\phi}\{f_2^n(y) - f_2^{(0)}(y)\}\right] \quad (5)$$

Here, the suffixes 1 and 2 denote the upward and downward velocity particles, and $\triangle y$ is the lattice width, respectively. The particle velocities are given as $c_1 = 1$ in 1 direction and $c_2 = -1$ in 2 direction.

After the collision stage, the equilbrium distribution functions for $f_1^{(0)}$ and $f_2^{(0)}$ can be rewritten as,

$$f_1^{(0)}(y) = \left(1 - \frac{1}{2}g\right)f_1^n(y) \quad (6)$$

$$f_2^{(0)}(y) = f_2^n(y) + \frac{1}{2}gf_1^n(y) = \left(1 + \frac{1}{2}g\right)f_2^n(y) \quad (7)$$

$$(\because f_1^n(y) = f_2^n(y))$$

The variation of the distribution function at the time step n and n+1 have to be "0" so that the difference solution does not change with time. Namely, the big parenthesized passage of the difference equations (4) and (5) have to be "0". Then, the related expressions are written as,

$$\begin{aligned} f_1^n(y) &= f_1^n(y-\triangle y)(1+\theta)^{-1} \\ &= f_1^n(y-\triangle y)\left(1 - \theta + \frac{1}{2}\theta^2 + \cdots\right) \end{aligned} \quad (8)$$

$$\begin{aligned} f_2^n(y) &= f_2^n(y-\triangle y)(1-\theta)^1 \\ &= f_2^n(y-\triangle y)(1-\theta) \end{aligned} \quad (9)$$

Here, θ is $(1/2\phi)g\triangle t$ and $\theta \ll 1$. From the eqs. (8) and (9), it is well known that an error is θ^2 in $f_1^n(y)$ and $f_2^n(y)$.

In Fig. 3, the gravity g is made to change from 0.01 to 0.03, and the size of an error in the flow velocity in each case is logarithmically plotted to the gravity g. Here, v_e represents an error of the flow velocity. From the figure, it is well known that $\log v_e$ becomes the 1st function of $\log g$, and the gradient shows 2. Namely, an error of the flow velocity changes in proportion to $g^2(\propto\theta)$. This corresponds with the analytical result, as stated above, that an error of θ^2 order exists in a steady state of the gravitational field between the distribution function f_1^n and f_2^n. Tsutahara et al.(1998)[14] has confirmed that the error of α occurs, when a similar analysis on gravitational field in a steady state is carried out in LB method.

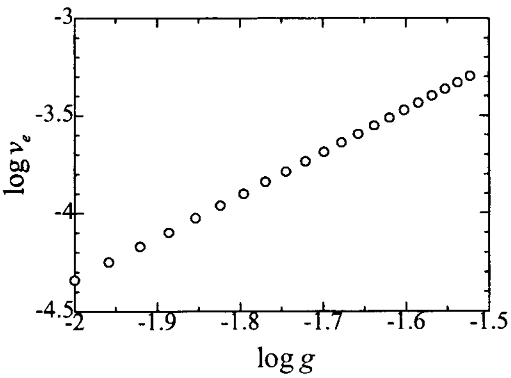


Fig. 3 Gravity dependence of velocity error in FDLB method

However, in FDLB method, when the effect of gravity or other body force is considered, it seems to be possible to carry out the numerical simulation with an error which is less than LB method.

4. Stratified Flows

4.1 Entropy and Potential Density

In the two-dimensional model, the pressure p is defined as $p=e\rho$ related to the density ρ and the internal energy e, and the entropy s from the related thermodynamics is obtained, respectively by,

$$s=c\log\left(\frac{p^{1/r}}{\rho}\right) \tag{10}$$

where the specific heats r is given by $(D+2)/D$, and D is the number of spatial dimensions. Therefore the related equation is expressed as,

$$s \propto \log e - \log\rho. \tag{11}$$

On the other hand, in geophysical fluid, the concept of potential density is often used, and it is a concept equal to the entropy stratification. The fluid is in a stable state when the potential density decreases upward, whereas the fluid is unstable in its reverse condition. In other words, the distribution of the potential density corresponds to the density distribution.

Here, the Froude number Fr is defined by using the density distribution at the channels of the upper and lower sides. In case that there is no disturbance in the flow, the physical quantity is represented by the suffix B. Then, the potential density is expressed as,

$$\rho_B=\rho_0\left(\frac{p_0}{p_B}\right)^{1/r} \tag{12}$$

where the suffix 0 represents the standard position. The suffies 1 and 2 as the standard positions denote the lower side and the upper side in the channel, respectively.

Accordingly, the buoyancy frequency N and the Froude number Fr of the flow are defined as follows.

$$N=\left(-\frac{g}{d}\frac{\rho_{B1}-\rho_{B2}}{\rho_{B1}}\right)^{1/2} \tag{13}$$

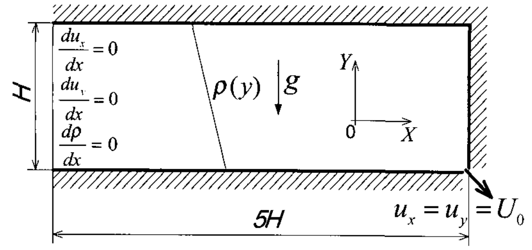


Fig. 4 Simulated two-dimensional line sink flow

$$Fr=\frac{Q}{Nd^2} \tag{14}$$

Here, d is the channel height and Q is the flowrate of the fluid.

4.2 Line Sink Flow in a Stratified Fluid

A phenomenon of the line sink flow in a entropy stratified fluid is simulated. A two-dimensional channel with solid walls established to the top, the bottom and the right side wall is considered, and the left side extended infinitely, as shown in Fig.4.

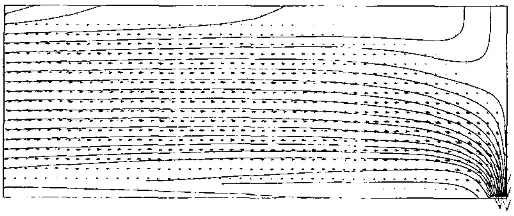
The stratified fluid in this region drain out at the sink that is located at the right corner starting from a quiescent state. At the solid walls, a reflection boundary condition of particle is applied. Namely, particles reaching the solid wall are rebounded with a velocity distribution which given by the equilibrium distribution function. The velocity of sinking is set to 0.02 in the entire region, and the width of the sink is made to 3 nodes. The gravitational force and the time step are $g=0.02$ and $\Delta t=0.01$, and the relaxation time coefficient ϕ varies at each calculation. The viscosity coefficient μ relating the relaxation time coefficient is changed by the calculation, and is chosen to keep a numerical stability. At the initial conditions, the fluid density is uniformly set to be $\rho=1.0$, and the internal energy e has the mean value between the top wall and the bottom wall. At first, the calculation starts without the sinking under the gravity, and then the sinking starts when the fluid almost reaches the steady state.

The fluid of the lower part receives a compression when the gravity works, then the internal energy (temperature) increases toward the upper part. Due to the energy diffusion(temperature conduction), the entropy of the upper part increases because of internal energy which is delivered to the upper part, and the non-uniformity of the entropy in the entire region occurs. Consequently, the entropy stratification can be built up by changing the boundary condition of the internal energy at the upper and lower part.

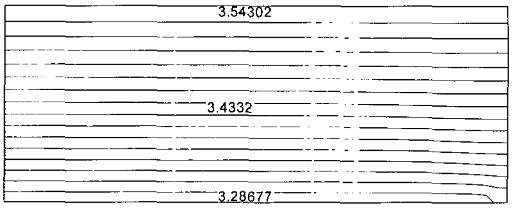
5. Results and Discussion

The following results show that the fluid flow almost reaches the steady state.

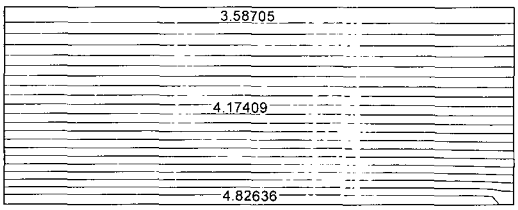
Figure 5(a) shows the velocity field and streamlines under the uniform entropy. The internal energies at the upper(suffix 2) and lower(suffix 1) wall are fixed at $e_2=0.83$ and $e_1=0.87$, respectively. Even though the density increases toward the lower part due to the gravity, the entropy is uniformly distributed in the whole region (see Figs. (b) and (c)). In this case, the fluid flows to the sink point from the whole region, and it coincides with a flow where the density is uniformly distributed in an incompressible fluid. In the case of uniform entropy stratification, the flow is under the condition of neutral stability, and therefore, as shown in Fig. 5, the flow drains out from the whole region.



(a) Velocity vectors and streamlines



(b) Entropy



(c) Density

Fig. 5 Flow field in an uniform entropy stratified flow

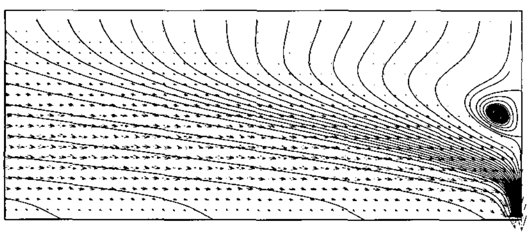
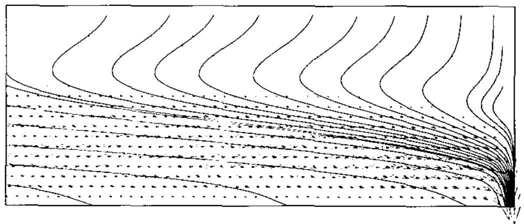
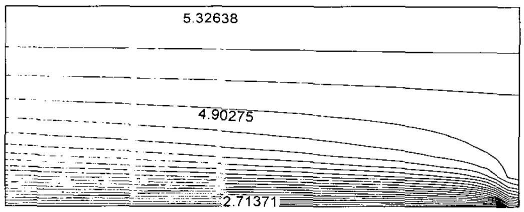


Fig. 6 Velocity vectors and streamlines in the stable entropy stratification

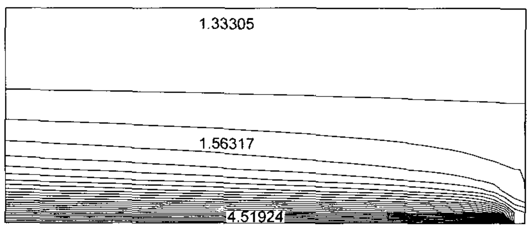
Figure 6 shows the velocity vectors and streamlines, which is the stable entropy stratification but not the sufficient entropy stratification. The internal energies at the upper and lower wall are made to be $e_1=0.5$, $e_2=1.0$, respectively. In the Figure, we certainly note that a dividing stream line appears and a stagnation flow occurs at the upper right portion, where it makes a circulating flow; however, the flow drains out the sink node at the whole region.



(a) Velocity vectors and streamlines



(b) Entropy



(c) Density

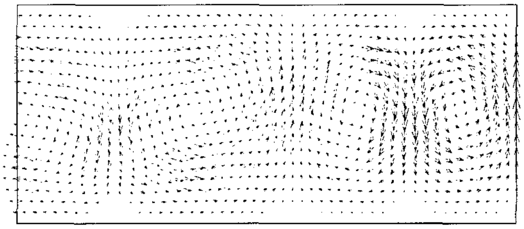
Fig. 7 Flow field in a strong stable entropy stratification

Considering the Froude number Fr defined in Eq. (15), it becomes 0.4×10^{-2}

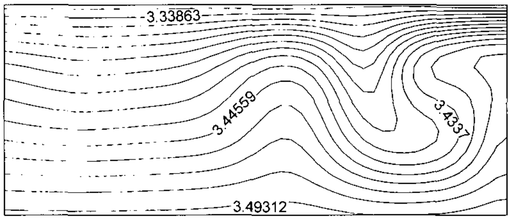
and has a very small value. In case of an inviscid · incompressible flow, the selective withdrawal phenomenon occurs as the fluid has a small Froude number. However, the fluid drains out at the whole region, and the Reynolds number Re obtained by the flowrate and the kinematic viscosity is $Re=50$. It seems that the effect of the viscosity is big while the compressible effect of the fluid appears. In an incompressible fluid, a vorticity appears due to the pressure variation and density gradients, which contributes to the selective withdrawal phenomenon. In the mean while, in case a fluid expansions, the effect of the sink propagates in a form of isotropic expansion wave, which expands the sinking mouth substantially.

Figure 7 shows the flow field of strong stable entropy stratification, where the internal energies at the upper and lower walls are put with $e_1=0.5$, $e_2=1.5$, respectively. The separated stream line of fluid is clearly seen in Fig. 7(a), where the fluid of the lower half is withdrawn to the sink node. This is a phenomenon equal to the selective withdrawal often observed in a reservoir, which established the temperature stratification.

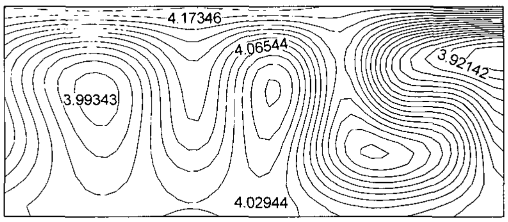
Figure 8 shows an unstable entropy stratification when the internal energies at the upper and lower walls are put with $e_1=0.95$ and $e_2=0.75$, respectively. The convection pattern like Benard convection appears in the velocity vector (a) and the density distribution (c). The effect of the sink is hidden in the large fluctuation of a convection pattern owing to the instability.



(a) Velocity vectors



(b) Entropy



(c) Density

Fig. 8 Flow field in an unstable entropy stratification

6. Conclusions

This study introduced the FDLB method capable of modeling hydrodynamic flow with simple equilibrium distribution function and the results are summarized as follows:

(1) A simple one-dimensional model of an incompressible fluid in FDLB method was tested in order to confirm the effect of the body force. At the evaluation of the flow velocity, it was proved that errors of the vector magnitudes in FDLB method was less than that of LB method.

(2) Using the finite difference method in a compressible lattice Boltzmann fluid

model, the line sink flow which is built in the entropy stratification was studied. In the case of the uniform entropy stratification, the flow was under a neutral stability and drained to the sinking node from the whole region. A separated stream line appeared and a stagnation flow occurred at the upper part of the sinking node, when the stable entropy stratification, but not the sufficient entropy stratification, is built in. However, in the case that the flow field shows the strong stable entropy stratification, a separated stream line has clearly been shown, and the selective withdrawal phenomenon which is often observed in a reservoir has appeared. In the flow field with the unstable entropy stratification, a convection pattern like Benard convection appeared, but the effect of the sink was not seen in the large fluctuation of the convection pattern due to the instability.

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References

[1] N. H. Brookes and R. C. Y. Koh, "Selective Withdrawal from Density Stratified Reservoirs", J. Hydraul. Div. Am. Soc. Civ. Eng., 95, pp 1369-1400, 1969.

[2] W. R. Debler and C. M. Vest, "Observations of a Stratified Flow by Means of Holographic Interferometry", Pro. r. Soc. London Ser., A358, pp 1-16, 1977.

[3] T. W. Kao, "Selective Withdrawal Criteria of Stratified Fluids", J. Hydraul. Div. Am. Soc. Civ. Eng., 102, pp 717-729, 1976.

[4] C. S. Yih, "Stratified Flows", Academic Press, 1980.

[5] P. J. Bryant and I. R. Wood, "Selective Withdrawal from a Layered Fluid", J. Fluid Mech., 77, pp 581-591, 1976.

[6] N. Cao, S. Chen, S. Jin and D. Martinez, "Physical Symetry and Lattice Symmetry in the Lattice Boltzmann Method", Phys. Rev. E, 55, pp 21-24, 1997.

[7] T. Seta, K. Kono, D. Martinez and S. Chen, "Lattice Boltzmann Scheme for Simulating Two-Phase Flows", JSME, B, 65-634, pp 1955-1963, 1999.

[8] U. Frisch, D. d'Humieres, B. Hasslacher, P. Lallemand, Y. Pomeau and J. P. Rivet, "Lattice Gas Hydrodynamics in Two and Three Dimensions", Complex Systems, 1, pp 599-632, 1987.

[9] F. J. Alexander, S. Chen and J. D. Sterling, "Lattice Boltzmann Thermo-Hydrodynamics", Phys. Rev. E, 47(4), pp 2249-2252, 1993.

[10] Y. Kato, K. Kono, T. Seta, S. Chen and D. Martinez, "AMADEUS Project and Microscopic Simulation of Boiling Two-Phase Flows by the Lattice Boltzmann Method", J. Mod. Phys. C, 8, pp 843-858, 1997.

[11] B. Chopard and M. Droz, "Cellular Automata Modeling of Physical Systems", Cambridge University Press, 1998.

[12] D. A. Wolf-Gladrow, "Lattice-Gas

Cellular Automata and Lattice Boltzmann Models", Lecture Notes in Mathematics, Springer, 2000.

[13] H. K. Kang, M. Tsutahara, K. D. Ro and Y. H. Lee, "Numerical Simulation of Shock Wave Propagation using the Finite Difference Lattice Boltzmann Method", KSME International Journal, Vol 16, No 10, pp 1327-1335, 2002.

[14] M. Tsutahara, S. D. Feng, T. Kataoka and N. Takada, "Numerical Simulation of a Thermal Convection by the Lattice Boltzmann Method", JSME, B, 64-623, pp 1966-1972, 1998.