Journal of the Korean Society of Marine Engineers, Vol.27, No.4, pp.467~478, 2003(ISSN 1226-9549)

Mathematical Model of Shock Absorber for Performance Prediction of Automobile

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(Manuscript : Received OCT 1, 2002 ; Revised APR 2, 2003)

Key words : shock absorber, twin tube type, dynamic characteristics, mathematical model

Abstract

Automotive shock absorber may not be regarded as only one(simple) damping machine because it is composed of many components, and shows non-linear damping characteristics. No matter how advanced form of shock absorber is developed, the oil shock absorber can not be neglected, because their structures are based on the oil shock absorber. Therefore, it is essential to accurately analyze the dynamic characteristics of oil shock absorber. It stands mainly for damper valve tuning which nowadays is still exhaustively done by means of ride work. In this study, damping mechanism and dynamic characteristics for oil shock absorber of twin tube type are analyzed, based on the mathematical model considering internal flow and pressure. For the reliability of numerical prediction, the database is constructed within the limit of adequate reliability. Finally, the programmed system that gives out necessary specification by inputting damping specification and tolerance is to be constructed.

INTRODUCTION

A shock absorber in an automobile is a device which absorbs or reduces shocks or inherent vibration to spring, improving ride comfort. A shock absorber transforms the up- and-down kinetic energy into heat energy, and hydraulic type is the most general form.

Many developed forms of shock absorbers have been introduced[1~2]. A gas shock absorber has nitrogen gas in the traditional hydraulic type. A stroke sensitive shock absorber[3], which changes damping characteristics according to the displacement of shock absorber, are developed. In this variable damping force shock absorber, the opening of orifice hole can be controlled by external device[4]. In addition, semi-active shock

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absorber, where the control of orifice opening is performed by solenoid valve, is put into practical use. On the other side, according to the development of shock absorber, additional devices are necessary, and more advanced form than variable damping force shock absorber results in cost up of automobile. Therefore, according to the level of automobile , different shock absorber is selected. Low-priced small automobile uses hydraulic or gas shock absorber, middle-priced automobile uses variable damping force type shock absorber, and high-grade large automobile adopts semi-active shock absorber.

Shock absorber in an automobile has been used as a standard component for the past several decades, but it is regarded as single component and analyzed in the field of dynamics, vibration and control parts. But in reality, it is composed of many components, and it shows non-linear damping characteristics according to velocities[5~10]. Therefore, it may not be regarded as only one damping machine. No matter how advanced form of shock absorber is developed, the importance of the oil shock absorber can not be neglected, because all forms of the developed shock absorbers bases their structure on oil shock absorber. Therefore it is essential to analyze the dynamic characteristics of the oil shock absorber.

In this study, for obtaining damping mechanism and dynamic characteristics for oil shock absorber of twin tube type, the mathematical model considering internal flow and pressure is proposed. For the reliability of numerical prediction, the database is constructed within the limit of adequate reliability. Finally, programmed system that gives out necessary specification by the input of damping specification and tolerance is to be constructed.

MATHEMATICAL MODEL OF SHOCK ABSORBER

ASSUMPTIONS FOR MATHEMATICAL MODELING

Following assumptions are necessary for modeling shock absorber mathematically (Fig.1).

1. Leakage of valve does not exist.
2. clearance leakage does not exist.
3. Cavitation does not exist.
4. There is no damping lag due to pressure drop through orifice.
5. Temperature and density of oil is constant.
6. Oil in orifice is incompressible.

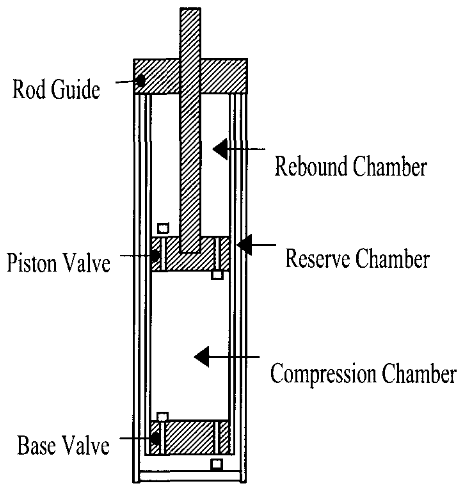


Fig. 1 Configuration of typical shock absorber for vehicles

DAMPING FORCE ACTION ON PISTON

Oil flow in a shock absorber occurs due to the pressure difference between rebound, compression and reserve chamber. Force difference between two ends of piston sections is the damping force(Fig.2).

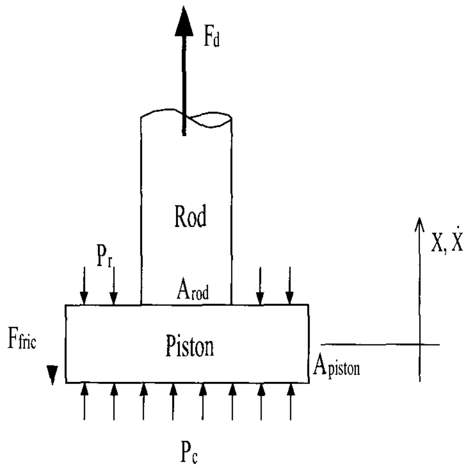


Fig. 2 Free-body diagram of piston and rod.

$$F_d = P_r(A_p - A_{rod}) - P_c A_p + F_{fric} \qquad (1)$$

Where, F_d is damping force, P_r is rebound chamber pressure, P_c is compression chamber pressure, A_p is piston area, A_{rod} is rod sectional area, F_{fric} is piston mechanical friction arising from the piston seal, cylinder tube and oil seal. Friction force is affected by vertical contact force, contact surface condition, so it is difficult to calculated by mathematical model. In this study, approximate value from experimental data is used.

MATHEMATICAL MODEL OF FLOW CHAMBER

The law of mass conservation is the continuous equation, which is defined by the fact that the net of accumulating mass flux in control volume and outgoing mass flux from the control volume is equal to incoming mass flux into control volume[11].

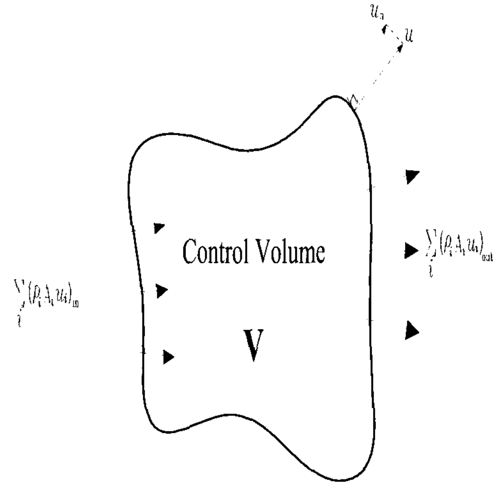


Fig. 3 Control volume in a flow field.

$$0 = \frac{d}{dt}\int_v \rho dV + \int_{As} \rho \vec{u} \cdot \vec{dA_s} \qquad (2)$$

$$= \frac{d}{dt}\int_v \rho dV + \rho_{out} u_{out} A_{out} - \rho_{in} u_{in} A_{in}$$

where u_n is the velocity normal to the control surface, A_s is the control surface and V is the control volume.

If ρ is assumed to be constant at arbitrary time through all of domain, $\rho_{in} = \rho_{out} = \rho$,

$$\frac{d}{dt}(\rho V) + \rho(u_{out} A_{out} - u_{in} A_{in}) = 0 \qquad (3)$$

$$Q_{in} - Q_{out} = \frac{dV}{dt} + \frac{V}{\rho}\frac{d\rho}{dt} \qquad (4)$$

when the compressibility of oil $\beta = \frac{1}{\rho}\frac{dp}{dt}$ is applied to Eq.(4),

$$Q_{in} - Q_{out} = \frac{dV}{dt} + \beta V \frac{dp}{dt} \qquad (5)$$

Eq.(5) can be expressed in terms of pressure variation rate as Eq.(6)

$$\beta V\frac{dp}{dt} = -\frac{dV}{dt} + Q_{in} - Q_{out} \qquad (6)$$

Therefore variation rate of pressure in the control volume is related to volume change of control volume to time and flow entering and leaving the control volume.

Compression Chamber Flow

Volume change and flow relation in compression chamber is expressed as follows in Fig.4.

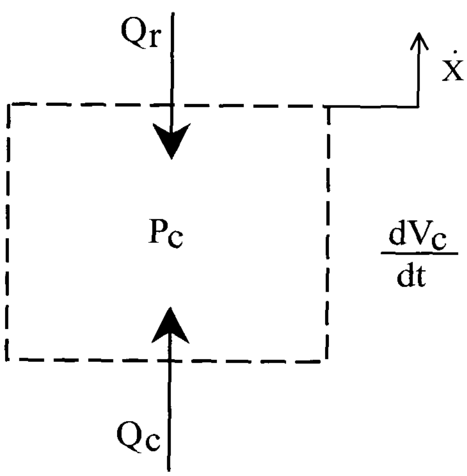


Fig. 4 Control volume of compression chamber.

The volume variation rate of the compression chamber due to piston movement is expressed as

$$\frac{dV_c}{dt} = A_p x \qquad (7)$$

Where V_c is volume in the compression chamber, and x is piston velocity.

Thus Eq.(6) can be modified to

$$\beta V_c\frac{dP_c}{dt} = -A_p x + (Q_r + Q_c)sgn(x) \qquad (8)$$

Where Q_r is flow rate across the piston valve and Q_c is flow rate across the base valve.

Rebound Chamber Flow

In rebound chamber, volume change and flow relation is expressed as follows in Fig.5:

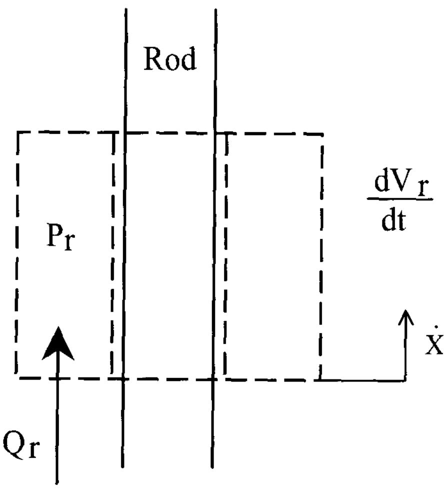


Fig. 5 Control volume of rebound chamber.

The volume variation of the rebound chamber due to piston movement is expressed as

$$\frac{dV_r}{dt} = -(A_p - A_{rod})x \qquad (9)$$

Thus, the changing rate of pressure in the rebound chamber is expressed as

$$\beta V_r\frac{dp_r}{dt} = (A_p - A_{rod})x - Q_r sgn(x) \qquad (10)$$

Reserve Chamber Flow

Compressed air and oil coexist in reserve chamber. Compressed air pressure in reserve chamber is given as Eq.(11) by the ideal gas law.

$$p_{res}V_{air}=m_aRT \tag{11}$$

Where P_{res} is the pressure in the reserve chamber, V_{air} is the air volume in the reserve chamber, m_a is the air mass, R is the gas constant and T is the air temperature.

Since the reserve chamber is isolated, m_a and T is assumed to be constant.

$$P_{res}V_{air}=const \tag{12}$$

The volume variation of the reserve chamber due to the piston movement is determined by subtraction the oil volume entering and exiting the reserve chamber from the air volume at a reference location of the piston.

$$V_{air}=V_{ao}-\int Q_c dt \tag{13}$$

From the assumption of constant temperature of the air, pressure change due to reserve chamber volume variation can be obtained by Boil' law which relates pressure to volume.

$$P_{res}=\frac{m_aRT}{V_{ao}-\int Q_c dt} \tag{14}$$

Therefore pressure variation rate of compression and rebound chamber is obtained with initial pressure of reserve chamber pressure. Thus obtained ordinary differential equation about pressure is solved using Runge-Kutta Method to get pressure change after infinitesimal time. Pressure variation of each chamber is related to flow rate passing through orifice, so reasonable flow equation for the orifice should be selected.

VALVE FLOW RATES

Orifice flow

Generally, flow in orifice is obtained using Bernoulli Equation which is applied to fluid following streamline. This theorem is energy conservation equation applied to energy of fluid. At the steady state when ideal fluid flows in a tube, the energy of fluid passing through a certain section(↓) in the tube is equal to that passing through section(↑) in the same tube during infinitesimal time $\triangle t$.

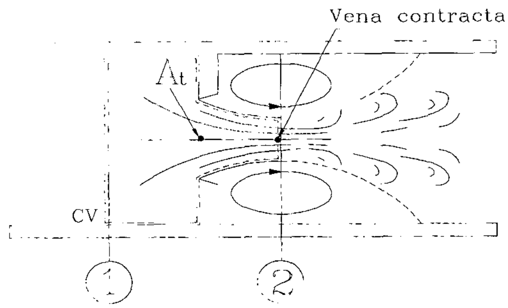


Fig. 6 Flow pattern through an orifice.

From the energy equation,

$$\frac{v_1^2}{2g}+z_1+\frac{P_1}{\gamma}=\frac{v_2^2}{2g}+z_2+\frac{P_2}{\gamma}+h_l \tag{15}$$

Neglecting the friction loss ($h_l=0$) and assuming that

$z_1=z_2$,

$$v_2^2-v_1^2=\frac{2}{\rho}(P_1-P_2) \tag{16}$$

From continuity equation,

$$A_1v_1=A_2v_2 \tag{17}$$

Eq.(15) can be expressed as

$$v_2=\sqrt{\frac{2(p_1-p_2)}{\rho[1-(A_2/A_1)^2]}} \tag{18}$$

Finally, the flow rate through orifice is

$$Q_{th}=v_2A_2=\frac{A_2}{\sqrt{1-(A_2/A_1)2}}\sqrt{\frac{2(p_1-p_2)}{\rho[1-(A_2/A_1)^2]}} \quad (19)$$

Above equation indicated theoretical flow, but because of several conditions, flow rate cannot be measured directly using the equation.

First, since velocity v_2 is the theoretical velocity which dose not include the friction loss, the actual velocity is less than the theoretical one. So we can define velocity coefficient C_v.

$$v_{2a}=C_vV_2 \quad (20)$$

Second, it is difficult to know the actual area of the section↑ due to developing vena contracta.

So using C_c and A_t

$$A_2 = C_vV_t \quad (21)$$

Therefore the actual flow rate through orifice can be expressed as

$$Q=v_{2a}A_2=C_cv_{2a}A_t=C_cC_vA_tv_2$$

$$=\frac{C_cC_vA_t}{\sqrt{1-C_c^2(A_t/A_1)^2}}\sqrt{\frac{2(p_1-p_2)}{\rho}} \quad (22)$$

There are many other factors affection flow rate. For example, pressure loss due to friction and errors due to fluid inertia, viscosity, the assumption of one-dimensional flows are important factors. So the discharge coefficient C_d is introduced as Eq.(22), it includes all factors[12~13].

$$C_d=\frac{\text{actual flowrate}}{\text{theoretical flowrate}}=\frac{C_cC_v}{\sqrt{1-C_c^2(A_t-A_1)^2}} \quad (22)$$

Therefore the flow rate through orifice is expressed simply as

$$Q=C_dA_t\sqrt{\frac{2(p_1-p_2)}{\rho}} \quad (23)$$

Generally the discharge coefficient of orifice is experimental value determined by geometric shape, size, flow velocity, density. It is a function of Reynolds number and orifice geometry.

Viersma introduced approximate graph for C_d and R_e . Using Experimental equation Eq.(25) in the case of Low R_e and R_{et} in Eq.(26), predicted by Von Mises, in the case of high R_e .

$$R_{et}=\left(\frac{0.611}{\delta}\right)^2 \quad (24)$$

δ is 0.2 for circular orifice and 0.157 for rectangular slit

$$C_d=\left[1.5+13.74\left(\frac{L}{DR_e}\right)^{0.5}\right]^{-0.5}, \frac{DR_e}{L}>50$$

$$C_d=\left[2.28+64\frac{L}{DR_e}\right]^{-0.5}, \frac{DR_e}{L}<50 \quad (25)$$

Eq.(25) can be applied to a long circular orifice, and hydraulic diameter is used for other shapes.

Valve supported by spring

Flow through orifices in piston valve and base valve of a shock absorber is supported and controlled by spring.

$$\sum F_y=m\ \dot{y} \quad (26)$$

$$F_p+F_v+F_s-F_c-F_d=m\dot{y}$$

$$m y+C_v\dot{y}+k\dot{y}+\begin{Bmatrix} k_cy, & y<0 \\ 0, & y>0 \end{Bmatrix}$$

$$-2C_fC_d^2\triangle p\frac{\pi^2(d_{vi}+d_{vo})^2}{A_{in}}y2 \quad (27)$$

$$=\triangle pA_v-F_{sp}$$

Eq.(26) is expressed simply as

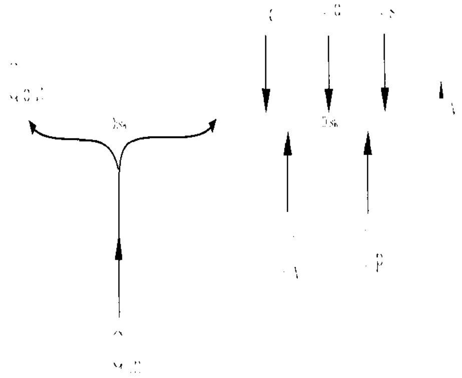


Fig. 7 Simplified model of valve supported by spring.

$$m\ddot{y}+C_v\dot{y}+ky+f_1(y)+f_2(y)=F \tag{28}$$

Considering the dynamic characteristics of valve, Eq.(28) becomes ordinary differential equation. However in actual shock absorber analysis, Eq.(28) is simplified as Eq.(29).

$$A_v\triangle p=F_{sp}+ky \tag{29}$$

Disc valved don't open until an opening force $A_v\triangle p$ due to pressure difference is reached to preload F_{sp} . The displacement of disc valve is y.

$$y=\begin{Bmatrix} (A_v\triangle p-F_{sp}), & A_v\triangle p>F_{sp} \\ 0, & A_v\triangle p\le F_{sp} \end{Bmatrix} \tag{30}$$

The displacement of disc valve is limited to $y_{\max}$.

PERFORMANCE ANALYSIS OF SHOCK ABSORBER

STRUCTURE OF SHOCK ABSORBER

When shock absorber is mounted to a chassis in an automobile, it can be classified into two types with respect to functional side. One is conventional shock absorber which have only function of damping, and the other is suspension strut which has function of reinforcing the strength of suspension as well as damping. With respect to internal structure, shock absorber can be classified into three kinds; One is twin tube type, another is gas mono-tube type and the third is gas twin tube type where gas is injected into twin tube type of shock absorber (Fig.8).

Twin tube type

It is most general and has longest history, but nowadays it si replaced by gas twin tube type gradually. It shows small impact from road because it does not contain gas, but in case of hard driving or driving on rough road, it shows aeration or cavitation, and has the demerit of deterioration performance due to unstable damping force. Internal cylinder tube is filled with oil and piston rod, and air of atmospheric pressure fills external tube. Reserve chamber indicates the space between cylinder tube and outer tube. Orifice(the passage of oil) is installed piston valve part and base valve part respectively.

Gas mono tube type

Gas of high pressure is injected into gas chamber which functions the reserve chamber in twin tube type. The volume change by piston rod is compensated by the movement of free piston. There is a reserve chamber which is injected with high pressure of nitrogen gas under the free piston, and when volume difference occurs between upper chamber and lower chamber, it is compensated buy the movement of free piston.

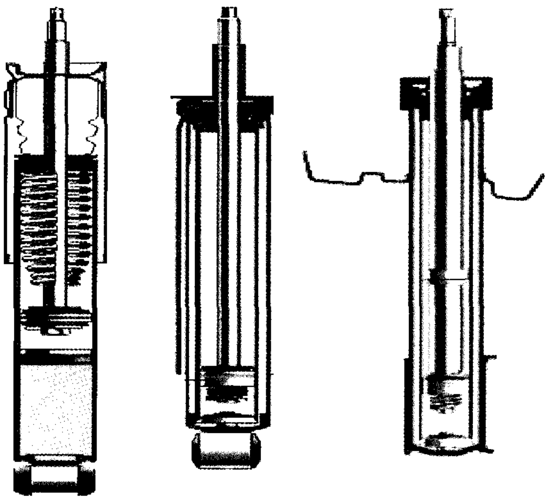
Oil is perfectly separated from gas, so aeration and cavitation does not occur, and stable damping force is available ever under hard driving. In addition there is no limit in install angle, and there is no problem when vertical or horizontal position is changed in installation. On the other hand, high-pressure gas is apt to transmit the impulse from the road surface, and high quality of seal is necessary for oil leakage prevention relatively difficult manufacturing raise the cost.

Gas twin tube type

Defects of twin tube and gas mono tube types are tried to overcome by this type. Nowadays the number of automobiles which adopts gas twin tube type shock absorber of low-pressure gas increase. It has sample principle as twin tube type. Low-pressure nitrogen gas is injected into reserve chamber, when oil in rebound chamber flows into compression chamber through piston valve, and oil is pressured by nitrogen gas in order not to bring about cavitation. If oil in compression chamber flows into rebound and reservoir chamber, it is difficult to bring about cavitation, and stable damping force is obtained type pressure of injected nitrogen gas is low, so impulse from road surface is not easily transmitted, and total length becomes short. However, in case of hard driving it has demerit of aeration.

RESULTS OF NUMERICAL ANALYSIS OF DAMPING PERFORMANCE AND DISCUSSION

Pressure variation rate equation according to each flow is derived from flow equation about compression and rebound stroke in shock absorber. Substitution of varied pressure for damping Eq. (1) gives out damping movement equation. Flowchart in Fig.9 shows the procedure of obtaining damping performance from this equation.



(a) (b) (c)

(a) Mono tube type (b) Twin tube type (c) Strut type

Fig. 8 Kinds of shock absorber.

Firstly, from reading the input data such as valve dimension and physical properties, cross section of flow by valve opening corresponding to pressure difference is obtained. Then the pressure of flow room which are converted to input and output of flow by up-and-down movement of piston and pressure difference is obtained using Runge-Kutta method. In addition, changed discharge of coefficient of each orifice and valve clearance is obtained for that instant. For different input condition, another calculation is performed to bring out damping force action on piston. This work is repeated for the variation of displacement.

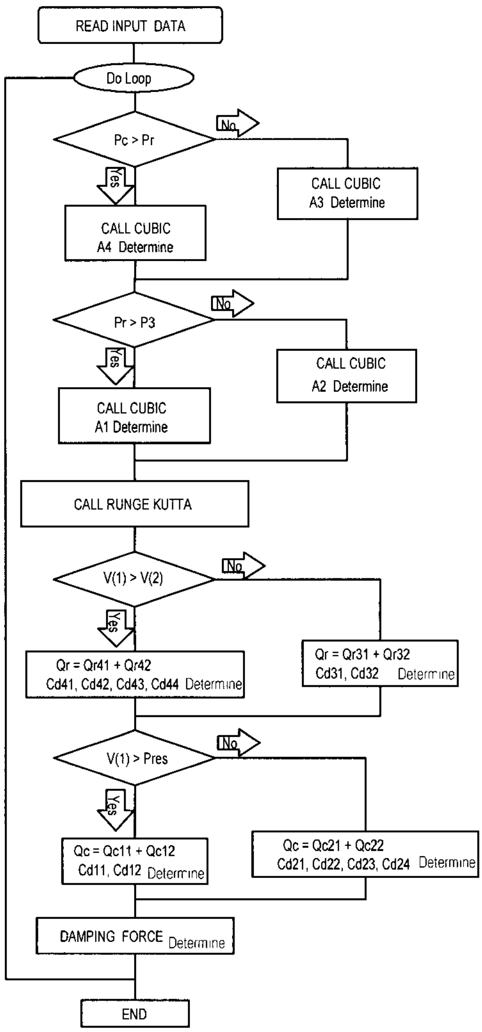
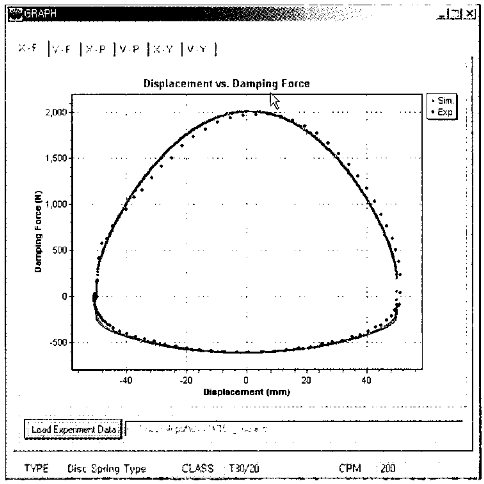
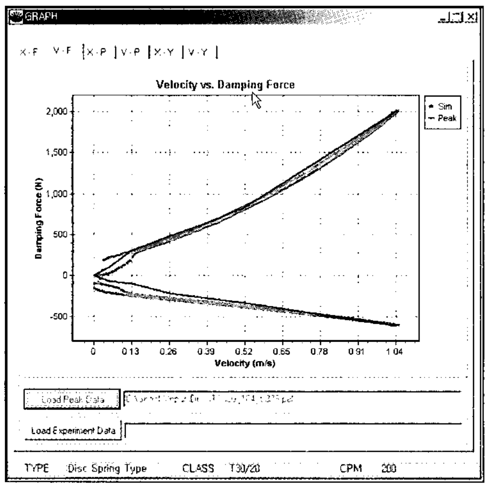


Fig. 9 Flow chart of shock absorber analysis program.

The results of simulation and experiment are shown in Fig.10. There is a good agreement in displacement vs. damping force graph(a), except the measuring error due to time delay, and there also is a good agreement in velocity vs. damping force graph(b), except inconsistency in low velocity range. In this range, friction force is the governing factor to damping performance because in low velocity range, pressure difference between both end of piston is small.



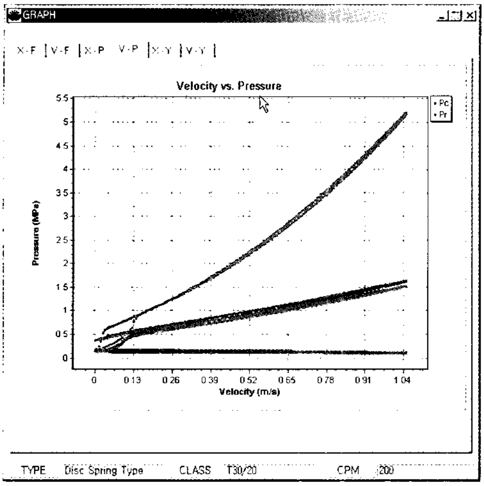
(a) Displacement vs. Damping Force



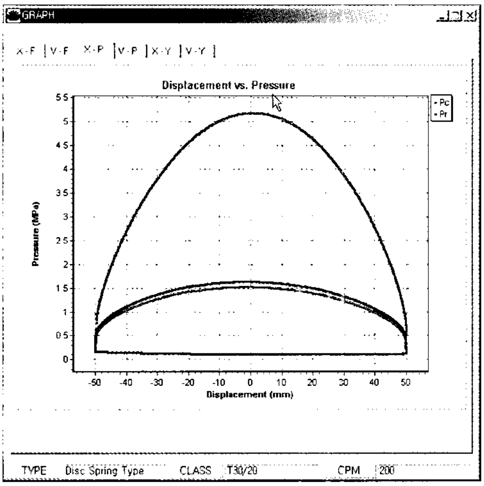
(b) Velocity vs. Damping Force

Fig. 10 Comparison between simulation and experimental results.

Fig.11 shows pressure distribution according to piston displacement and velocity in compression and rebound chamber.



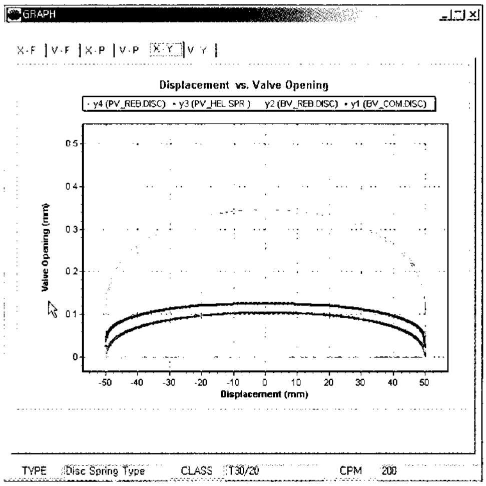
(a) Displacement vs. Pressure



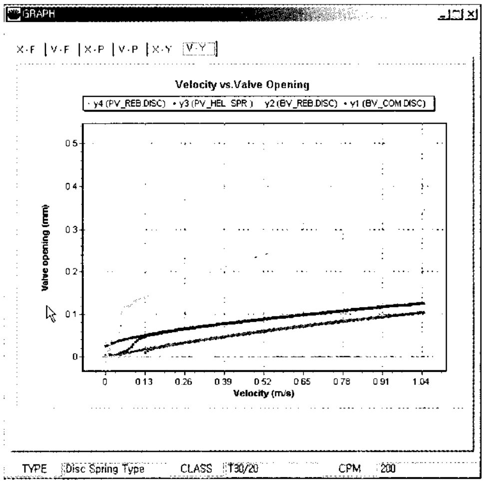
(b) Velocity vs. Pressure

Fig. 11 Pressure distribution at compression & rebound chamber.

Fig.12 shows the opening length of a each important valve. The most widely opening base valve can not exceed about 0.6mm, rebound valve in piston valve part does not open over 0.4mm. This tells us the importance of flatness and dimension accuracy of sintered piston and base valve. The opening amount of shim disc in piston and base valve is around 0.2mm, so the flatness of valve seat should be controlled very tightly, and tightening torque of piston should also be controlled tightly.



(a) Displacement vs. Opening



(b) Velocity vs. Pressure

Fig. 12 Opening length of each valve during compression processes.

CONCLUSION

To establish design technology of shock absorber for automobile, the validity of computer simulation was proved by performing experiments, with the model of twin tube type oil shock absorber which is widely used constructed program, database are stored, and this database system makes it possible for engineer to input necessary damping specification and to print out specification sheet in a few minutes.

In the development of new model car, tuning of shock absorber of which is essential component in suspension system, is cone by special tuner's damping performance specifications. Calibration to required specification by trial and error of tuner required several days. However constructed database system makes it possible in a few seconds. Only experienced tuning engineer could calibrate automobile shock absorber damping performance which are dependent upon topography of each country. It was very difficult task for the engineer of low level experience to put together components of shock absorber tuned to stable driving and comfortable ride comfort by experienced tuner. Such difficulties are solved by constructed database system for automobile shock absorber, and any engineer capable of assembling skill of shock absorber can calibrate damping performance which was only performed by experienced tuning engineer before.

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DEFINITIONS

F_d	: Damping force [N]
F_{fric}	: Friction force [N]
P_r	: Pressure in rebound chamber [Pa]
P_c	: Pressure in compression chamber [Pa]
P_{res}	: Pressure in reserve chamber [Pa]
A_p	: Piston cross-sectional are $[m^2]$
A_{rod}	: Rod cross-sectional area $[m^2]$
H_{11}	: Flow rate coefficient for base valve during compression stroke
H_{21}, H_{22}	: Flow rate coefficient for base valve during rebound stroke
H_{31}	: Flow rate coefficient for piston valve during rebound stroke
H_{41}, H_{42}	: Flow rate coefficient for piston valve during compression strike