Korean Journal of Remote Sensing, Vol.19, No.3, 2003, pp.233~246

Predictive Spatial Data Fusion Using Fuzzy Object Representation and Integration: Application to Landslide Hazard Assessment

No-Wook Park*, Kwang-Hoon Chi*, Chang-Jo Chung, Byung-Doo Kwon*****

Korea Institute of Geoscience and Mineral Resources*

Geological Survey of Canada**

Department of Earth Sciences, Seoul National University***

Abstract : This paper presents a methodology to account for the partial or gradual changes of environmental phenomena in categorical map information for the fusion/integration of multiple spatial data. The fuzzy set based spatial data fusion scheme is applied in order to account for the fuzziness of boundaries in categorical information showing the partial or gradual environmental impacts. The fuzziness or uncertainty of boundary is represented as two kinds of fuzzy membership functions based on fuzzy object concept and the effects of them are quantitatively evaluated with the help of a cross validation procedure. A case study for landslide hazard assessment demonstrates the better performance of this scheme as compared to traditional crisp boundary representation.

Key Words : Spatial Data Fusion, Fuzzy Boundary, Categorical Information.

1. Introduction

Since the 1990s, there has been an increased concern regarding the integrated analysis of multi-source/sensor spatial data. Various remote sensing images represent the spatial distribution of energy or physical characteristics from the earth surface in different wavelength ranges of the electromagnetic spectrum. Other sources of spatial data may include topography, some thematic maps of geology, geophysics, geochemistry, vegetation, forest and soils of an interested area. To make optimized decisions, better use must be made of available information acquired from different sources. Since each source gives us partial knowledge of the underlying phenomena, a single data source may not provide all the information required for decision making, while multiple data related to certain phenomena may help in the extraction of more information with higher accuracy and less uncertainty.

Geo-spatial data fusion is the formal framework that expresses the means and tools for the alliance of data originating from different sources (Wald, 1999). Data fusion is therefore by definition an enormous and complex field, comprising issues ranging from registration and pixel-level fusion of data for improving the spatial resolution of space-borne imagery to decision

Received 3 March 2003; Accepted 20 May 2003.

level fusion by using previously computed information stored in a GIS.

Much research has showed the advantage of spatial data fusion over traditional single source/sensor analysis in many application fields. In remote sensing community, data fusion techniques for the classification or change detection of remote sensing images have been extensively investigated in the past years and many papers that address the development of methodologies for the use of multi-sensor/source image have been reported (Lee *et al.*, 1987; Serpico *et al.*, 1996; Solberg *et al.*, 1996; Bruzzone *et al.*, 1999; Solaiman *et al.*, 1999; Park *et al.*, 2002). In geological applications, much effort has been devoted to any prediction tasks such as exploration of unknown geologic objects or future geological hazard assessment (Moon, 1990; Chung and Fabbri, 1998, 1999; Lee *et al.*, 1999; Choi *et al.*, 2000; Chi *et al.*, 2001). Since these applications deal with the unknown future events using spatial data, we will use the term "predictive spatial data fusion" throughout this paper.

Unlike the situation where we only deal with data from a single source, however, one of the most serious problems faced in the multi-source/sensor data fusion is the information content and relative reliability of each data set with respect to the application target. Since data come from various sources/sensors, the data inevitably have varying degrees of reliabilities for an observation and it may be impossible to maintain a consistent scale and the corresponding level of detail. For example, due to a simplification of reality and uncertainty in mapping procedures, the map boundary in categorical maps is expressed as a crisp one. However, many categorical maps have in reality indeterminate boundaries and the inaccuracy in boundary positions, depending on the map scale and resolution (Burrough and McDonnell, 1998; Zhang and Kirby, 1999). Hence, such input information regarding the relative reliabilities and uncertainties of the sources should be properly assessed and accounted for during data fusion processes.

While this concept of uncertainty analysis is well known for statistics/geostatistics community (Heuvelink *et al.*, 1989; Goovaerts, 1997), it has only recently received attention in geoscience. Related to the boundary problem in categorical maps, vague or fuzzy boundary representation has been put forward to describe the uncertainties in the perspective of a GIS (Wang and Hall, 1996; Cheng, 2002), but links between this theory and spatial data fusion for environmental impact research or geological hazard assessment have

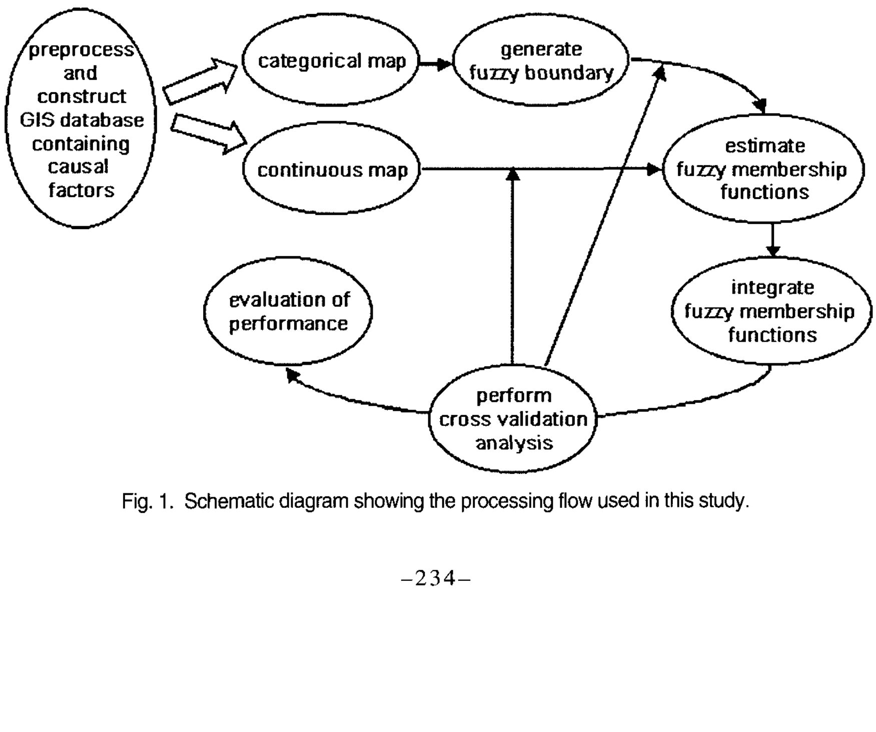


Fig. 1. Schematic diagram showing the processing flow used in this study.

not until recently been elaborated.

The motivation of this paper is to alleviate this trend by quantitatively investigating the uncertainties in spatial data fusion with multiple geoscience data sets. The core objective of this paper is to develop an effective scheme for quantitative uncertainty analysis in predictive spatial data fusion task. Among many types of uncertainties, of particular interest are the effects of the uncertainties of input data sets on the final integrated results.

In this paper, to account for the fuzziness or uncertainties of boundary in categorical maps, we propose and apply "fuzzy object" representation and integration based on fuzzy set theory. First, we generate the fuzzy boundary in categorical maps and the corresponding prediction maps based on fuzzy set theory. Then, the effects of fuzzy boundary on the final prediction result are evaluated with the help of cross-validation based on random spatial partitioning (Fig. 1).

This paper is structured as follows. In the next section, we present the general concept of fuzzy object representation and integration. Then, a case study for landslide hazard assessment is presented to illustrate the schemes proposed here. Finally, we conclude with discussion and remarks.

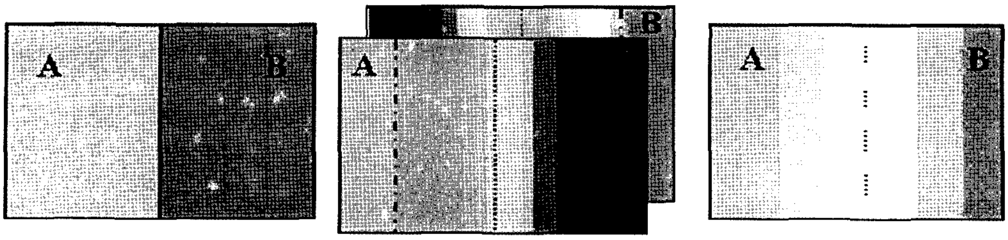
2. Fuzzy Object Representation

Spatial data are the results of qualitative and quantitative observation of spatial phenomena (Unwin, 1981). Among various types of spatial data, categorical data are fundamental sources of spatial information in a GIS. Categorical map information depicts the distributions of discrete attributes in the form of exhaustive, exclusive area units by crisp boundary lines (e.g. geological map, soil map, forest map, etc). Categorical map information is usually obtained by digitizing, vectorization, and rasterization in a GIS. Through these processes, the boundaries are represented cartographically by precisely defined lines of zero width (Mark and Csillag, 1989). However, such a representation method has difficulties in dealing with, for example, soil type mixture in a boundary position, which cannot be properly described by a crisp boundary. Many have in reality indeterminate boundaries and the inaccuracy in boundary positions. Since they are distributed continuously in space and time and measurement procedures generally produce data with a limited accuracy, the errors are compounded into the database including the original maps themselves, and lead to the uncertain description of geographical entities depending on the map scales and resolutions of them.

Suppose we have two categorical maps: the scale of one map is 1:25,000 and that of the other map is 1:50,000. If we assume that each map was mapped with a line of standard width (e.g. 5mm) and the resolution is 5m, a sharp boundary on a 1:25,000 scale map covers 125m and 25 pixels, and the boundary on a 1:50,000 scale map corresponds to 250m and 50 pixels. The traditional approach ignores these uncertain or fuzziness of boundary and useful information about the nature of spatial change is lost.

In the traditional predictive spatial data fusion approach, spatial data layers including original categorical maps and categorized continuous maps are first overlaid in order to generate the unique condition sub-areas (Chung and Fabbri, 1993). The traditional approach assumes that the boundaries of all maps have zero width and no uncertainty. However, if categorized maps are originated from maps having different scale one another, the unique condition sub-areas inevitably have uncertain or fuzzy boundary width, not zero width, so these uncertainties may affect the final prediction results.

Considering these conditions, we try to reflect the fuzziness or uncertainty of boundaries into the predictive spatial data fusion task. The fuzzy set theory can provide us with a natural method of quantitatively processing



(a) (b) (c)

Fig. 2. (a) Crisp object, (b) fuzzy representation model, (c) fuzzy object.

multiple data sets to reflect natural phenomena or irregular behaviors (Zadeh, 1965; Zimmermann, 1996). First, in order to apply the proposed schemes, using fuzzy concept, each class in certain categorical map is converted to partial and multiple memberships of all the candidate classes (Fig. 2(b)). By taking the maximum membership value, final fuzzy object or boundary can be generated (Fig. 2(c)). When we generate the multiple memberships of all the classes, the scale and resolution of the map can be considered. With a membership grade, a boundary can describe not only the location but also a change rate in phenomena at the boundary. A membership grade can also be interpreted as the level of plausibility or appropriateness to draw a boundary line at the location indicated (Wang and Hall, 1996).

The proposed method consists in two steps for data representation. Each step generates the fuzzy membership function. However, the fuzzy membership function for each step has the different meanings.

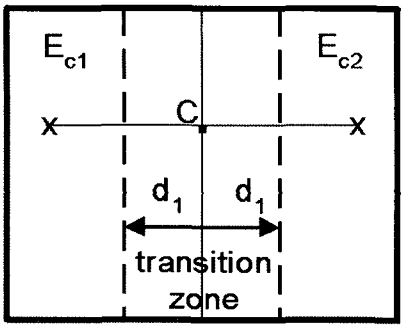
At the first step, we construct the fuzzy membership functions that account for the fuzziness at boundary positions. For the simplicity of explanation, suppose that we have one categorical map having two categories or attributes, E_{c1}, E_{c2}. In this study, we assume that each category in categorical maps has a core area and a transition zone. The fuzzy transition zone, d (= $2d_1$) (Fig. 3(a)), can be computed from polygon boundaries by considering the scale and resolution of the categorical map. Then, in order to express the characteristics that show the external gradation of membership function values from inside the transition zone to the outside, we define two fuzzy membership functions ($\mu_{E_{c1}}(x)$, $\mu_{E_{c2}}(x)$) of two categories as a semantic function.

$$\mu_{E_{c1}}(x) = \begin{Bmatrix} 1 & x < C - d_1 \\ 0.5 \sim 1 & C - d_1 \le x < C \\ 0.5 & x = C \\ 0 \sim 0.5 & C < x \le C + d1 \\ 0 & C + d_1 < x \end{Bmatrix} \tag{1}$$

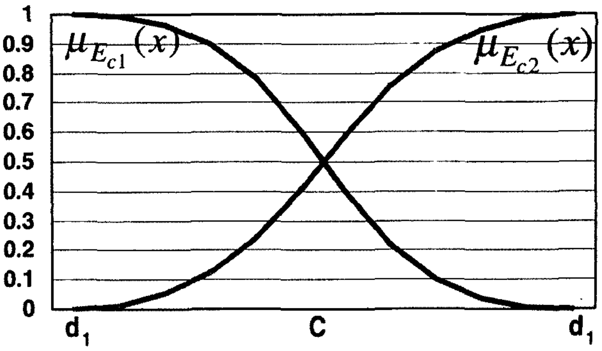
$$\mu_{E_{c2}}(x) = \begin{Bmatrix} 1 & x < C - d_1 \\ 0 \sim 0.5 & C - d_1 \le x < C \\ 0.5 & x = C \\ 0.5 \sim 1 & C < x \le C + d1 \\ 0 & C + d_1 < x \end{Bmatrix} \tag{2}$$

The parameters of the membership function are selected so that the locations corresponding to the original drawn boundary are at the crossover value, 0.5. The membership function is then applied so that those sites well within the original boundary have a membership value of 1, those sites inside, but near the boundary have a membership value between 0.5 and 1, and those sites outsides the boundary have a membership value below 0.5 concomitant with their distance from the boundary (Fig. 3(b)).

At the second step, we construct the fuzzy membership function for landslide hazard index. In our approach for landslide hazard assessment, a fuzzy model is based on the conceptual idea of expressing the landslide hazard in terms of possibility (fuzzy membership function). In practice, construction of fuzzy



(a)



(b)

Fig. 3. (a) Configuration of fuzzy transition zone, (b) fuzzy membership representation of boundary.

membership functions heavily depends on the problem to be solved. Also, it is very difficult to construct a certain type of semantic model (e.g. linear, bell-type, etc), especially for categorical data. Our main objective for landslide hazard assessment is to estimate relative hazard level within the study area. That is, we wish to separate the hazardous sub-areas affected by landslides and the non-hazardous sub-areas not affected by landslides. A difference in proportions of fixed size may have greater importance when both proportions are close to 0 or 1 than when they are near the middle of the range. For example, suppose we compare two frequency distribution functions of the hazardous and the non-hazardous sub-areas in terms of the proportion of areas within each class in spatial data. The difference between 0.010 and 0.001 may be more noteworthy than the difference between 0.510 and 0.501. In such cases, the ratio of proportions is also a useful descriptive measure. By adopting this idea, in this paper, we constructed the fuzzy membership functions by using likelihood ratio (Agresti, 1990; Chung and Fabbri, 1998). The likelihood ratio can highlight the difference between the frequency distributions of the hazardous and the non-hazardous sub-areas. The likelihood ratio ranges from zero to infinity. The more the likelihood ratio exceeds 1, the stronger the relationship between two patterns will be. However, the fuzzy membership function value should be a number in the range [0,1] with 1 representing full membership and 0 non-membership. To rest the likelihood ratio to the range [0,1], we used the following logistic-type ratio relation.

$$\mu_i = \frac{\lambda_i}{1+\lambda_i} \Leftrightarrow \lambda_i = \frac{\lambda_i}{1-\lambda_i} \tag{3}$$

where, μ_i is a fuzzy membership value and λ_i is an likelihood ratio value.

After getting landslide hazard index functions ($LHI_{E_{c1}}$, $LHI_{E_{c2}}$), then the final fuzzy membership functions which reflect the gradual variation of an attribute over the boundary between two dissimilar map units are obtained by computing a weighted estimate of the relative hazard index values over the boundary zone, considering the boundary membership functions of the two polygons (Fig. 4).

$$\mu_{FF}(x) = \frac{\sum_i LHI_{E_{ci}} * \mu_{E_{ci}}}{\sum_i \mu_{E_{ci}}} \tag{4}$$

where, $\mu_{FF}(x)$ is a final landslide hazard index membership function and a fuzzy membership function for boundary.

After each data layer of target information denoted from fuzzy theory is obtained, we can integrate them by using fuzzy combination operators. Basic algebraic relationships of inclusion, union, intersection,

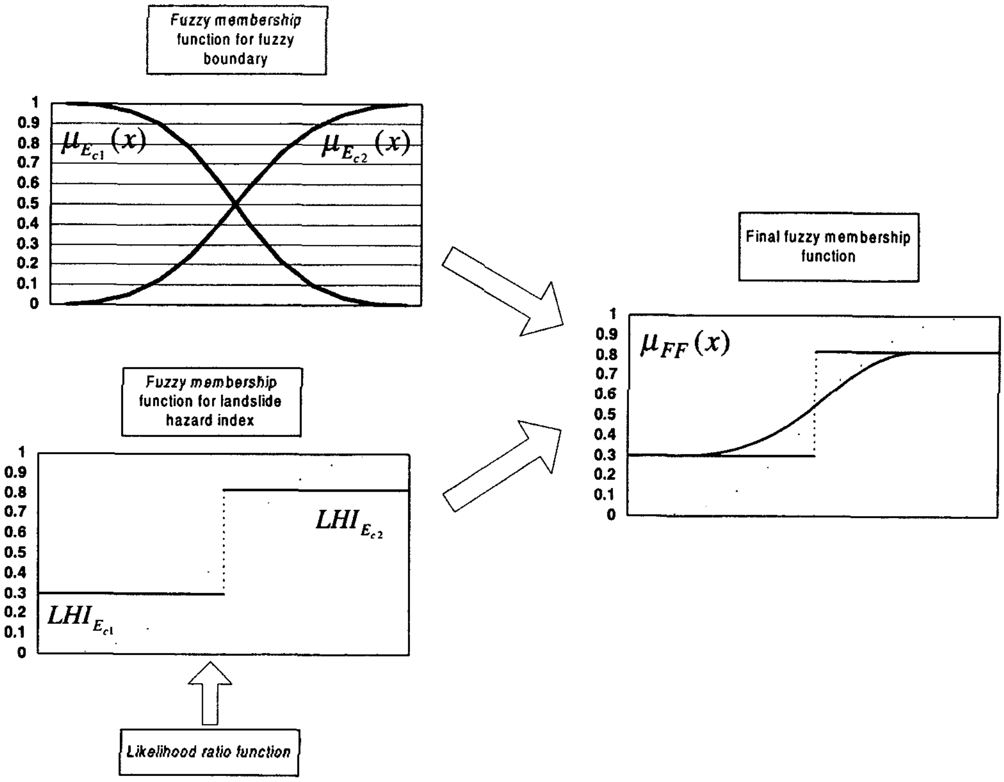


Fig. 4. Procedures for computing the final fuzzy membership function using fuzzy object representation.

complement and convexity are extended to fuzzy sets, making applications of fuzzy set theory easier to real problems. When two membership functions $\mu_A(x)$ and $\mu_B(x)$ are combined, some useful fuzzy combination operators for geological applications are as follows (An *et al.*, 1991; Moon, 1998; Chung and Fabbri, 2001).

Fuzzy OR

$$\mu_{OR}(x) = \text{MAX}\,[\mu_A(x), \mu_B(x)] \tag{5}$$

Fuzzy AND

$$\mu_{AND}(x) = \text{MIN}\,[\mu_A(x), \mu_B(x)] \tag{6}$$

Fuzzy Algebraic Sum

$$\mu_{SUM}(x) = 1 - \prod_{i=1}^{2} (1-\mu_i(x)) \tag{7}$$

Fuzzy Algebraic Product

$$\mu_{PRODUCT}(x) = \prod_{i=1}^{2} \mu_i(x) \tag{8}$$

Fuzzy γ operator

$$\mu_\gamma(x) = [\mu_{SUM}(x)]^\gamma \times [\mu_{PRODUCT}(x)]^{1-\gamma},\ 0 \le \gamma \le 1 = \tag{9}$$

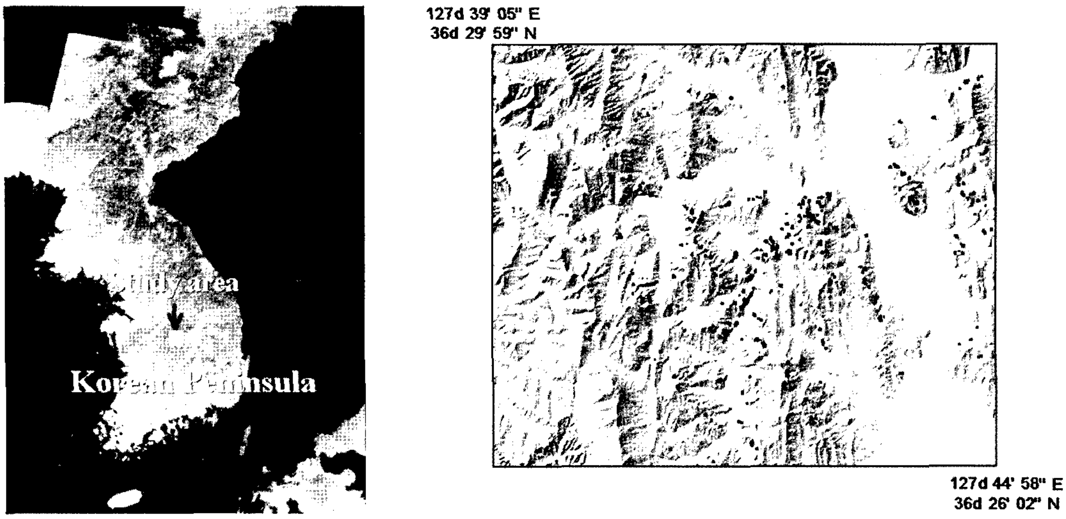
Fuzzy bounded sum operator

$$\mu_{BS}(x) = \min\{1, \sum_{i=1}^{2} \mu_i(x)\} \tag{10}$$

3. Case Study

1) Study Area and Data Sets

For a case study, the Boeun area in Korea, which had considerable landslide damage following heavy rain in 1998, was selected as the study area (Fig. 5). We used some data sets from the spatial database that had been constructed by our previous research (Chi *et al.*, 2002). Using the aerial photos taken before and after landslides



(a) (b)

Fig. 5. (a) Location map of the study area, (b) landslide scar distributions draped over the shaded relief map.

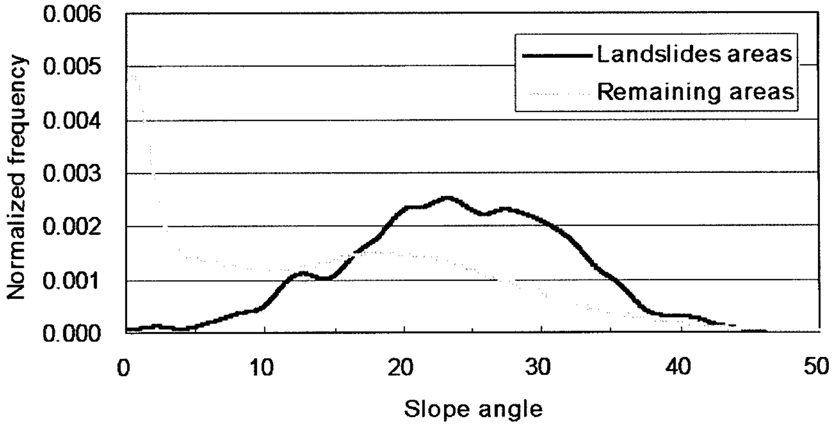
occurrence and field verification, we upgraded the past landslide scarp distributions. Finally, in total 459 landslides were mapped. The study area covers approximately 62 km^2 and consists of 1720×1444 pixels, with a pixel size of 5m by 5m. The spatial database consists of 6 layers. They are (1) the spatial distribution of past landslide scarps; (2) slope; (3) aspect; (4) forest type (1:25,000); (5) soil material (1:25,000); and (6) geology (1:50,000). As we pointed out in the introduction, the scales of spatial data sets are inconsistent.

2) Data Representation and Integration

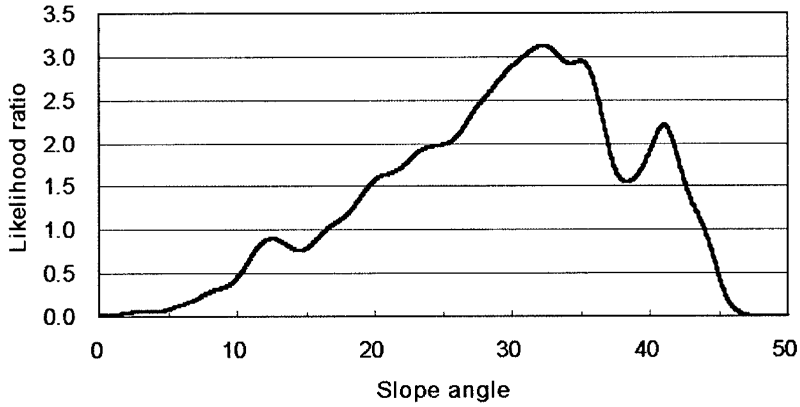
Based on fuzzy object concept and fuzzy set theory, we constructed landslide hazard index functions for quantitative landslide hazard assessment. The spatial data sets consisted of categorical data and continuous data. In the traditional landslide hazard research, any continuous data was classified to obtain a categorical map containing a few class labels. In this case, the boundaries in categorical maps of slope and aspect have the different intrinsic natures, compared to other categorical maps. So we separately processed the categorical data sets and continuous data sets.

For representation of categorical maps, we applied the fuzzy object concept. First, to account for the fuzziness of boundaries, we generated the fuzzy boundaries in categorical map. As a semantic fuzzy membership function for fuzzy boundary, a half Gaussian bell-type function was used. Then in the transition zones, fuzzy landslide hazard index functions based on likelihood ratio were calculated using a weighted estimate over the boundary zone.

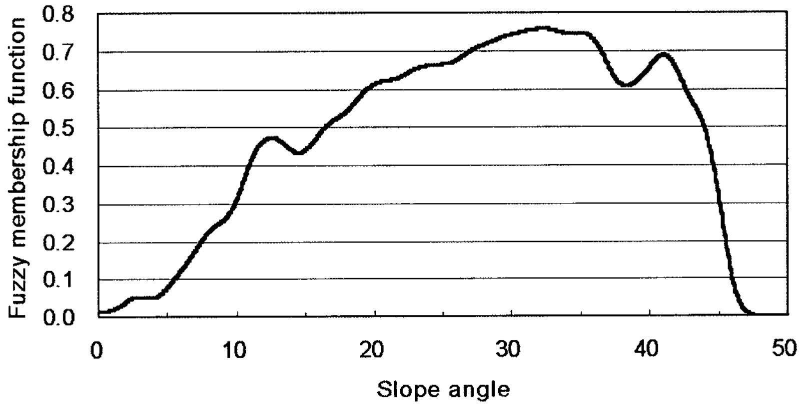
For representation of continuous maps, first we applied the likelihood ratio functions. But instead of converting them to categorical maps, we used original continuous scale maps. For continuous data, the likelihood ratio was based on two frequency distribution functions of areas affected by past landslides and areas not affected by past landslides. To compute the empirical frequency distribution function, we have employed the kernel method. The density estimation based on kernel functions (also called the Parzen window approach) is a well-known nonparametric approach that has been shown to be able to provide an asymptotic, unbiased, consistent estimate of the true



(a)



(b)



(c)

Fig. 6. (a) Empirical frequency distributions of landslides areas and remaining areas, (b) likelihood ratio function (c) fuzzy membership function of a slope map.

distribution (Parzen, 1962; Silverman, 1986). The kernel method derives the empirical frequency distribution function from the superposition of kernel functions centered on data samples and acting as smoothing operators. In our case, we apply this approach to estimate two empirical frequency distributions by adopting kernel functions of the Gaussian type. A value 2% of data range of the spread parameter in the Gaussian kernel function was selected experimentally as a result of the training phase. Fig. 6 shows the likelihood ratio function for the slope map. By taking a ratio, the difference between two empirical frequency distributions can be highlighted. Like representation of categorical maps, the logistic-type ratio relation was applied to convert the likelihood ratio to the fuzzy membership function ranging from 0 to 1.

After preparing the landslide hazard index membership functions of all input spatial data, we integrated them using the fuzzy algebraic sum operator. In a fuzzy set approach for spatial data fusion, it is not easy to select an optimal fuzzy combination operator. However, because main purpose of this paper is to investigate the effectiveness of fuzzy object representation, not to select the best combination operator, we used the fuzzy algebraic sum operator empirically.

3) Results

To investigate the impacts of the width of fuzzy boundary on the prediction patterns, the procedure was repeated for a series of transition zones having various sizes (d_1 = 0, 6, 12, 24). If d_1 is 0, it means that the categorical map has a crisp boundary. The larger d_1 is, the wider the transition zone will be.

Once an experiment procedure has been obtained, the extent to which it is valid should be evaluated. To evaluate the prediction results quantitatively, we applied the cross-validation approach based on random spatial partitioning of past landslides. The analytical procedures for cross-validation consist of 4 steps.

Step 1) The past landslides were randomly divided into 2 disjoint sets of equal size $n/2$, where n is the total number of past landslides (e.g. 459 in this study).

Step 2) The prediction maps were generated 2 times using one group, each time with the remaining occurrences held out as a validation set.

Step 3) Using rank order statistics, each prediction map is expressed in terms of relative landslide hazard values in the study area.

Step 4) we computed the prediction rate curve (Chung and Fabbri, 1999; Chi *et al.*, 2001) by considering the prediction rates in all past landslides from 2 prediction maps.

The cross-validation results using only 3 categorical maps (forest type, soil type and geology) are shown in Fig. 7 and Fig. 8. As expected, according to the increase of the transition zone, the fuzzy membership functions have gradual changes or smoothing effects. But in case of d_1 =24, these effects are too much to find the original features. Prediction rate is the measurement of how well the prediction model predicts the distribution of future landslides. To calculate the prediction rate, we first counted the number of pixels of reference landslides in the landslide hazard level whose value is larger than (1 minus a certain value). Then the number was divided by the total pixel numbers of reference landslides in order to obtain a normalized prediction rate. The prediction rate curve is the cumulative version of the prediction rate. It has the form y = function (x), as shown in Fig. 8. Here, x, ranging from 0 to 1, is the percentage of relative landslide hazard and corresponds to the legend in the prediction map. Because landslide hazard has a ranking equal sub-area in the landslide hazard map, it is the same as the percentage of the study area used, and y is the percentage of occurrences predicted within the most favorable x of the study area. From Fig. 8, at the most hazardous areas (e.g. from top 0%~5%), prediction rates are quite different. In case of the traditional crisp

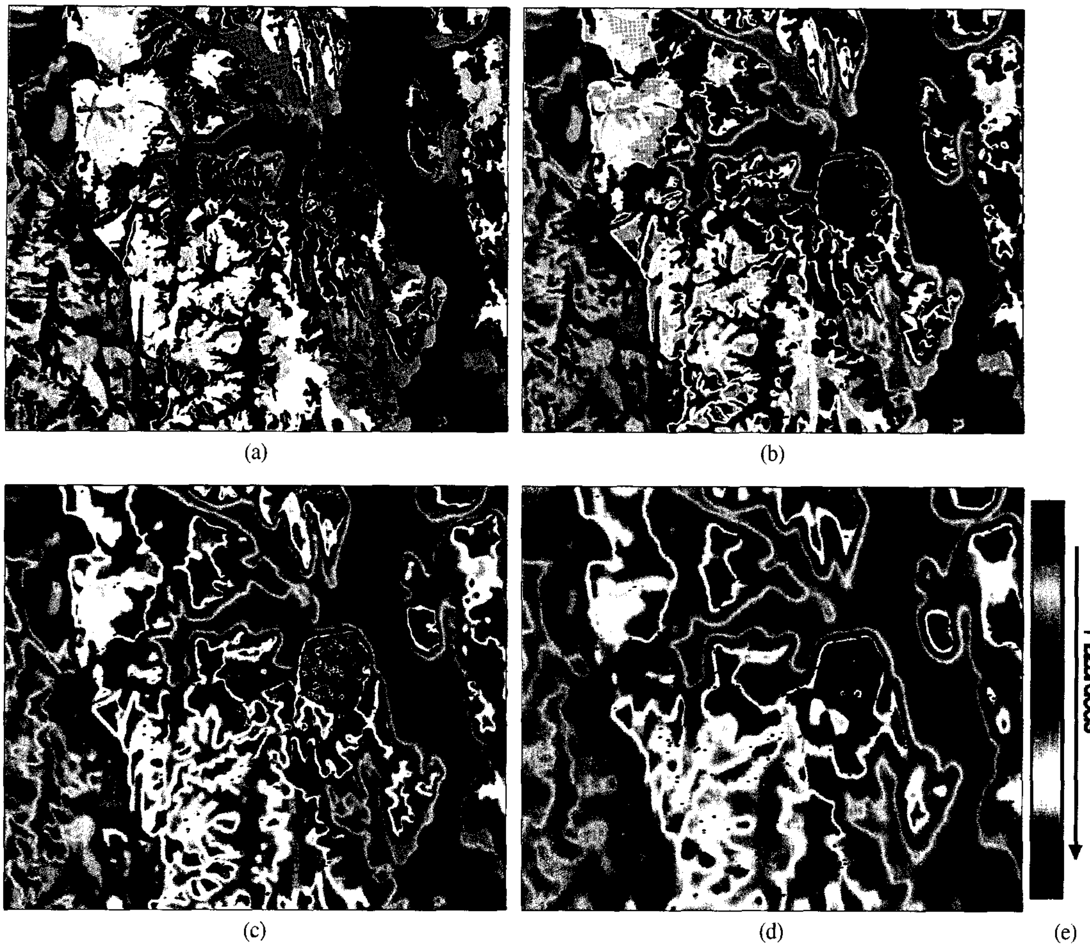


Fig. 7. Prediction maps using 3 categorical maps and group 1 landslide scars,
(a) d_1=0, (b) d_1=6, (c) d_1=12, (d) d_1=24, (e) hazard level. Black dots denotes the landslide scars.

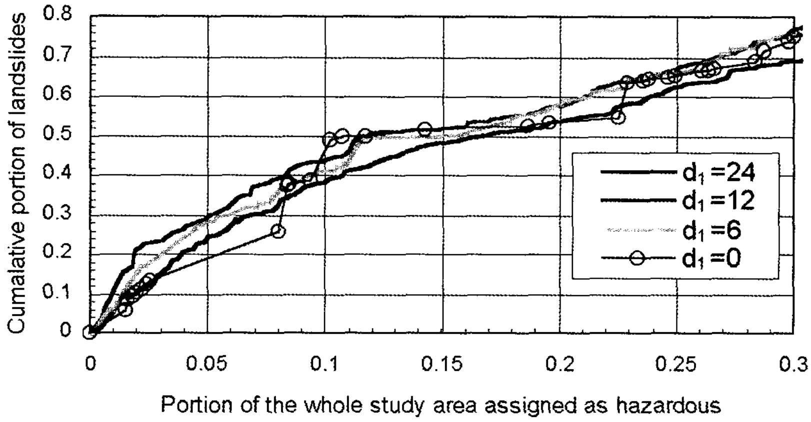


Fig. 8. Prediction rate curve based on 3 categorical maps only.

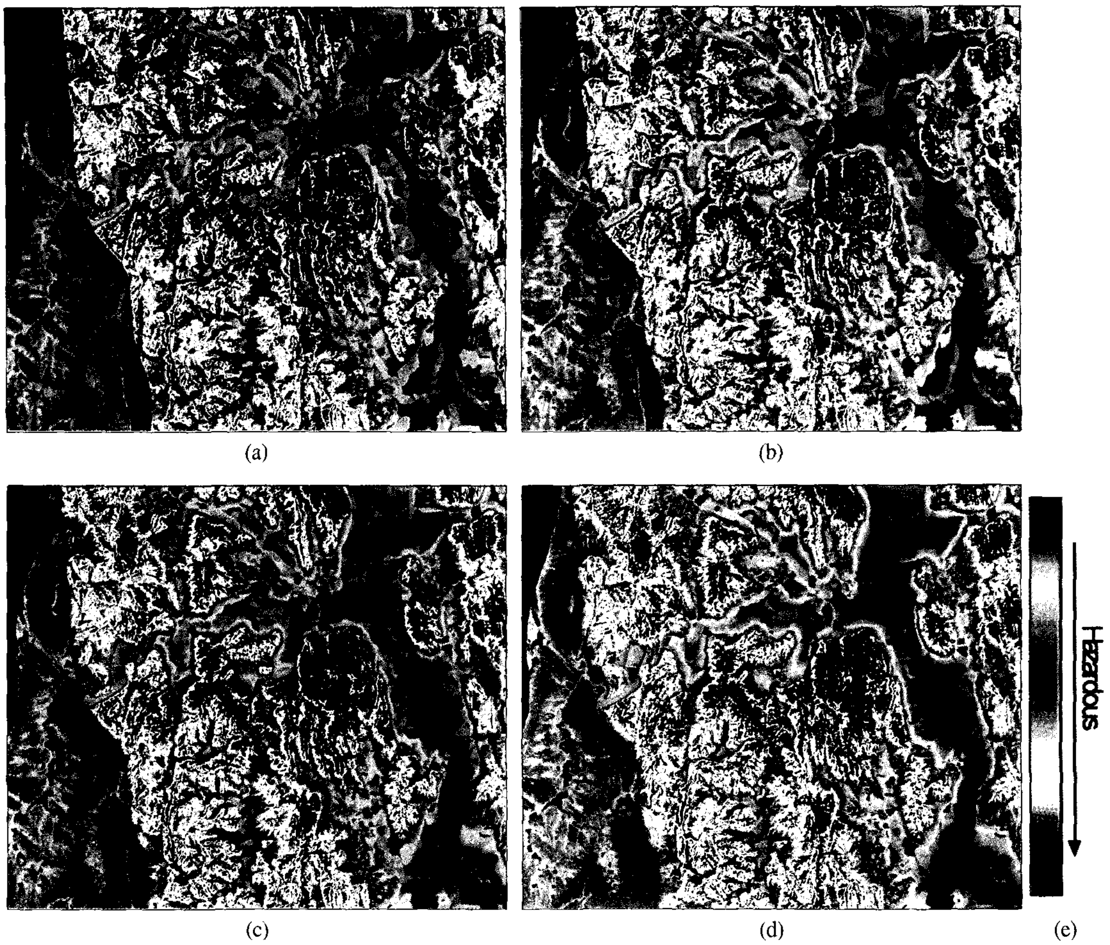


Fig. 9. *Prediction maps using 3 categorical maps, 2 continuous maps and group 1 landslide scars,* (a) d_1=0, (b) d_1=6, (c) d_1=12, (d) d_1=24, (e) hazard level. Black dots denotes the landslide scars.

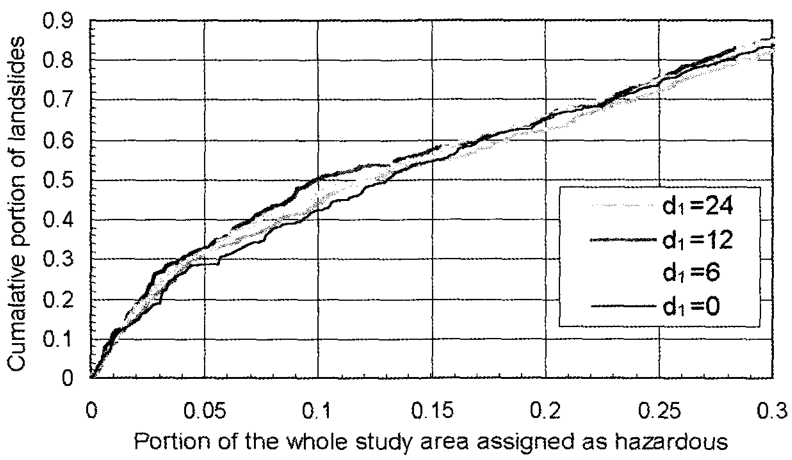


Fig. 10. Prediction rate curve using 3 categorical maps and 2 continuous maps.

boundary (d_l =0), if we take the most hazardous 5% of the area, then we estimate that 18.3% of landslides are located in the area. On the other hand, as d_l increases from 6 to 12, at the most hazardous 5% areas, the prediction rates are about 28.8% and 28.0%, respectively. This improvement of the prediction rates would be explained by the information content. Since the use of fuzzy boundary would include useful information about the nature of spatial change and spatial context, this effect results in improvement of the prediction powers. In case of d_l=24, though its prediction rate is higher than that of d_l=0, it did not show the highest prediction rate. A possible explanation is that the original categorical features, especially small size polygons, were smoothed too much and even disappeared by large transition zone (Fig. 7). Especially, if we assume that each map was mapped with a line of 5mm, a case of d_l=12 represents a 1:25,000 scale map with 5m resolution and a case of d_l=24 corresponds to the boundary on a 1:50,000 scale map. So the fuzzy boundary of the forest type, soil material and geology map would be represented well by setting d_l=12, d_l=12 and d_l=24, respectively. The experiment results partially agree with the map scale.

Finally, we carried out the cross-validation procedure using 3 categorical maps based on fuzzy boundary and 2 continuous maps (Fig. 9 and Fig. 10). When we added the continuous maps to the modeling procedure, the prediction results showed similar patterns and the overall prediction rate was slightly improved. By adding another maps related to landslide occurrences, especially the slope map, one of important factors for landslides, it might affect the final prediction results. Compared to the prediction results using only categorical maps, the discrepancy of prediction rates was reduced. But the order of the prediction capability is preserved. That is, in case of d_l=0, its prediction rate was the highest value and that of d_l=24 showed the worst result.

4. Conclusions

Fuzzy set theory allows the fusion of multiple spatial data in landslide hazard assessment through fuzzy membership function representation and combination using a fuzzy operator. Especially, this paper has incorporated fuzzy object representation into traditional fuzzy set based spatial data fusion. This approach can reflect the fuzziness of boundaries in categorical maps showing the different environmental impacts in data representation stage. Once fuzzy data representation had been implemented, a cross-validation approach was carried out in order to quantitatively investigate the effect of the transition zones on the prediction rates. The case study shows that use of fuzzy object and boundary can improve the prediction results in terms of prediction rates, compared to the traditional crisp boundary approach. This approach also can consider the map scale and resolution in the processing stage if a reasonable assumption is adopted. In this study, since our application target is to assess the relative landslide hazard level, we empirically compare the results in terms of prediction rates. Though it is still difficult to select an optimal transition zone size, the scale and resolution of the map can be a guideline. To strengthen the situation here identified, more research will be devoted to extensive experiments in several study areas.

Acknowledgments

This research is supported by the Korean Ministry of Science and Technology.

References

Agresti, A., 1990. *Categorical data analysis*, John Wiley & Sons.

An, P., W. M. Moon, and A. Rencz, 1991. Application of fuzzy set theory to integrated mineral exploration, *Canadian Journal of Exploration Geophysics*, 27(1): 1-11.

Bruzzone, L., D. F. Prieto, and S. B. Serpico, 1999. A neural-statistical approach to multitemporal and multisource remote-sensing image classification, *IEEE Transactions on Geoscience and Remote Sensing*, 37(3): 1350-1359.

Burrough, P. A. and R. A. McDonnell, 1998. *Principles of geographical information systems*, Oxford University Press.

Cheng, T., 2002. Fuzzy objects: their changes and uncertainties, *Photogrammetric Engineering & Remote Sensing*, 68(1): 41-49.

Chi, K. H., J. S. Shin, and N. W. Park, 2001. Quantitative analysis of GIS-based landslide prediction models using prediction rate curve, *Korean Journal of Remote Sensing*, 17(3): 199-210.

Chi, K. H., K. Lee, and N. W. Park, 2002. Landslide stability analysis and prediction modeling with landslide occurrences on KOMPSAT EOC images, *Korean Journal of Remote Sensing*, 18(1): 1-12.

Choi, S. W., W. M. Moon, and S. G. Choi, 2000. Fuzzy logic fusion of W-Mo exploration data from Seobyeog-ri, Korea, *Geosciences Journal*, 4(2): 43-52.

Chung, C. F. and A. G. Fabbri, 1993. The representation of geoscience information for data integration, *Nonrenewable Resources*, 2(2): 122-139.

Chung, C. F. and A. G. Fabbri, 1998. Three Bayesian prediction models for landslide hazard. In, Bucciantti, ed., *Proceedings of International Association for Mathematical Geology Annual Meeting (IAMG '98)*, Ischia, Italy, October 3-7, 1998, pp. 204-211.

Chung, C. F. and A. G. Fabbri, 1999. Probabilistic prediction models for landslide hazard mapping, *Photogrammetric Engineering & Remote Sensing*, 65(12): 1389-1399.

Chung, C. F. and A. G. Fabbri, 2001. Prediction models for landslide hazard zonation using a fuzzy set approach, In: Marchetti, M., Rivas, V., (Eds.), *Geomorphology & Environmental Impact Assessment*, Balkema Publishers, pp.31-47.

Goovaerts, P., 1997. *Geostatistics for Natural Resources Evaluation*, Oxford University Press.

Heuvelink, G. B. M., 1998. *Error propagation in environmental modeling with GIS*, Taylor & Francis.

Lee, K., N. W. Park, K. H. Chi, and B. D. Kwon, 1999. GIS-based spatial data integration and statistical analysis using multiple geoscience data sets: a case study for mineral potential mapping, *Korean Journal of Remote Sensing*, 15(2): 91-105.

Lee, T., J. A. Richards, and P. H. Swain, 1987. Probabilistic and evidential approaches for multi-source data analysis, *IEEE Transactions on Geoscience and Remote Sensing*, 25(3): 283-292.

Mark, D. M., and F. Csillag, 1989. The nature of boundaries on area class image, *Cartographica*, 21(1): 65-78.

Moon, W. M., 1990. Integration of geophysical and geological data using evidential belief function, *IEEE Transactions Geoscience and Remote Sensing*, 28(4): 711-720.

Moon, W. M., 1998. Integration and fusion of geological exploration data: a theoretical review of fuzzy logic approach, *Geosciences Journal*, 2(4): 175-183.

Park, N. W., W. M. Moon, K. H. Chi, and B. D. Kwon, 2002. Multi-sensor data fusion for supervised land-cover classification using Bayesian and geostatistical techniques, *Geosciences Journal*, 6(3): 193-202.

Parzen, E., 1962. On the estimation of a probability density function and the mode, *Annals of Mathematical Statistics*, 33: 1065-1076.

Serpico, S. B., L. Bruzzone, and F. Roli, 1996. An experimental comparison of neural and statistical non-parametric algorithms for supervised classification of remote-sensing images, *Pattern Recognition Letters*, 17: 1331-1341.

Silverman, B. W., 1986. *Density estimation for statistics and data analysis*, Chapman & Hall.

Solaiman, B., L. E. Pierce, and F. T. Ulaby, 1999. Multisensor data fusion using fuzzy concepts: application to land-cover classification using ERS-1/JERS-1 SAR composites, *IEEE Transactions on Geoscience and Remote Sensing*, 37(3): 1316-1326.

Solberg, A. H., T. Taxt, and A. K. Jain, 1996. A Markov random field model for classification of multisource satellite imagery, *IEEE Transactions on Geoscience and Remote Sensing*, 34(1): 100-113.

Unwin, D., 1981. *Introductory spatial analysis*, Methuen & Co. Ltd.

Wald, L., 1999. Some terms of reference in data fusion, *IEEE Transactions on Geoscience and Remote Sensing*, 37(3): 1190 -1193.

Wang, F. and G. B. Hall, 1996. Fuzzy representation of geographical boundaries in GIS, International *Journal of Geographical Information Systems*, 10: 573-590.

Zadeh, L. A., 1965. Fuzzy sets, *IEEE Information and Control*, 8(3): 338-353.

Zhang, J. and R. P. Kirby, 1999. Alternative criteria for defining fuzzy boundaries based on fuzzy classification of aerial photographs and satellite images, *Photogrammetric Engineering & Remote Sensing*, 65(12): 1379-1388.

Zimmermann, H. J., 1996. *Fuzzy set theory and its applications*, Kluwer Academic Publisher.