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Optimization of Magnet Pole of BLDC Motor by Experimental Design Method

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Abstract - The finite element method (FEM) is typically used in the process of motor design. However, the FEM requires computation time, Therefore, decreasing the number of FEM simulations may also decrease the simulation cost. Several optimal design methods overcoming this problem have been recently studied. This paper investigates the optimal design of the magnet pole of a BLDC motor through reducing simulation cost. The optimization minimizes the magnet volume and limits the average and cogging torques to certain values. In this paper, the response surface methodology and Taguchi's table for reducing the number of FEM simulations are used to approximate two constraints. The optimization result shows that the presented strategy is satisfactorily performed.

Keywords: BLDC motor, experimental design method, fractional factorial design, optimization, response surface methodology

1. Introduction

The brushless DC Motor is highly efficient and has a high ratio of power to volume by using high energy density magnets [1,2]. The magnet pole of a BLDC motor is an important part of the design. The FEM is typically used in the design of a BLDC motor. The FEM enables comparatively accurate computation of design variables but requires extensive computation time. Therefore, decreasing the number of FEM simulations may decrease the simulation cost. This paper investigates the optimization of the magnet pole of a BLDC motor minimizing the magnet volume via reducing simulation cost.

Many optimization methods for nonlinear problems with constraints have been studied. Deterministic methods, where constrained nonlinear problems are generally converted to unconstrained nonlinear problems with penalty and barrier functions offer optimization methods such as the multi-dimensional search method, the gradient and conjugated gradient methods, and the conjugated directions method [3,4]. Gradient and conjugated gradient methods requiring derivatives of the objective function can converge quickly but may reach local optimization. In addition, these methods have difficulty calculating the gradient of the objective function, especially in simulations utilizing FEM. The conjugate direction method is robust but converges slowly and requires many FEM simulations. The neural network, genetic algorithm, and evolution strategies have been recently used in stochastic methods, which are likely to converge a global optimum but require many FEM simulations [5–7]. Several studies utilizing these methods for motor design have been performed, but these strategies have difficulty in the design process due to numerous FEM simulations.

Recently, experimental design methods overcoming these problems have been studied [8–12]. In these studies, Taguchi's table and the response surface methodology are used for an approximation of the object function, which accelerates the convergence of the optimization and decreases the number of FEM simulations.

This paper studies the optimization of the magnet pole of the BLDC motor where the magnet volume is minimized and the average and cogging torques are constrained with the given values of the initial motor model. This optimization problem has two nonlinear constraints calculated from FEM simulation in the design process. The experimental design method including Taguchi's table and the response surface methodology is utilized in this optimization. In the presented optimization, two constraints are approximated to polynomial functions of the second order. The original optimization problem is changed to a simpler problem, and the number of FEM simulations largely decreases.

2. Torque Calculation of BLDC Motor

FEM is used to calculate the developed torque of the initial model of a BLDC motor where the specification and cross-sectional view are shown in Table 1 and Fig. 1.

S60 iron material is used for the stator and rotor of the BLDC motor, and its B-H curve is shown in Fig. 2.

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Table 1 Specifications of the initial model

output power	150W	phases	3
rated current	8.333A	connection	Y
rotational speed	1800rpm	torque constant	0.095Nm/A
axial length of core	40mm	magnet thickness	5mm
pole arc degree	120edeg	remnant flux density	1.12T
stator phase resistance	0.162Ω	line to line inductance	0.303mH

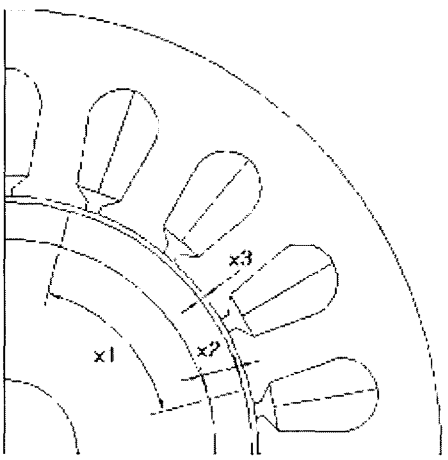


Fig. 1 Cross-sectional view of the initial model

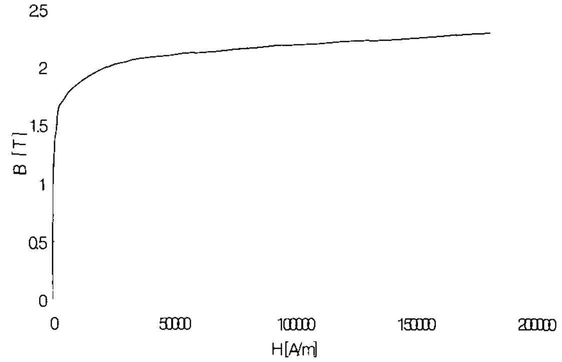


Fig. 2 B-H curve of iron material

Fig. 3 shows the finite element mesh using triangle elements for the model. The entire region is discretized because there is no electric and magnetic symmetry in the rotation of the rotor.

Fig. 4 shows the cogging torque waveform obtained in the initial model. In Fig. 4, the peak cogging torque is 0.0332 Nm. FEM simulation has calculated this cogging torque due to the slot teeth in case the currents of the stator coils are zeros. The software package used for the FEM simulation is Magsoft's FLUX2D.

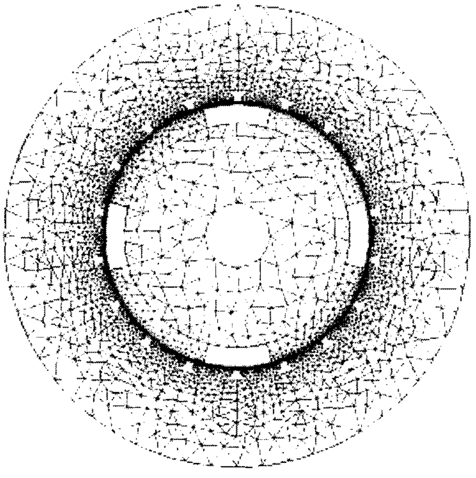


Fig. 3 Finite element mesh for the initial model

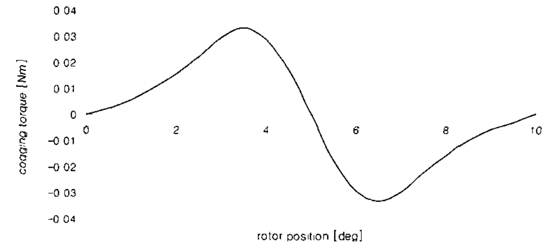
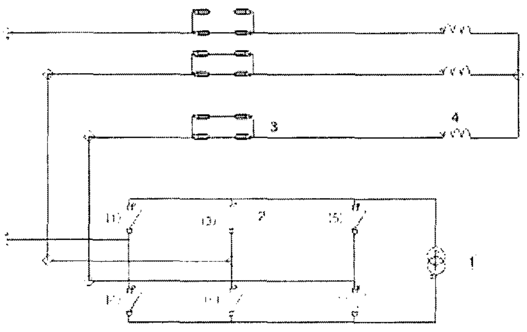


Fig. 4 Cogging torque waveform of the initial model

The external circuit for the BLDC motor drive, shown in Fig. 5., is composed of a constant current source, a six-step inverter, stator coils with two parallel circuits, and end-turn inductors. A three-phase full-bridge circuit for the inverter is connected to wye-connected phase windings of the motor. End-turn inductors are considered due to two-dimensional analysis of FEM.



No.	component	value
1	current source	8.333A
2	switch	position dependent
3	coil	42 turns, 0.162Ω
4	end-turn inductance	0.035mH

Fig. 5 External circuit for the BLDC motor drive

The switches of the six-step inverter are assumed to be ideal. DC current is fed from the supply to the motor via two lines for an interval 60°. During this interval, the third line carries no current and is idle. At the end of each 60° period, the current commutates from one of the conducting lines into the idle line. Line current waveforms and conduction intervals of switches are shown in Fig. 6.

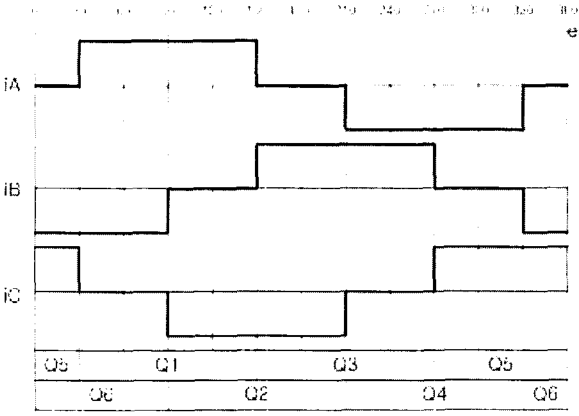


Fig. 6 Line current waveforms and conduction intervals of inverter switches

The developed torque of the initial model of the BLDC motor calculated by the FEM under the above conditions and with the motor speed of 1800rpm is shown in Fig. 7 (the average torque is 0.68059Nm).

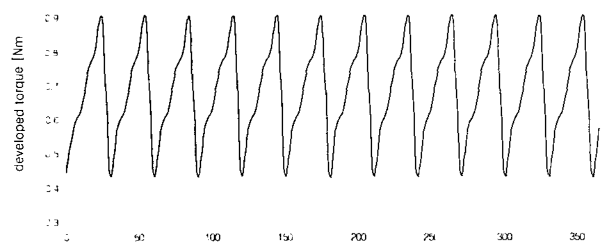


Fig. 7 Torque waveform of the initial model

3. Optimal Design of Magnet Pole

This paper investigates the optimization of the magnet pole of a BLDC motor through reducing low simulation cost. The experimental design method including Taguchi's table and the response surface methodology is utilized for reducing the number of FEM simulations.

3.1 Object Function and Constraints

In the optimization of the magnet pole of a BLDC motor, the magnet volume is minimized and the average and cogging torques are constrained with the given values of the initial motor model, which are obtained in Section 2. Design variables related to the magnet volume and torques are the pole arc degree (x_1), magnet thickness (x_2), and airgap length (x_3) shown in Fig. 1.

The object function is the volume of a magnet pole, which can be written as (1).

$$V_m(x_1,x_2,x_3)=\pi\frac{x_1}{360}l\,[\,(r_i-x_3)^2-(r_i-x_3-x_2)^2\,] \quad (1)$$

where r_i (=20.9mm) and l (=40mm) are the inner radius of the stator and the axial length of the magnet, respectively.

The values of design variables and magnet volume of a commercial BLDC motor used as an initial model in this paper are shown in Table 2.

Table 2 The initial values of design variables of the BLDC motor

x_1	x_2	x_3	V_m
60deg	3mm	0.5mm	2375mm^3

The constraints in this optimal design are the design variables and the average and peak cogging torques calculated in Section 2. From considering the initial model, the constrained ranges of the constraints are as follows.

$$T_{av}(x_1,x_2,x_3) \geq 0.68059\,Nm$$
$$T_{pc}(x_1,x_2,x_3) \leq 0.0332\,Nm$$
$$40° \langle\, x_1 \,\langle\, 60°$$
$$2\,mm \,\langle\, x_2 \,\langle\, 4\,mm$$
$$0.4\,mm \,\langle\, x_3 \,\langle\, 0.6\,mm$$

3.2 Approximation by Response Surface Methodology

The average and peak cogging torques are nonlinear functions whose values are calculated from the FEM simulation. Therefore, the derivatives of these constraints are difficult to obtain, and conventional optimization strategies require many FEM simulations. In this paper, these constraints are approximated by the response surface methodology (RSM) for reducing the number of FEM simulations. The RSM is a mathematical and statistical technique and has proved its usefulness in motor design where the object function is approximated by the RSM [10–12].

Two constraints of the average and peak cogging torques are approximated to polynomial functions of the second order as follows.

$$\begin{aligned}T_{av}(x_1,x_2,x_3)&=a_0+a_1x_1+a_2x_2+a_3x_3+a_4x_1^2\\&\quad+a_5x_2^2+a_6x_3^2+a_7x_1x_2+a_8x_1x_3+a_9x_2x_3\end{aligned} \quad (2)$$

$$\begin{aligned} T_{pc}(x_1, x_2, x_3) = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 {x_1}^2 \\ + b_5 {x_2}^2 + b_6 {x_3}^2 + b_7 x_1 x_2 + b_8 x_1 x_3 + b_9 x_2 x_3 \end{aligned} \tag{3}$$

The coefficients of (2) and (3) are the effects of the design variables. Such an approximation removes the discontinuities due to the discretization of the analysis region into finite elements and reduces largely the number of FEM simulations. The coefficients are usually determined by the least squares regression analysis for fitting the response surface approximation to the responses in sampling points [8].

3.3 Fractional Factorial Design

In the experimental design method for an optimization, design variables are factors and the value of the function is response. During an experiment, every factor takes some values called levels. The order of the response polynomial approximation is equal to the number of levels minus one. The number of experiments largely increases as the number of factors and levels increases. However, performing all experiments or simulations is too time-consuming. Taguchi proposed a set of tables using orthogonal arrays for fractional factorial designs [13,14]. Taguchi's L^9 orthogonal array, where no more than the second higher order interactions are considered, is shown in Table 3.

Table 3 Taguchi's table of L^9

	x_1	x_2	x_3
1	1	1	1
2	1	2	2
3	1	3	3
4	2	1	2
5	2	2	3
6	2	3	1
7	3	1	3
8	3	2	1
9	3	3	2

The fractional factorial design using Taguchi's table of L^9 needs nine experiments. The real values of levels 1, 2, and 3 are 40°, 60°, and 80° in x_1, 3mm, 3mm, and 4mm in x_2, and 0.4mm, 0.5mm, and 0.6mm in x_3.

4. Simulation Results and Discussions

The coefficients of (2) and (3) have been obtained by the least square method from the responses of nine experiments.

$$\begin{aligned} T_{av} = -0.07176 + 0.03344114 x_1 + 0.2756128 x_2 \\ -2.75983 x_3 - 0.00034046 {x_1}^2 - 0.080525 {x_2}^2 \\ -6.9615 {x_3}^2 - 0.007037 x_1 x_2 - 0.0746004 x_1 x_3 \\ +1.452607 x_2 x_3 \end{aligned} \tag{4}$$

$$\begin{aligned} T_{pc} = 0.1810047 - 0.0024819 x_1 + 0.0076983 x_2 \\ -0.16748 x_3 + 0.000016439 {x_1}^2 - 0.032404 {x_2}^2 \\ -3.0439 {x_3}^2 - 0.0017248 x_1 x_2 + 0.0161907 x_1 x_3 \\ +0.62793 x_2 x_3 \end{aligned} \tag{5}$$

Response surfaces of the average and peak cogging torques in (4) and (5) are shown in Figs. 8 and 9 where the airgap length (x_3) is 0.5mm.

From the approximation of (4) and (5), the solution of the constrained optimal problem described in Section 3.1 has been obtained by using Matlab. The optimal result is shown in Table 4. The average and peak cogging torques calculated from (4) and (5) are 0.6806Nm and 0.0332Nm.

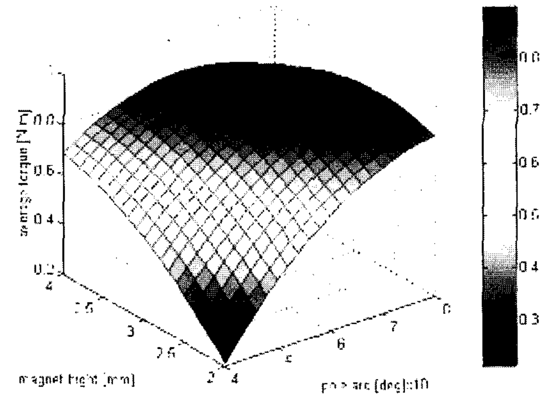


Fig. 8 Response surface of the average torque ($x_3 = 0.5$mm)

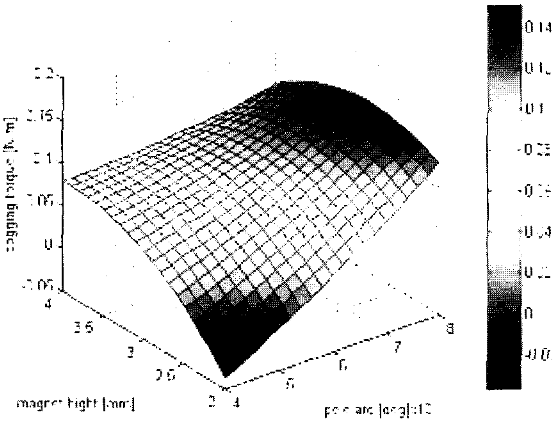


Fig. 9 Response surface of the peak cogging torque ($x_3 = 0.5$mm)

Table 4 The optimal values of design variables of the BLDC motor

X_1	x_2	x_3	V_m
66.28deg	2mm	0.533mm	1792mm^3

The magnet volume of the optimal model has decreased by 24.5% compared to the initial model. The rotor shapes of the initial and optimal models are shown in Fig. 10. The initial model is on the left and the optimal model is on the right.

This optimal solution obtained from the experimental design is evaluated by the FEM simulation. Fig. 11 shows the cogging torque waveform obtained in the motor model with the optimal values of the design variables. In Fig. 11, the peak cogging torque is 0.0319 Nm. This cogging torque due to the slot teeth has been calculated by FEM simulation in case the currents of the stator coils are zeros.

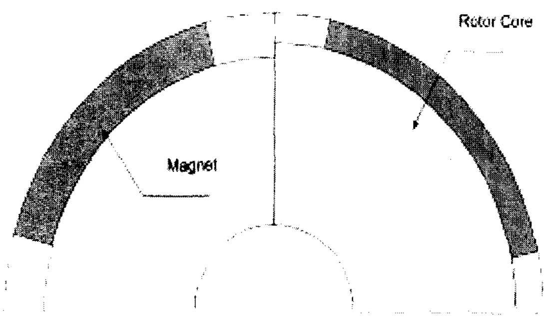


Fig. 10 Rotor shapes of the initial and optimal model

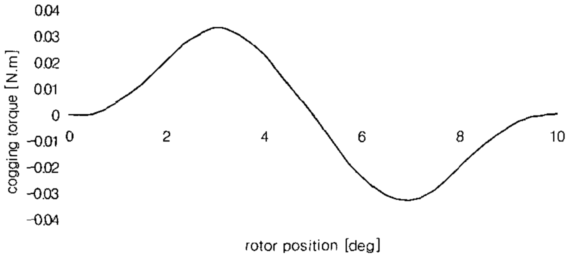


Fig. 11 Cogging torque waveform of the optimal model

Fig. 12 shows the developed torque waveform obtained by FEM simulation in the motor model with the optimal values of the design variables. This developed torque has been calculated under the same conditions as described in Section 2. In Fig. 12, the average torque is 0.6701Nm.

The differences between the average torques and between the peak cogging torques calculated by the approximation function and the FEM are only 1.5% and 3.9%, respectively. The simulation results show the presented strategy is satisfactorily performed.

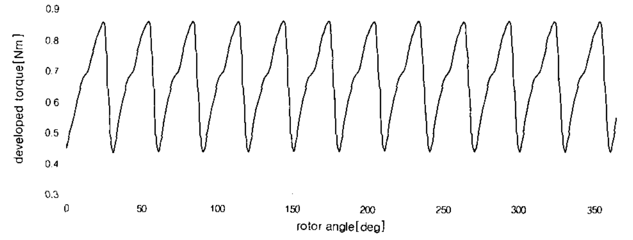


Fig. 12 Developed torque waveform of the optimal model

5. Conclusions

This paper investigates the optimization of the magnet pole of a BLDC motor where the magnet volume is minimized and the average and cogging torques are constrained with the given values of the initial motor model. This optimization problem has two constraints, the values of which are calculated from FEM simulation in the design process. An experimental design method including Taguchi's table and the response surface methodology is utilized in this optimization. In the presented optimization, two constraints of the average and peak cogging torques are approximated to polynomial functions of the second order. The original optimization problem is changed to a simpler problem and the number of FEM simulations largely decreases. The simulation results show the magnet volume of the optimal model decreases by 24.5% compared to the initial model, and the given constraints on the average and peak cogging torques are well satisfied.

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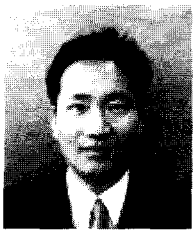

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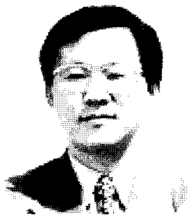

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