Magnet Design using Topology Optimization

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Abstract - The topology optimization for the magnet design is studied. The magnet design in the C-core actuator is investigated by using the derived topology optimization algorithm and finite element method. The design sensitivity equation for the topology optimization is derived using the adjoint variable method and the continuum approach.

Keywords: topology optimization, permanent magnet, sensitivity analysis, magnetostatics

1. Introduction

In spite of its short research history, topology optimization has attracted many engineers and mathematicians. In the case of electromagnetic systems, traditional optimization methods such as shape optimization focus on the improvement of the current design for the better performance. On the other hand, topology optimization generates a conceptual design that could, if necessary, be used as an initial design for the shape optimization.

The principle of topology optimization on electromagnetic systems is the same as that of the structural system [1]. Topology optimization for electromagnetic systems using the density method has already been studied [2-3].

Sigmund presented a study of topology optimization in Micro-Electro-Mechanical-Systems (MEMS), but an electrostatic force in the electric field was used due to its simple manufacturing [4]. In MEMS, generating large forces with the electrostatic actuation is difficult. In the electrostatic actuation, a large area and high voltage are required to get large forces. Generating large forces by using magnetic actuation is more practical.

Therefore, the study for the design of the magnetic actuation is now needed. Magnet design using topology optimization can be applied to the MEMS actuator. The final shape of the actuator, which is decided by the topology optimization, can be easier to make in the MEMS field than in the conventional manufacturing field.

The design sensitivity expressions are analytically derived using the continuum approach and the adjoint variable method (AVM). The continuum design sensitivity analysis (DSA) is already known to be accurate and efficient [5].

Hence, this research focuses on the development of theory and the application into practical examples. The sensitivity equation using the continuum method is derived and the sensitivity calculation is performed using a commercial finite element method (FEM) tool, ANSYS.

The optimization routine is implemented using a sequential linear programming (SLP) in design optimization tool (DOT), which is one of the commercial optimization codes. Then, to show the efficiency of the proposed method, the topology optimization of the C-core actuator is solved to reduce the volume while maximizing the energy.

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Received October 15, 2002; Accepted March 24, 2003

2. Sensitivity of Topology Optimization

2.1 Topology Optimization

An objective of topology optimization is to find an optimum material distribution that maximizes or minimizes a prescribed quantity while satisfying given constraints. A general topology optimization problem takes the form

$$\begin{aligned} &\text{maximize(minimize)} \; f(A,\mu,H_c) \\ &\text{subject to} \qquad a_\Omega(A,\bar{A}) = l_\Omega(\bar{A}) \quad \text{for all } \bar{A} \in \tilde{A} \end{aligned} \tag{1}$$

where $f(A,\mu,H_c)$ is the objective function, such as the magnetic energy, the magnetic force or torque, or the uniform flux; A is the vector potential; $\bar{A}$ is the virtual vector potential; $\tilde{A}$ is the space of the virtual vector potential; μ is the permeability; and $a_\Omega(A,\bar{A})$ is the energy bilinear form; $l_\Omega(\bar{A})$ is the load linear form, are functions of the permeability, μ, the coercive force, H_c, and the system output, A.

2.2 Design Sensitivity in Electromagnetic System

An integral objective function form in electromagnetic systems may be written as

$$\psi = \iiint_{\Omega} g(A, \nabla A, u)\, d\Omega \qquad (2)$$

where u is the design vector of the permeability, the current density, and the coercive force and ∇A is the gradient of the vector potential.

$$u = [J_s, \mu, H_c]^T \qquad (3)$$

The adjoint equation for the adjoint variable λ is

$$a_u(\lambda, \bar{\lambda}) = \iiint_{\Omega} [g_A \bar{\lambda} + g_{\nabla A} \nabla \bar{\lambda}]\, d\Omega \qquad (4)$$

which must hold for all admissible virtual vector potentials or electric field densities $\bar{\lambda} \in \tilde{A}$.

Using the variational form of the objective function of (2) and the direct differentiation result, the sizing design sensitivity equation is [5]

$$\begin{aligned} \psi' &= \iiint_{\Omega} [g_A A' + g_{\nabla A} \nabla A' + g_u \delta u]\, d\Omega \\ &= \iiint_{\Omega} g_u \delta u\, d\Omega + \iiint_{\Omega} [g_A A' + g_{\nabla A} \nabla A']\, d\Omega \\ &= \iiint_{\Omega} g_u \delta u\, d\Omega + l'_{\delta u}(\lambda) - a'_{\delta u}(A, \lambda) \end{aligned} \qquad (5)$$

2.3 Maxwell and Variational Equations in Magnetostatic Field

The magnetostatic field can be described using the set of Maxwell's equations.

$$\nabla \times H = J_s, \quad H = \frac{1}{\mu}(B - \mu_0 M), \quad \nabla \cdot B = 0 \qquad (6)$$

where H, B, and μ_0 are the magnetic field intensity, the magnetic flux density, and the permeability of free space, respectively.

The vector M represents the magnetization vector (A/m) in the permanent magnet. It is the zero-vector outside the permanent magnet region. The magnetization vector is related to the coercive force and the residual flux density (7).

$$H_c = \frac{\mu_0}{\mu} M = \frac{1}{\mu} B_r \qquad (7)$$

where H_c and B_r are the coercive force and the residual magnetic flux density, respectively.

By introducing a vector potential $B = \nabla \times A$ and (7), we have a single governing equation

$$\nabla \times (\frac{1}{\mu} \nabla \times A) = J_s + \nabla \times H_c \qquad (8)$$

To obtain the variational equation, multiplying both sides of (8) with the virtual vector potential $\bar{A}$, integrating over the domain, and applying boundary conditions leads to the variational equation becoming

$$a_{\Omega}(A, \bar{A}) = l_{\Omega}(\bar{A}) \quad \text{for all } \bar{A} \in \tilde{A} \qquad (9)$$

where $\tilde{A}$ is the space of the admissible vector potential, the energy bilinear form is

$$a_{\Omega}(A, \bar{A}) = \iiint_{\Omega} \left[(\nabla \times A) \cdot (\frac{1}{\mu} \nabla \times \bar{A}) \right] d\Omega \qquad (10)$$

and the load linear form is

$$l_{\Omega}(\bar{A}) = \iiint_{\Omega} \left[J_s \cdot \bar{A} + H_c \cdot (\nabla \times \bar{A}) \right] d\Omega \qquad (11)$$

2.4 Topology Sensitivity in Magnetostatic Field

The adjoint equation of (4) can be changed as (12) because the gradient of the virtual vector potential $\nabla \bar{\lambda}$ becomes zero.

$$a_u(\lambda, \bar{\lambda}) = \iiint_{\Omega} [g_A \bar{\lambda}]\, d\Omega \qquad (12)$$

If the equivalent source current is

$$J_{eq} = g_A \qquad (13)$$

then the adjoint response λ is the response of (8) but the right side of (8) is replaced with the equivalent source current of (13).

$$\nabla \times (\frac{1}{\mu} \nabla \times \lambda) = J_{eq} = g_A \qquad (14)$$

Using the design sensitivity formula of (5) and variations of the energy bilinear (10) and the load linear (11) forms, the sensitivity with respect to the permeability is

$$\frac{\partial \psi}{\partial \mu} = \iiint_{\Omega} \left[g_{\mu} + \frac{1}{\mu^2} (\nabla \times A) \cdot (\nabla \times \lambda) \right] d\Omega, \qquad (15)$$

the sensitivity with respect to the source current is

$$\frac{\partial \psi}{\partial J_s} = \iiint_\Omega \left[g_{J_s} + \lambda \right] d\Omega \ , \qquad (16)$$

and the sensitivity with respect to the coercive force is

$$\frac{\partial \psi}{\partial H_c} = \iiint_\Omega \left[g_{H_c} + (\nabla \times \lambda) \right] d\Omega \ . \qquad (17)$$

3. Density Method

To represent the porous material, suppose a fictitious material whose properties, like the coercive force H_c, can be represented as a P-powered function of ρ , that is,

$$H_c = H_{c_0} \rho^P \qquad (18)$$

$$\int_\Omega \rho(x) d\Omega \leq V \ , 0 \leq \rho \leq 1 \ , x \in \Omega \qquad (19)$$

where P is the penalization factor, ρ is the density variable, H_c is the coercive force, and H_{c_0} is the initial value of coercive force. If ρ is 1, the fictitious material becomes the magnet. Otherwise, the fictitious material becomes the air.

4. Numerical Example

The objective of a C-core actuator is the attraction force for the magnet to move the blade as shown in Fig. 1. So it is important to design the magnet's shape to maximize the force. This design can be an optimization problem in which the force is maximized in a specific volume. Because the energy variation is equal to the force, we can define the objective function as the magnetic energy [3].

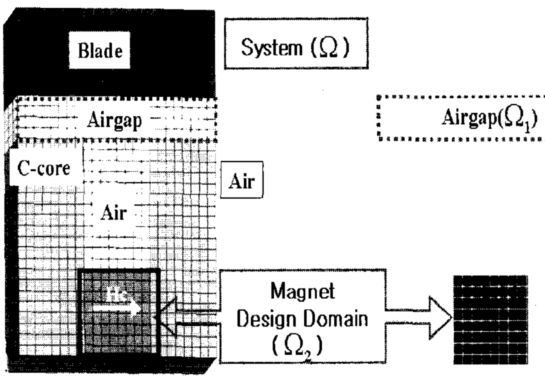


Fig. 1 C-core Actuator

Therefore, in this paper, the objective function is the magnetic energy in the airgap; the constraint is the volume of the magnet; the design variable is the density variable of coercive force of each element in the magnet domain; the initial value of the coercive force (H_{c_0}) is 871562.78 (A/m) and the relative permeability of the magnet is 1.05.

If the value of ρ in an element goes to 1, the element becomes the magnet. Otherwise, the element becomes the air.

In this example, the topology optimization formulation is written as

$$\textit{Maximize :} \ \ W = \int \int_{\Omega_1} \frac{1}{2} B \cdot H d\Omega \ \ (\Omega_1 : Airgap) \qquad (20)$$

$$\textit{Subject to :} \ \ g = \frac{\int \int_{\Omega_2} \rho A t d\Omega}{V_r} - 1 \leq 0 \qquad (21)$$

$$\textit{Bounded to :} \ \ 0 \leq \rho \leq 1, \text{for all } \rho \in \Omega_2 \qquad (22)$$

where V_r is the volume to remain after the topology optimization, Ω_1 is the system domain to calculate the system energy, and Ω_2 is the design domain to calculate the volume constraint. The object function is the total magnetic system energy, and the constraint is the volume of the magnet.

The width of the core and the blade is 20 mm. The length of the core and the blade are 60 and 50mm, respectively.

Figures 2 and 3 show the result of the topology optimization for the magnet design with $V_r=0.8V_{initial}$ and $P=3$. V_r and $V_{initial}$ are the remained and the initial volume of the magnet, respectively.

Figure 2 (a) shows only the design domain of the C-core. Figure 3 shows the contour plot for the topology results in Fig. 2.

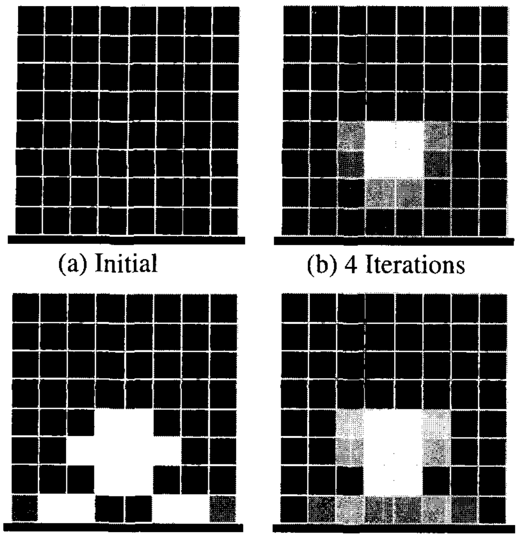
The initial magnetic energy is 44.857 (J/m), and the final magnetic energy is 43.527 (J/m). Therefore, the magnetic system energy is decreased about 2.9% as shown in Table 1.

However, the volume of the magnet of the optimal designed model is decreased 20% compared to the initial model, that is 20% of the volume of the magnet can be saved while maintaining the nearly current magnetic energy.

5. Conclusion

Topology optimization of the magnet in electromagnetic systems is studied, and topology optimization sensitivity is derived by the continuum method. The program for the topology optimization is developed. The SLP in DOT is used as an optimization algorithm.

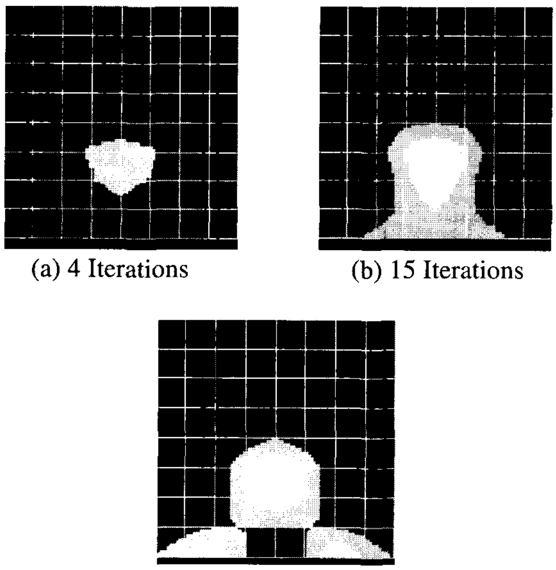
The developed program is confirmed by the C-core that is optimized to maximize the magnetic energy with the given volume. This research can be used for magnet topology optimization in MEMS.



(a) Initial (b) 4 Iterations

(c) 15 Iterations (d) Final (39 Iterations)

Fig. 2 The Result of Optimization



(a) 4 Iterations (b) 15 Iterations

(c) Final (39 Iterations)

Fig. 3 The Contour Plot of Topology Result

Table 1 Comparison

	Initial	Optimal	Error
Energy [J/m]	44.857	43.527	2.9 %
Volume [%]	100	80	20 %

Acknowledgements

This work was supported by a grant (R12-1998007000-0) from RRC-HECS, CNU.

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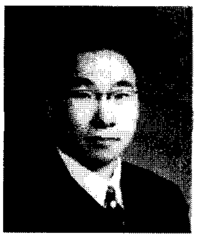

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