

Inversion of Material Coefficients for Numerical Analysis of Piezoelectric Actuators Using a Three-Dimensional Finite Element Method

Hyun-Woo Joo*, Chang-Hwan Lee*, Jong-Seok Rho* and Hyun-Kyo Jung*

Abstract - In this paper, the impedance of a piezoelectric transducer is calculated using the three-dimensional finite element method. The validity of numerical routine is confirmed experimentally. Using this numerical routine, the effects of material coefficients on piezoelectric actuators characteristics are analyzed. The material constants, which make significant effects, are selected and the relations between material constants are studied. Using these processes, three variables of material constants for a piezoelectric transducer are selected and the design sensitivity method is adopted as an inversion scheme. The validity of the inversion scheme for a piezoelectric transducer is confirmed by applying the proposed method to the sample piezoelectric transducer.

Keywords: electrical impedance, inversion target, finite element method, material constants, piezoelectric actuator

1. Introduction

As communication and information technologies are developed, piezoelectric materials are widely used in many applications. The piezoelectric actuator is an attractive component for electronic devices because of the possibility of miniaturization. Moreover, piezoelectric material has other merits, such as low cost, space efficiency, ease of manufacturing, and light weight. The Mason Model is the established standard model for piezoelectric transducers, which is representing a piezoelectric device by using electro-mechanical three-port. However, this model treats the piezoelectric devices one-dimensionally [1]. For two- or three-dimensional simulation of piezoelectric media, the complete set of fundamental equations governing piezoelectric media should be solved. In numerical analysis of piezoelectric actuators, the material constants, which are composed of the elasticity coefficient, piezoelectric coupling coefficient, and dielectric coefficient, are essential and important. So far, these coefficients are estimated via expensive and time-consuming experiments on test samples and the large numbers of constants make finding the coefficients difficult [2].

In this paper, resonance characteristics of piezoelectric actuators are analyzed using the three-dimensional finite element method. This numerical routine is applied to a sample piezoelectric transducer, and the validity of the finite element routine is confirmed experimentally. In addition,

the material constants inversion method for numerical analysis of piezoelectric actuators is proposed. Resonance characteristics of piezoelectric media are changed by even small variations of each variable. So by using the general inversion method, difficulty of convergence of the objective function and inaccuracy of the optimization direction is expected. Here, to save calculation time in the inverse procedure for piezoelectric transducers, the number of variables is reduced by analyzing the effects of constants on piezoelectric characteristics, and the research for relations between material coefficients makes inversion of the piezoelectric actuator coefficient possible.

2. Formulation

The matrices in Equation (1) are the basis for the derivation of the finite element model, which relates mechanical and electrical quantities in piezoelectric media [3].

$$\begin{aligned} T &= c^E S - e^T E \\ D &= e S + \epsilon^S E \end{aligned} \quad (1)$$

where T is the vector of mechanical stresses, S is the vector of mechanical strains, E is the vector of electric field, and D is the vector of dielectric displacement.

The material tensors c^E , e , and ϵ^S appearing in (1) are the elasticity coefficient, piezoelectric coupling constants, and dielectric constants, respectively. According to the structure and polarization of piezoelectric material, these material

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Received February 19, 2002; Accepted October 29, 2002

coefficients show a symmetry and sparsity pattern as shown in the following matrices [2].

$$c^E = \begin{bmatrix} cE_{11} & cE_{12} & cE_{13} & 0 & 0 & 0 \\ cE_{12} & cE_{11} & cE_{13} & 0 & 0 & 0 \\ cE_{13} & cE_{13} & cE_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & cE_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & cE_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & (cE_{11} - cE_{12})/2 \end{bmatrix}$$

$$e = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{bmatrix}$$

$$\varepsilon^S = \begin{bmatrix} \varepsilon S_{11} & 0 & 0 \\ 0 & \varepsilon S_{11} & 0 \\ 0 & 0 & \varepsilon S_{33} \end{bmatrix}$$

The description of the propagation of elastic waves in a piezoelectric medium and the description of piezoelectric resonators are based on the fundamental laws of continuum mechanics on one hand, and Maxwell's equation describing the electromagnetic field on the other. From Maxwell's equation, the electric field is related to the electrical potential by

$$E = -\nabla\phi \quad (2)$$

and the mechanical strain to the mechanical displacement by

$$S = Bu \quad (3)$$

where

$$B = \begin{bmatrix} \partial/\partial x & 0 & 0 \\ 0 & \partial/\partial y & 0 \\ 0 & 0 & \partial/\partial z \\ \partial/\partial y & \partial/\partial x & 0 \\ 0 & \partial/\partial z & \partial/\partial y \\ \partial/\partial z & 0 & \partial/\partial x \end{bmatrix}. \quad (4)$$

The elastic behavior of piezoelectric media is governed by Newton's law [3]

$$\rho \ddot{u} = \nabla \cdot T \quad (5)$$

where is density of the piezoelectric medium.

Assuming that piezoelectric media has no free volume charge inside it, then the electric behavior for piezoelectric media described by Maxwell's equation is

$$\nabla \cdot D = 0. \quad (6)$$

Equations (5) and (6) are the basic relationships used in the mathematical description of vibrations of piezoelectric resonators.

Applying Hamilton's variational principle to piezoelectric media [4], energy functional is like (7–8).

$$\delta \int L dt = 0 \quad (7)$$

$$L = E_{kin} - E_{el} + E_d + W \quad (8)$$

where the operator denotes the first-order variation and with Kinetic energy defined as

$$E_{kin} = \frac{1}{2} \iiint \rho u'^2 dV, \quad (9)$$

Elastic energy defined as

$$E_{el} = \frac{1}{2} \iiint S' T dV, \quad (10)$$

Dielectric energy defined as

$$E_d = \frac{1}{2} \iiint D' E dV, \quad (11)$$

and the energy generated by external mechanical or electrical source defined as

$$W = \iiint u' f_B dV + \iint u' f_s dA - \iint \Phi_{qs} dA + \sum u' F_p - \sum \Phi Q_p \quad (12)$$

where u is the the vector of particle velocity, V is the volume of the piezoelectric medium

f_B	vector of mechanical body forces	[N/m ³]
f_s	vector of mechanical surface forces	[N/m ³]
F_p	vector of mechanical point forces	[N]
A_F	area where forces are applied	[m ²]
Q_s	point charges	[As]
A	area where charges are applied	[m ²]

To apply the finite element method into any system,

employing interpolation functions for computing small discrete elements is essential. With interpolation function for displacement and electrical potential, then Equations (2) and (3) can be written as

$$E = -\text{grad } \Phi = -\text{grad}(N_\phi \hat{\Phi}) = -B_\phi \hat{\Phi} \quad (13)$$

$$S = Bu = BN_u \hat{u} = B_u \hat{u}. \quad (14)$$

By substituting Equations (13) and (14) into Equations (9)–(11) and applying Hamilton's principle, Equation (7), then the following linear differential equations for describing a single piezoelectric finite element are obtained [3].

$$-\omega^2 Mu + j\omega D_{uu} u + K_{uu} u + K_{u\phi} \Phi = F_B + F_S + F_P \quad (15)$$

$$K'_{u\phi} u + K_{\phi\phi} \Phi = Q_S + Q_P \quad (16)$$

K_{uu}	mechanical stiffness matrix
D_{uu}	mechanical damping matrix
$K_{u\phi}$	piezoelectric coupling matrix
$K_{\phi\phi}$	dielectric stiffness matrix
M	mass matrix
F_B	mechanical body forces
F_S	mechanical surface forces
F_P	mechanical point forces
Q_S	electrical surface charges
Q_P	electrical point charges

The damping behavior is determined by the damping matrix. This damping matrix can be assembled from the damping properties of the structure, which are usually frequency dependent. An arbitrary frequency dependence of the damping requires many damping coefficients. So it would require large computational effort to consider the full damping coefficient. In a practical case, only two damping coefficients, that is α and β in equation (15), are considered. Typical values of α and β are $\alpha=7.5$ and $\beta=2E-5$ for 1MHz [3].

3. Simulation Results for Piezoelectric Transducers

The electrical impedance is resonance characteristic quantities that can be verified experimentally since electrical impedance measurements can easily be carried out using a network analyzer. From the electrical impedance calculation, resonance and anti-resonance characteristics for piezoelectric transducers can also be obtained. Using the finite element formulation of piezoelectric transducers,

electrical impedance can be calculated by using a ratio between external charge and electrical potential on the electrode, like Equation (17) [1].

$$Z(\omega) = \frac{\Phi(\omega)}{j\omega Q_e} \quad (17)$$

Fig. 1 shows the impedances from three-dimensional calculation and experimentation for a test model piezoelectric transducer, whose experimental impedance is obtained from [3]. From these results, the finite element analysis routine in this paper is clearly accurate. Material data used in this paper is piezoelectric material VIBRIT 420 referred to in [3].

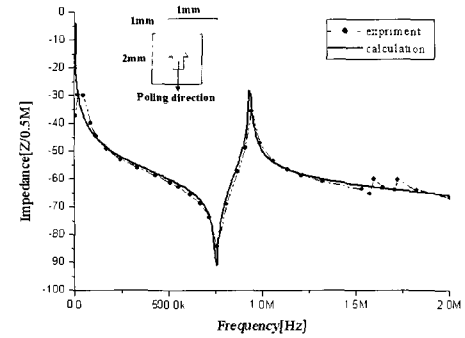


Fig. 1 Impedance calculations for a piezoelectric transducer

4. Numerical Analysis of Material Constants

4.1 Effects of material coefficients for impedance

In this paper, the inversion procedure of piezoelectric material constants for three-dimensional numerical analysis includes the investigation of the effects of material constants on the characteristics of a targeted piezoelectric system. From this investigation, the variables are decided then the variables converge toward matching the calculated impedance to the objected impedance.

To use the three-dimensional numerical analysis and design method for a piezoelectric system, ten material constants are necessary. Measuring piezoelectric material constants for numerical analysis is difficult because the experiments are expensive and time-consuming, limiting the analysis and design of piezoelectric system. Although the inversion schemes of piezoelectric material constants have been recently studied, finding all the material constants is difficult because of frequency sweep and calculation time requirements.

This paper shows that the number of material constants as inversion variables can be reduced by investigating the

effects of material constants on the characteristics of the piezoelectric system. Then the inversion is performed by using a proper inversion scheme such as the simple iteration and design sensitivity method according to the number of variables.

At first, the inversion of the piezoelectric transducer's material constants is performed. The effects of material constants on resonance and anti-resonance characteristics of a piezoelectric transducer are investigated. In this paper, the material constants are classified into three groups. The first group consists of some material constants, which have no effect on the characteristics of the piezoelectric transducer. This group, not considered as variables of inverse scheme, includes c_{44}^E , e_{51} , e_{31} , and ϵ_{11}^S in a z-direction poling piezoelectric system. The second group has dependent relations, that is, the material constants in this group have values related with main element by numerical factors. This group includes, c_{11}^E , c_{12}^E , and c_{13}^E mainly c_{33}^E . The third group has independent relations and includes c_{33}^E , e_{33} , and ϵ_{33}^S .

Fig. 2 shows the impedance comparison between the experimental and simulation results by varying the values of c_{44}^E , e_{51} , e_{31} , and ϵ_{11}^S . These constants clearly have no effect on the characteristics of piezoelectric transducers. So these constants are included in first group.

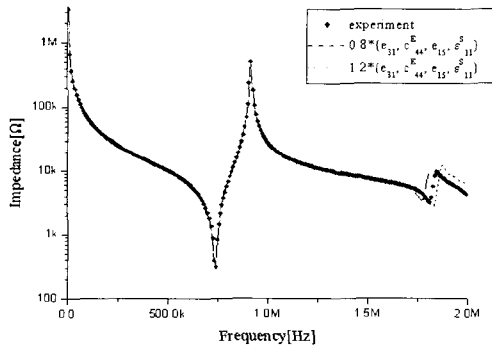


Fig. 2 Impedance comparison between experimental and simulation results multiplied by some factor to c_{44}^E , e_{51} , e_{31} , and ϵ_{11}^S

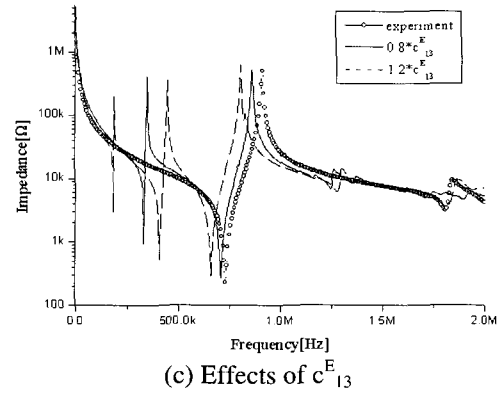
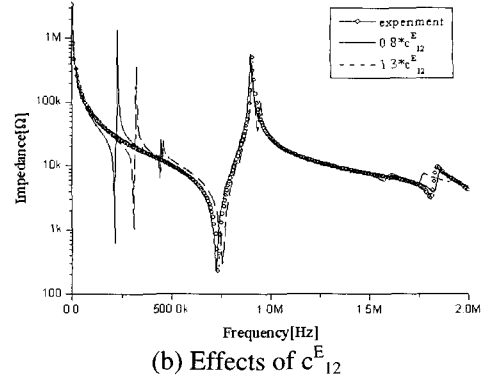
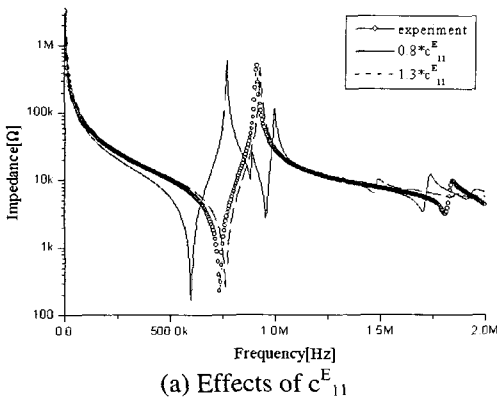


Fig. 3 Effects of variation of material constants on the resonance characteristics of piezoelectric transducer

Fig. 3 shows the effects of variations of c_{11}^E , c_{12}^E , and c_{13}^E on the characteristics of piezoelectric transducer. These constants show many complicated trends and are difficult to define as dependent variables.

Fig. 4 depicts the effects of variations of c_{33}^E on a piezoelectric transducer. From this result, c_{33}^E can make dominant effects on resonance and anti-resonance characteristics of piezoelectric transducer.

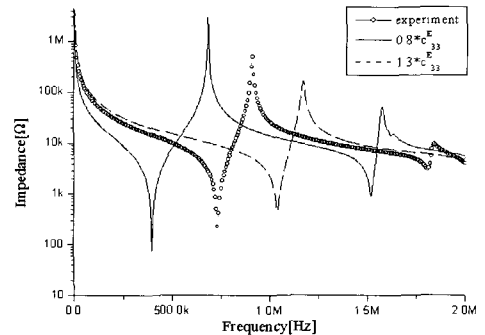


Fig. 4 Effects of c_{33}^E to the resonance characteristics

Strictly speaking, all c^E of the elements have complicated inter-relationships that cannot be defined clearly [4]. So, in this paper, c_{11}^E , c_{12}^E , and c_{13}^E are included in second group and c_{33}^E is defined as main element of this group. That is, values of c_{11}^E , c_{12}^E , and c_{13}^E are decided by multiplying some

factors to the main element c_{33}^E

Fig. 5, for example, when c_{13}^E is given by acceptable factor of c_{33}^E , shows the effects of variation of these factors on piezoelectric transducer. From this results, it is assumed that the variation of c_{11}^E , c_{12}^E , and c_{13}^E when the acceptable range of relation factors, can make little effects on characteristics of piezoelectric transducer.

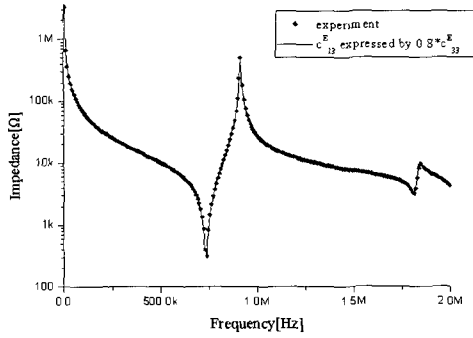
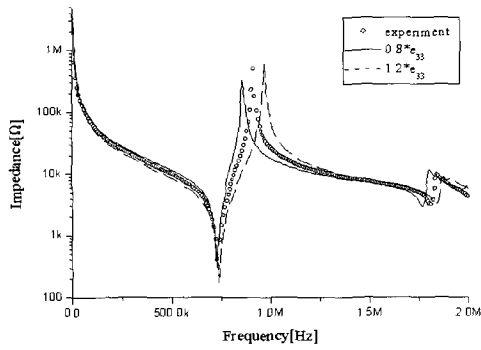
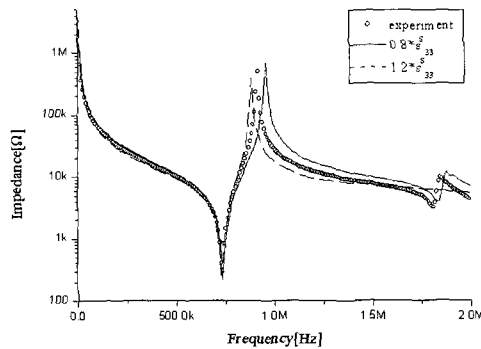


Fig. 5 Impedance comparison between experimental and simulation results when c_{13}^E is expressed by ratios to c_{33}^E

Fig. 6 depicts the effects of variation of e_{33} and ϵ_{33}^S or the piezoelectric transducer. These constants are included in the third group. From Fig. 6, changes of e_{33} and ϵ_{33}^S cause variations of anti-resonance frequency.



(a) Effects of e_{33}



(b) Effects of ϵ_{33}^S

Fig. 6 Effects of variation of material constants on the resonance characteristics of piezoelectric transducer

From these results as shown in Figs. 2–6, the more influential of the 10 material constants to the resonance characteristics of the piezoelectric transducer can be selected. So, the number of variables for the inverse scheme is significantly reduced and a general numerical method for inverse scheme can be adopted.

4.2 Inverse problem by sensitivity scheme

The sensitivity analysis is widely used for inverse problems because it can deal with many design variables and it has a very fast convergence rate. Fig. 7 contains a flow chart of the sensitivity scheme for the material coefficients inverse problem of the piezoelectric actuator. In this analysis, the objective function is resonance and anti-resonance frequency and the design variables are c_{33}^E , e_{33} , and ϵ_{33}^S .

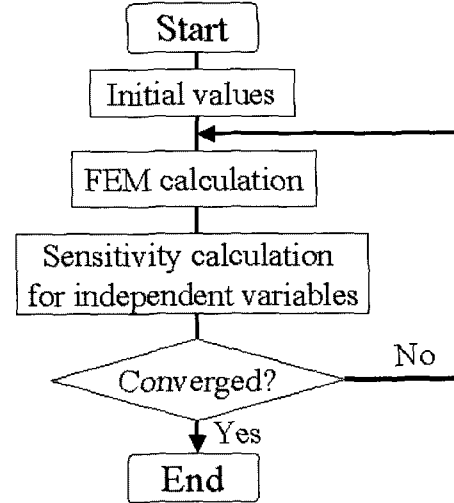


Fig. 7 Flow chart for sensitivity analysis

For renewing coefficients using the sensitivity analysis as explained in Fig. 7, the steepest-descent method is adopted in this paper.

$$M' = M - \alpha \frac{F}{\left| \frac{dF}{dM} \right|^2} \frac{dF}{dM} \quad (18)$$

where M , F , α , and $\left| \frac{dF}{dM} \right|^2$ indicate variables, objective function, relaxation factor, and norm for increment of variables, respectively [5].

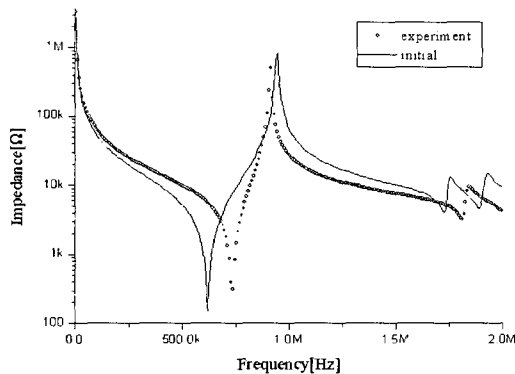
Fig. 8(a) depicts impedance comparisons between experimental and simulation results with initial values of three variables as shown in Table 1. With the initial coefficients, the nearly identified impedance result, shown in Fig 8(b), is obtained by using the inverse scheme. The preestimate, which c_{11}^E , c_{12}^E , and c_{13}^E can be expressed by in pro-

portion to c_{33}^E by the Poisson ratio under certain ranges, is acceptable for piezoelectric transducer as shown in Fig. 8(b). However, the material constants we required, did not have the same values as those obtained experimentally, but the values that gave the same impedance results as the experimental ones. So, though the values differ from the measured ones, the numerical results for piezoelectric actuators using these material coefficients are the same as with the measured ones. Therefore, inverse coefficients as shown in Table 1 can be used for other simulations of piezoelectric transducers without finding all of material coefficients in Equation (1) experimentally.

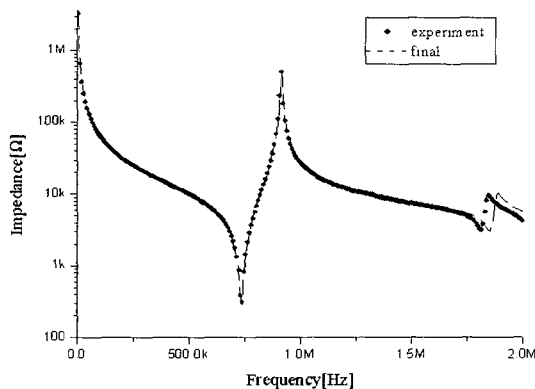
Fig. 9 shows the application of inverse coefficients to piezoelectric transducers. From this result, innovating an inverse procedure for piezoelectric transducers is reasonable.

Table 1 Comparison between initial and final constants for piezoelectric transducer

variable	c_{33}^E	e_{33}	ϵ_{33}^S
Initial value	10.3E10	15.7	5.5E-9
Final value	15.0E10	13.2	7.0E-9



(a) Impedance comparison between experimental and numerical results with initial coefficients



(b) Impedance comparison between experimental and numerical results with final coefficients

Fig. 8 Inversion of piezoelectric transducer material constants for matching impedance result with experimental results

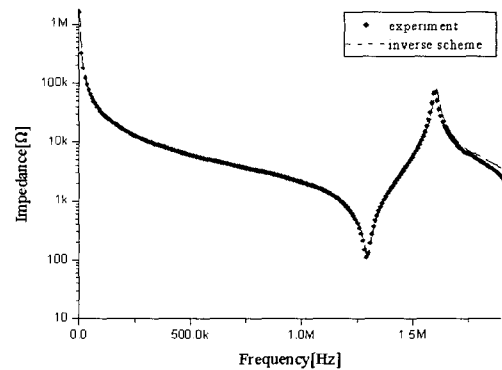


Fig. 9 Impedance comparisons of test model for piezoelectric transducer

5. Conclusion

In this paper, the impedances of a piezoelectric transducer are calculated using a three-dimensional finite element method. The validity of the numerical routine is confirmed experimentally. Using this numerical routine, the effects of material coefficients on resonance characteristics of piezoelectric transducers are analyzed, and the number of variables is significantly reduced, saving calculation time for the inversion procedure. The inversion method for reduced material coefficients of a piezoelectric actuator using reduced variables is proposed, and its validity is confirmed by applying the proposed inversion scheme to the sample piezoelectric transducer.

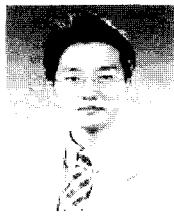
References

- [1] R. Lerch and H. Kaarmann, "Three-Dimensional Finite Element Analysis of Piezoelectric Media," *Proceedings of IEEE Ultrasonics Symposium*, 1987.
- [2] B. Kaltenbacher, M. Kaltenbacher, R. Lerch, and R. Simkovic, "Identification of material tensors for piezoceramic materials," *IEEE Ultrasonic Symposium*, Vol. 2, pp. 1033-1036, 2000.
- [3] Reinhard Lerch, "Simulation of Piezoelectric Devices by Two- and Three-Dimensional Finite Elements," *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control*, vol. 37, pp. 233-247, May 1990.
- [4] A.J. Moulson and J.M. Herbert, *Electroceramics*, Chapman and Hall, 1990.
- [5] P. Neittaanmaki, M. Rudnicki, and A. Savini, *Inverse Problems and Optimal Design in Electricity and Magnetism*, Clarendon Press, 1996.



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