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# A Simple Approach of Estimating the Shear Strength Parameters for Unsaturated Soil-Aggregate Systems

# 불포화 지반재료의 전단강도정수 추정을 위한 간편법

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## 요 지

본 연구에서는 불포화 토질역학 이론에 근거하여 지반재료의 안정처리로 인해 유발되는 전단강도계수의 변화를 추정하는 방법론을 제안하였다. 지반재료의 유효 점착력과 유효 내부마찰각들이 suction측정값들과 일축압축강도 실험결과를 활용하여 추정되었으며, 안정처리제의 사용량에 따른 효과도 비교하였다. 또한 유전상수 측정실험 결과를 통해서 안정처리에 따른 재료의 suction 변화를 알 수 있었으며, 제안된 방법이 불포화 지반재료의 강도정수를 추정할 수 있음을 보여주고 있다.

## Abstract

This paper presents the results of a study that was performed to evaluate trends of shear strength parameters in stabilization of unbound soil-aggregate systems based on the theory of unsaturated soil mechanics. Two important shear strength parameters, effective cohesion and effective angle of internal friction were estimated by the proposed approach using the results from suction measurements and unconfined compressive strength test. In addition, the effect of different addition rates of stabilizing agent was compared. Due to the stabilization process, an increase in suction potential on engineering properties of geomaterials was observed by using dielectric constant measurements. In conclusion, the results from this study show that the proposed approach can be simply used for predicting shear strength parameters of the stabilized geomaterials.

**Keywords** : Angle of friction, Cohesion, Geomaterials, Shear strength, Soil suction, Stabilization, Unsaturated soils

## 1. Introduction

Stabilization for unbound soil-aggregate systems has been used extensively to overcome deficiencies in materials, adverse environmental conditions, and improve engineering properties such as swelling potential, permeability, compressibility, or temperature susceptibility. To overcome deficiencies in soil or aggregate properties, various soils can be mixed together as mechanical stabilization or with stabilizing admixtures as chemical stabilization. In terms of utilizing these admixtures, it is important for engineers to select an appropriate stabilizer, because the behavior of each admixture and the corresponding stabilization mechanism is vastly different.

Recently, the measurement of soil suction and the use

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of unsaturated soil mechanics are demanded in many disciplines of pavement engineering areas because moisture distribution and state in the soil-aggregate systems are important in studying the behavior of the pavement. The importance can be accounted for stabilized pavement materials as well as for unbound one.

The overall goal of this study is to evaluate physical and mechanical properties and to verify the stabilizing mechanism based on sound theory. Various laboratory tests, unconfined compressive strength test, suction measurement using filter paper, and dielectric measurement were performed. Based on the results, shear strength theory for unsaturated soils was employed to verify the expected stabilizing mechanisms.

## 2. Shear Strength for Unsaturated Soils

In a saturated soil, shear stress can only be resisted by the skeleton of solid particles, therefore, shear strength is expressed as a function of effective normal stress as follows (Craig, 1997):

$$\tau_{ff} = C' + \sigma_{ff}' \tan\phi' = C' + (\sigma_f - u_{w})_f \tan\phi' \qquad (1)$$

where

$\tau_{ff}$ = shear stress on the failure plane at failure,

$C'$ = effective cohesion,

$(\sigma_f - u_w)_f$ = effective normal stress on the failure plane at failure,

$\sigma_{ff}$ = total normal stress on the failure plane at failure,

$u_{wf}$ = pore-water pressure at failure, and

$\phi'$ = effective angle of internal friction.

Fredlund and Morgenstern (1977) proposed that shear strength for unsaturated soils is an extended form formulated in terms of two independent stress state variables, $(\sigma_f - u_a)$ and $(u_a - u_w)$ from the shear strength equation for saturated soils.

$$\tau_{ff} = C' + (\sigma_f - u_a)_f \tan\phi' + (u_a - u_w)_f \tan\phi^b \qquad (2)$$

where

$(\sigma_f - u_a)_f$ = net normal stress state on the failure plane at failure,

$u_{af}$ = pore-air pressure on the failure plane at failure,

$(u_a - u_w)_f$ = matric suction on the failure plane at failure, and

$\phi^b$ = angle indicating the rate of increase in shear strength relative to $(u_a - u_w)_f$.

When the soil is saturated, pore-air pressure approaches pore-water pressure. That is to say, $u_a$ is equal to $u_w$ at saturation. Therefore, the matric suction term becomes zero, and the net normal stress term is effective stress. The failure envelope can be expressed in a three-dimensional manner to show two stress state variables, $(\sigma - u_a)$ and $(u_a - u_w)$. The frontal plane represents a saturated soil where the matric suction is zero. On the frontal plane, the $(\sigma - u_a)$ axis becomes $(\sigma - u_w)$ due to saturation.

Lytton (1995) has modified Eq. (2) using Lamborn's (1986) reversible thermodynamics principles, representing effective stress that is generated on the unsaturated soil's mineral skeleton due to tension in the pore water, as follows:

$$\tau_{ff} = C' + [(\sigma_f - u_a)_f + f\theta(u_a - u_w)_f] \tan\phi' \qquad (3)$$

where

$\tan\phi^b = f\theta \tan\phi'$.

$f\theta$ = degree of saturation, $\tan\phi^b / \tan\phi'$, totally saturated soils have 1.0,

$\theta$ = volumetric moisture content, and

$f$ = a parameter related to degree of saturation, unsaturated soils have 1.0.

The relationship of suction versus degree of saturation is illustrated in Fig. 1. From the application of Lamborn's theory to unsaturated soils, the shear strength equation can be represented as a linear function in a two-dimensional space as expressed in Fig. 2. The failure envelope is a function of shear stress and effective normal stress, with parameters of effective cohesion, $C'$, and internal friction angle due to net normal stress, $\phi'$, defining the envelope.

Based on above relationship, the Lamborn's reversible

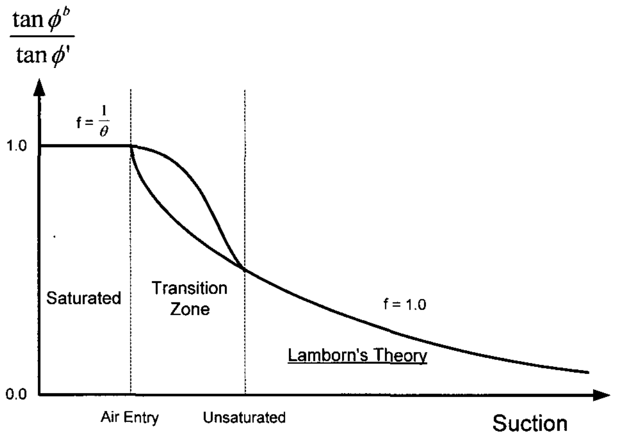

Fig. 1. Degree of saturation versus matric suction

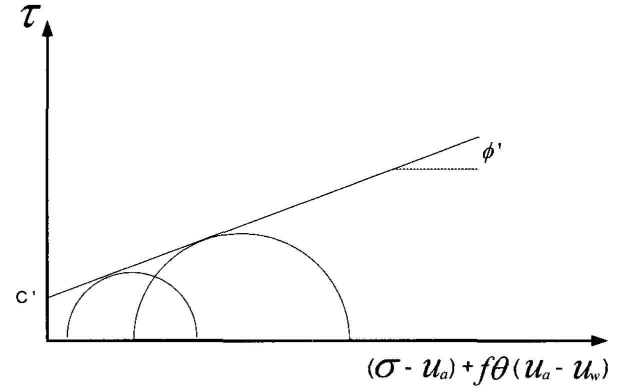

Fig. 2. Simplified Mohr-Coulomb failure envelope for unsaturated soils

thermodynamics principles can be described in the somewhat familiar form as follows:

$$\sigma'_w = -f\theta u_w \tag{4}$$

where

$\sigma'_w$ = the stress on the soil mineral skeleton due to the water, and

$u_w$ = the matric suction, a negative number, corresponding to tensile stress in the pore water.

This formulation has applications in estimating the shear strength of the soil, as it is expressed in the shear strength equation for unsaturated soils. The stress $\sigma'_w$ contributes to the strength increment due to the tensile stress in the pore water and the resultant pulling force between soil particles. This stress is viewed as a confining pressure affecting all directions of a soil specimen. In a typical unconfined compressive strength test, confining pressure is not intentionally applied to a soil specimen. However, unsaturated soil specimens have confining pressure themselves due to the suction characteristic. Therefore, the minor principal stress is $-f\theta u_w$ because it is confining pressure, and the corresponding major principal stress becomes $q_u - f\theta u_w$ in the typical unconfined compressive strength test. The $q_u$ represents the uniaxial compressive strength at the unconfined compressive strength test. Therefore, the unconfined strength and suction measurement can be used for estimating two strength parameters, effective cohesion and internal friction angle.

## 3. Materials

For this study, a petroleum-resin-based (PRB) material has been used as a stabilizing agent. The PRB can be conveniently classified as an emulsified petroleum resin. Because of high viscosity in its concentrated form and stable characteristic with water, the PRB is diluted with water. Dilution ratio of 4:1 (four parts of water and one part of concentrate) and/or 6:1 (six parts of water and one part of concentrate) was used.

Limestone aggregate was used for the PRB treatment. Because the PRB was known as anionic, negatively charged, the positively charged materials such as limestone are expected to be compatible. The aggregate gradation was selected in accordance with the one suggested by Marian and Raymond (1996) as shown in Fig. 3. The limestone aggregate was non-plastic and showed 9 % optimum moisture content at a maximum

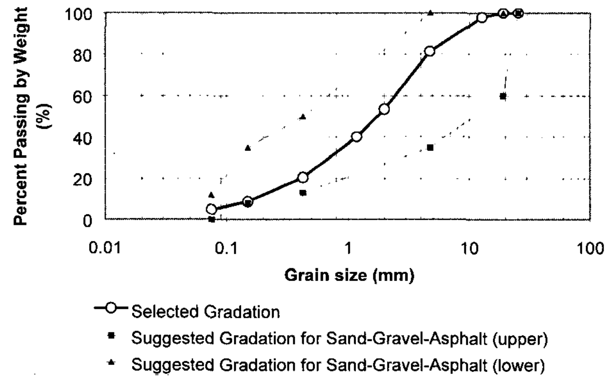

Fig. 3. Grain size distribution of limestone aggregate

dry density of 2.11 g/cm$^3$. The maximum sieve size for the limestone aggregate was 15.9 mm.

## 4. Laboratory Tests and Results

Unconfined compressive strength and soil suction were measured for estimating strength parameters and for verifying expected stabilization mechanism. Dielectric constants were also measured to provide supporting idea for the expected stabilization mechanism. Various treated and untreated specimens were tested for comparison.

### 4.1 Unconfined Compressive Strength Test

Product contents of 1.5%, 4%, and 6% and dilution ratios of 4:1 and 6:1 were examined, respectively. After oven drying of the limestone, each specimen was compacted at the same moisture content (9%), and then each compacted specimen was cured at 40℃ for 7 days. Two different curing conditions, sealed and unsealed, were tested to investigate the curing effect on strength. Moisture in sample cannot evaporate under the sealed condition.

Compressive strength was measured at a rate of 2% per minute in a strain-controlled test. Fig. 4 shows results of the unconfined compressive strength test. The compressive strength slightly increases under the unsealed curing condition when more stabilizing agent is added to the limestone base material. However, every specimen stress approached failure at approximately the same compressive under the sealed curing condition, although they were treated with different amounts and dilution ratios.

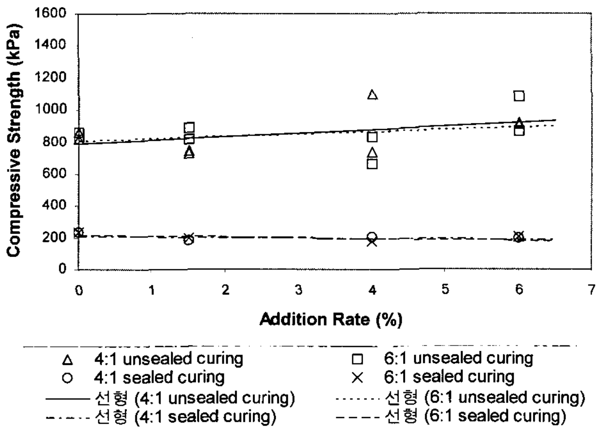

Fig. 4. Unconfined compressive strength results for limestone with different addition rates and dilution ratios

Based on the results, it can be inferred that appropriate curing time is necessary for the stabilization to be effective in increasing strength. In fact, treatment with the PRB may reduce strength unless proper curing is maintained. This is similar to the behavior of typical emulsion-type stabilizer where evaporation of water is required during the construction and curing process.

### 4.2 Suction Measurement

Soil suction is an indicator for characterizing soil water storage and movement and provides information about soil parameters that are influenced by the presence of water (Park et al., 1999). Suction is composed of two components, matric suction and osmotic suction. In this study, total suction, which is the summation of matric and osmotic suction, was measured because of difficulty in measuring matric suction of aggregate samples. Among various methods for suction measurement, the filter paper method is employed in this study due to its simplicity.

Each specimen, untreated and treated, was mixed with same moisture content of 9%. Products content of 1.5%, 4%, and 6% and dilution ratios of 4:1 and 6:1 were applied. Curing was performed at 40℃ for 7 days, and two different conditions were used: sealed and unsealed. Fisherbrand 9-790A was used as filter papers. Since there is no corresponding calibration curve for the Fisherbrand 9-790A to determine suction values from the measured moisture content in filter papers, a calibration curve was made as shown in Fig. 5.

Using the pre-determined calibration curve, total suction values were calculated and plotted in Fig. 6. Total suction values of treated specimens with different additive rates (1.5%, 4%, and 6%) and dilution ratios (4:1, 6:1) were compared with those of untreated specimens. As can be seen in Fig. 6, suction increased with higher additive rates and treated specimens

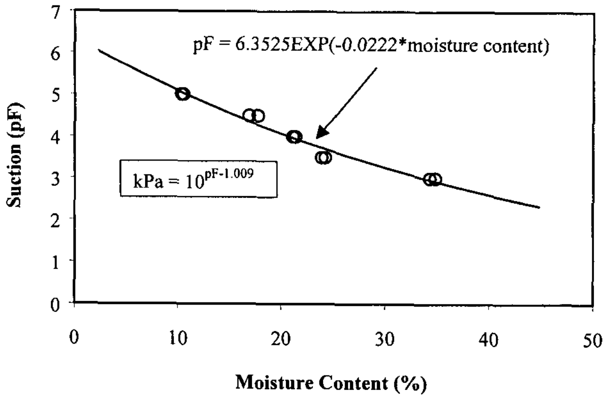

Fig. 5. Calibration curve for Fisherbrand 9-790A filter paper to evaluate soil suction

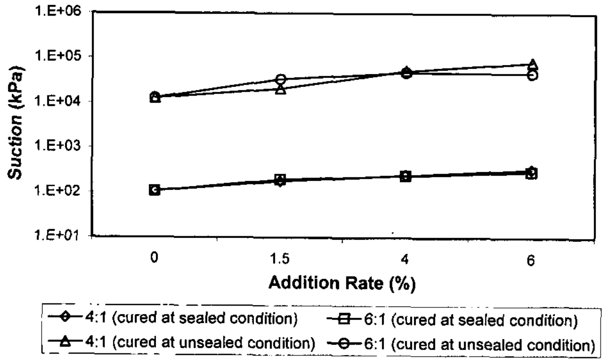

Fig. 6. Total suction values at different addition rates and dilution ratios

experienced higher suction than the untreated specimen. Specimens cured under the unsealed condition show much higher suction measurements than specimens cured under the sealed condition because they are much drier at the beginning of the test.

## 4.3 Dielectric Constant Measurement

The dielectric constant has recently been used as an indicator of the volumetric moisture content and the state of molecular bonding in granular materials (David and Annan, 1989). Unbound and stabilized granular materials with low dielectric constants normally have well-arranged and strongly adsorbed water molecules with high suction characteristic, and therefore seed potential strength properties. However, high dielectric constants indicate the presence of substantial free moisture in a material.

Among several methods for the dielectric constant measurement, so-called Tube Suction Test (TST) using a dielectric surface probe was used for this study. The tube suction test can be easily used to measure the moisture condition and the state of bonding of the water within the soil-aggregate system. The dielectric probe monitors the change of surface moisture due to capillary rise over time.

Limestone aggregate, which is natural or treated (6% of product content, 4:1 and 6:1 of dilution ratio), is compacted at 9 % of optimum moisture content in a 300 mm high and 150 mm diameter mold. The testing mold has 1 mm holes drilled at 13 mm spacing around the entire circumference of the cylindrical mold, approximately 6.4-mm from the bottom. Additional 1 mm holes are drilled in the base of the specimen. After drying each specimen, it was put in a reservoir of deionized water, and therefore the water penetrates the aggregate sample through the holes. Dielectric constant was measured six times at the different locations by the surface probe and the average of the four values except the maximum and minimum was recorded. The average value represents the moisture absorption and susceptibility of the aggregate samples over time.

Fig. 7 shows measurements of the cumulative amount of water absorbed for each specimen, untreated and treated with 6% and dilution ratios (4:1, 6:1) over time. It shows that treated specimens generally demonstrated less moisture absorption than the untreated specimen. Absorbed moisture is an indicator of moisture-absorption ability.

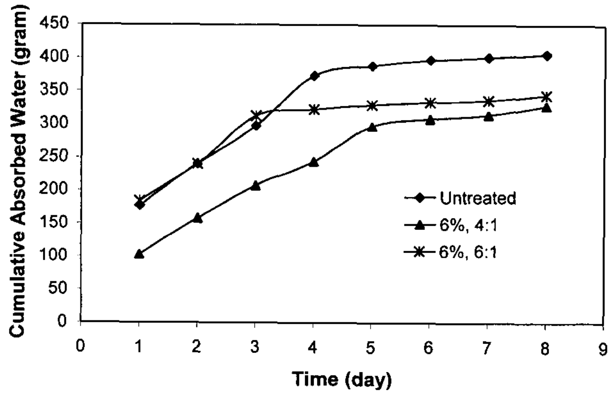

Fig. 7. Cumulative absorbed water amount over time

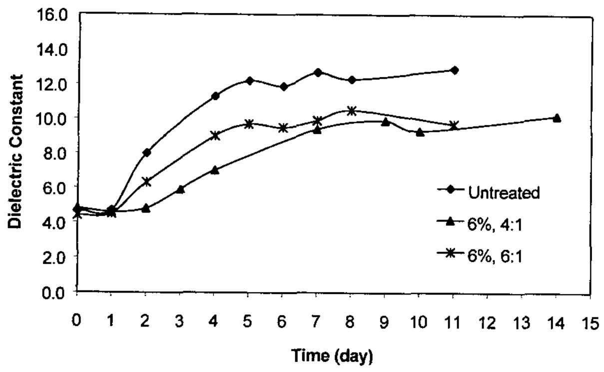

Fig. 8. Dielectric measurements over time

Dielectric constants were recorded and plotted in Fig. 8. The untreated specimen showed a rapid increase in dielectric constant and then approached a constant level thereafter. Lower dielectric constants were measured for treated specimens than for the untreated specimen. Based on the study by Scullion and Saarenketo (1995), a dielectric value below 10 is indicating a good condition of base materials. While materials having values above 10 may be susceptible to loss of stability and freeze-thaw damage due to significant levels of free water. From the results, the stabilizing agent treatment can be inferred to reduce permeability and/or moisture absorption sensitivity.

## 5. Estimating Shear Strength Parameters

Based on Lamborn's formulation for unsaturated soils and shear strength theory of the Mohr-Coulomb failure envelope, the changes in effective cohesion and effective angle of internal friction with different additive rates and dilution ratios were investigated using unconfined compressive strength data and suction data. For this analysis, matric suction was replaced by total suction due to difficulty in measuring the matric suction of aggregate base materials using filter paper method. Given an estimation of strength $(q_u)$, suction $(u_w)$, and volumetric moisture content $(\theta)$, the effective cohesion $(C')$ and the effective angle of internal friction $(\phi')$ can be calculated geometrically as illustrated in Fig. 9.

$$\phi' = \sin^{-1}\left(\frac{q_{u2} - q_{u1}}{2(\theta_1 u_{w1} - \theta_2 u_{w2}) + q_{u2} - q_{u1}}\right) \tag{5}$$

$$C' = \frac{q_{ui}}{2\cos\phi'} - \tan\phi'\left(\frac{q_{ui}}{2} - \theta_i u_{wi}\right) \tag{6}$$

where

$i$ = 1 or 2 (no summation).

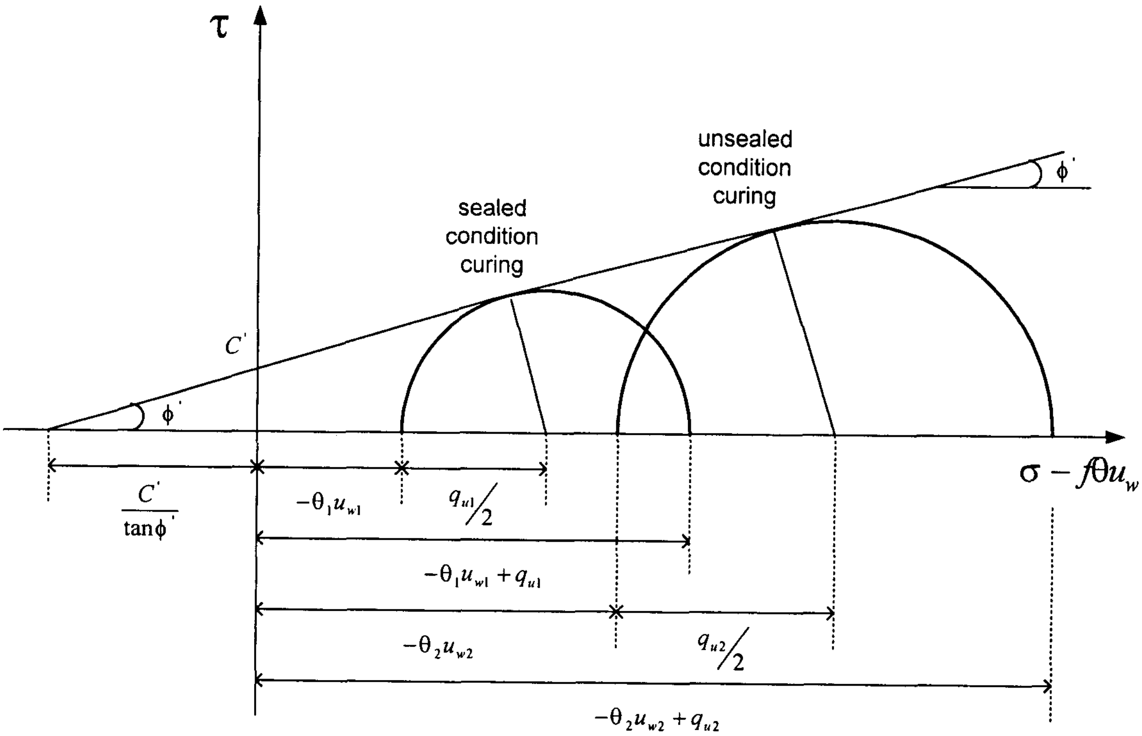

Fig. 9. Geometric illustration of Mohr-Coulomb failure envelope for determining the effective cohesion and angle of internal friction

Table 1. Summarized laboratory test data and resultant calculations of angle of internal friction and effective cohesion

| Product Content | Curing Condition | $q_u$ (kPa) | Suction (kPa) | $\theta$ | $-u_w$ (kPa) | $-f\theta u_w$ (kPa) | $C'$ (kPa) | $\phi'$ (degree) |
|---|---|---|---|---|---|---|---|---|
| Untreated | Sealed | 234.9 | 109.9 | 0.1705 | 109.6 | 18.7 | 26.25 | 46.99 |
| | Unsealed | 799.7 | 12912.2 | 0.009425 | 12998.2 | 122.5 | | |
| 1.5%, 4:1* | Sealed | 200.2 | 182.4 | 0.1710 | 184.0 | 31.5 | 3.09 | 46.28 |
| | Unsealed | 821.7 | 20941.1 | 0.007402 | 20887.4 | 154.6 | | |
| 4%, 4:1 | Sealed | 202.9 | 251.7 | 0.1712 | 254.0 | 43.5 | 36.35 | 28.54 |
| | Unsealed | 876.9 | 52601.7 | 0.007858 | 52466.6 | 412.3 | | |
| 6%, 4:1 | Sealed | 194.9 | 339.6 | 0.1714 | 342.7 | 58.7 | 47.34 | 19.70 |
| | Unsealed | 921.2 | 81470.4 | 0.009517 | 81261.3 | 773.4 | | |
| 1.5%, 6:1 | Sealed | 200.3 | 199.9 | 0.1711 | 201.8 | 34.5 | 4.95 | 45.73 |
| | Unsealed | 833.5 | 33962.5 | 0.004738 | 33875.3 | 160.5 | | |
| 4%, 6:1 | Sealed | 194.9 | 246.0 | 0.1715 | 248.2 | 42.6 | 21.51 | 31.83 |
| | Unsealed | 871.0 | 47973.3 | 0.007430 | 47850.1 | 355.6 | | |
| 6%, 6:1 | Sealed | 202.9 | 302.7 | 0.1717 | 298.5 | 51.2 | 29.20 | 29.76 |
| | Unsealed | 901.2 | 46881.3 | 0.008686 | 46760.9 | 406.1 | | |

Note: * dilution ratio with water (4 parts of water and 1 part of product concentrate).

Table 1 summarizes laboratory test results and corresponding calculations of $C'$ and $\phi'$ of each mixture with different additive rates and dilution ratios. Fig. 10 shows the effect of treatment on the effective internal friction angle. The estimated angle generally decreases with higher additive rates. Water-to-product ratio did not significantly affect the internal friction angle except at a 6% addition rate. Fig. 11 shows the effect of treatment on the effective cohesion. In general, the estimated effective cohesion increased as addition rate increased under the same dilation ratios. The effective cohesion of untreated specimens was higher than that of specimens treated with 1.5% due to lower compressive strength measured before curing condition. In addition, as expected, a dilution ratio of 4:1 showed higher effective cohesion than a dilution ratio of 6:1.

It is inferred that treatment with the PRB reduced friction resistance due to lubrication between aggregate particles and slightly increased cohesive strength because of the higher suction and thus the higher confining pressure. Accordingly, the net effect is to increase strength somewhat in spite of the lubrication of the particles.

The increase in suction potential after treatment and proper curing contributed to the decrease in moisture susceptibility as demonstrated by dielectric constant measurements. This is well-defined phenomenon as Gardner (1958) proposed an equation representing the relationship between the coefficient of permeability and the suction as follows:

$$k_w = \frac{k_s}{1 + a|h_T|^n} \tag{7}$$

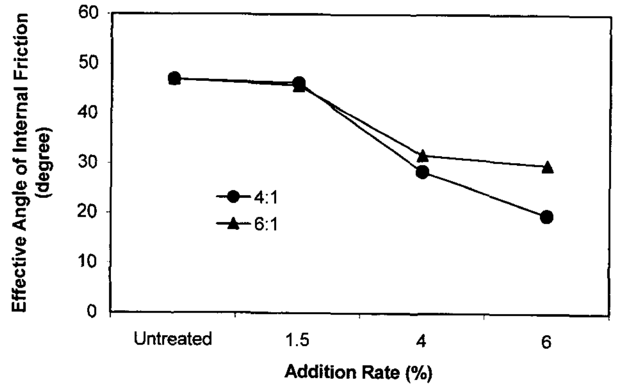

Fig. 10. Change of effective angle of internal friction at different addition rates and dilution ratios

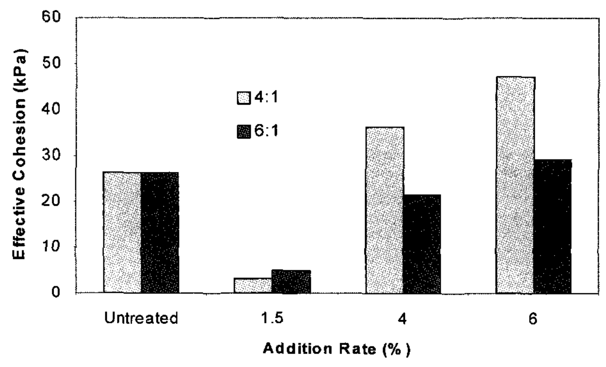

Fig. 11. Change of effective cohesion at different addition rates and dilution ratios

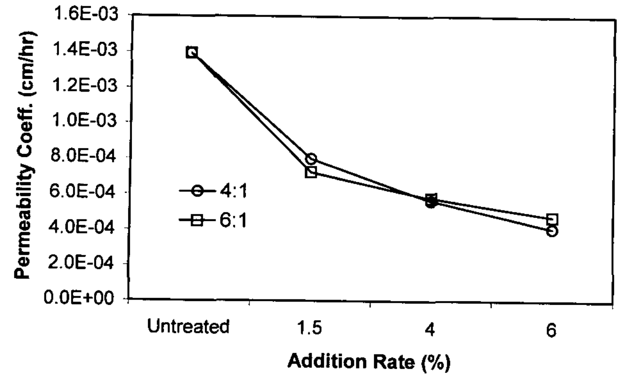

Fig. 12. Predicted permeability coefficients at different addition rates and dilution ratios

where

$k_w$ = coefficient of permeability for unsaturated soils,
$k_s$ = coefficient of permeability for saturated soils,
$a$ = constant from the relationship between the coefficient of permeability and the matric suction, breaking point of the function,
$n$ = constant from the relationship between the coefficient of permeability and the matric suction, slope of the function, and
$h_T$ = total suction.

The coefficient of permeability for each treated and untreated case can be predicted by assuming typical values of $k_s$, $a$, and n. Representative constant values for various soil types can be obtained from a study by Guymon et al. (1993). If the limestone aggregate was regarded as a well-graded gravel type, the permeability coefficients for unsaturated limestone with different additive rates can be predicted as shown in Fig. 12. The untreated specimen showed a higher potential for moisture susceptibility than treated specimens. In addition, permeability reduced as the additive rate increased.

## 6. Conclusions

Through various laboratory tests and strength analysis using unsaturated soil mechanics theory, stabilization mechanism from PRB treatment of unbound aggregates has been evaluated. Higher suction potential was recorded as the product additive rate increased. Higher suction results in an increase in tensile stress in the pore water and a consequent larger tensile force between soil particles. Treatment resulted in effective cohesion increase, less moisture susceptibility, and a reduction of effective angle of internal friction between aggregate particles. It can be thought that slight strength-gain and less moisture susceptibility by treatment results from the higher suction characteristics.

In consequence, the results demonstrated that the approach in evaluating soil-aggregate stabilization based on the unsaturated soil mechanics theory could be adopted due to its general basis. However, to further investigate the process of this approach, more detailed verification study should be made.

## Acknowledgements

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