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An Integrated Approach to Teaching and Learning College Mathematics

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The key features of our integrated approach to teaching and learning college mathematics include interactive and discussion-based teaching, small group work, computer as a tool, problem solving approach, open approach, mathematics in context, emphasis on mathematical thinking and creativity, and writing/communicating about mathematics. In this paper we report a few examples to illustrate the type of problems we use in our integrated approach.

1. INTRODUCTION

Several researchers have placed a great emphasis upon students' thinking, active learning, discovery learning, and interest in Mathematics (see Ahuja, Lim-Teo & Lee (1998), Andrew (1995), Choike (2000), Heid (1989) and Schroeder & Lester (1989)). That is why, the student-centered mathematical classrooms are now considered to be more effective in learning mathematics than the teacher-centered traditional classrooms. Also, Arcavi, Kessel, Meira & Smith (1998) supported a mathematical classroom culture, which requires the following characteristics:

1. Emphasize on processes as well as results,
2. Encourage and support various levels of oral and written mathematical communication,
3. Encourage and empower leadership and authority shared with students, and
4. Encourage reflective mathematical practice with thinking mathematically.

Schroeder & Lester (1989) suggested problem solving as an approach to learning mathematics. In this method, the teaching of a mathematical topic begins with a problem situation that embodies key aspects of the topic, and mathematical concepts and techniques are developed as reasonable responses to reasonable problems.

Some teachers in colleges and universities in the U.S. and elsewhere prefer the use of computer as a tool for teaching mathematics because of the availability of appropriate numerical, graphic, and symbolic capability of software (see Ahuja, Lim-Teo & Lee (1999a; 1999b), Ahuja & Jahangiri (2000), Heid (1998; 1999), Hickernall & Proskurowski (1985), Judson (1990), Kenelly & Eshinger (1998), Palmiter (1991)).

Some of them have revealed that by using the computer as a tool for performing the mathematical procedure, the students can be provided with an opportunity and time to work on real-life problems in mathematics. However, some of them agree that there should be less emphasis on algebraic manipulation skills and more emphasis on underlying concepts and mathematical thinking. Not many teachers feel comfortable to adopt a laboratory approach or the computer as a tool to teach mathematics. For example, Andrew (1995) reported, "Many of our students have pitiful skills in arithmetic, algebra and trigonometry. While our brightest might gain much from the Mathematica project, we must be vigilant lest the 'B–' or 'C+' students replace basic math skills with button pushing". Furthermore, Kenelly & Eslinger (1998) observed that many students blindly use computer software without understanding the underlying concepts and procedures.

Ahuja, Lim-Teo & Lee (1999a; 1999b) emphasized that calculus teachers should make use of the computer technology as a tool coupled with new and innovative pedagogical techniques, such as cooperative learning groups, students projects, writing/communicating about mathematics and real-life applications. Also, effective mathematics teaching requires understanding of what students know and need to learn and then challenging and supporting them to learn it well (NCTM 2000).

The main purpose of this paper is to report a few examples to illustrate the type of problems we use in our integrated approach in teaching and learning college mathematics. We highlight some of the ways we use to engage our students in a more meaningful and effective mathematical atmosphere. The key features of our approach include interactive and discussion-based teaching, small group work, use of computer as a tool, mathematics in context, problem solving approach, open approach, emphasis on mathematical thinking and creativity, and writing/communicating about mathematics.

2. BACKGROUND

The subjects of our project at the Kent State University are required by many departments of arts and science to take some courses in Developmental Mathematics, Beginning Algebra and College Algebra. These students are more interested in the use of mathematics than its justification or proofs of theorems. Because of the open admission policy, we get a great number of non-traditional students who have either forgotten everything and/or have math phobia.

Though some of these students have a keen desire to learn college mathematics, many of them feel like "captured students" who, much to their surprise and disappointment, are suddenly forced to study a subject they may have been fleeing for years, even if fresh out of high school; see Stein (1972). Through our observations and informed surveys, we have discovered that most of our students:

(i) Are mathematically unprepared — they have gaps in mathematical knowledge and understanding, poor recall and retention of mathematical knowledge.
(ii) Have perception of having rusty math skills.
(iii) Have a short attention span, poor attitude and lack of motivation.
(iv) Have a lack of meta-cognitive math skill.
(v) Are passive listeners and have mental blocks.
(vi) Have not enough time and show poor attendance patterns.
(vii) Have a kind of frustration and fear of failure, but
(viii) Have a keen desire to have a passing grade with very little efforts.

Such a diverse students with variety of their abilities and backgrounds are constrained by learned or acquired behavior patterns that inhibit advanced learning; see Fad & Ryser (1993) and Lim-Teo, Ahuja & Lee (2000). The traditional teaching method of chalk-talk-homework-exam does not work for such students. For example, see Ahuja, Lim-Teo & Lee (1999a; 1999b).

3. IMPORTANT FEATURES OF INTEGRATED APPROACH

An integrated approach to teaching and learning college mathematics offers an approach that is different from the traditional approach of chalk-talk-homework-exam. In this integrated approach we give more stress on the following ten points:

(i) Conceptual understanding <u>rather</u> than only computations.
(ii) Relational understanding <u>rather</u> than just instrumental understanding.
(iii) Exploring patterns and relationships <u>rather</u> than just memorizing formulas.
(iv) Variety of pedagogical strategies <u>rather</u> than just chalk and talk.
(v) Variety of non-traditional assessments <u>rather</u> than just traditional tests/exams.
(vi) Effective and meaningful learning <u>rather</u> than just learning for test/exams.

(vii) Listening (hearing, interpreting) to students' thinking rather than only telling (speaking, explaining).
(viii) Cooperative learning rather than just individualistic learning.
(ix) Making sense of mathematics using real life applications rather than just explaining abstract concepts.
(x) Helping students to develop an appreciation of the power of mathematics rather than a negative view of math.

For interactive and discussion-based teaching, new material is introduced either with a class discussion or via teacher-made worksheets. The teacher poses problems and questions for discussions or investigations. During group work, the teacher observes group interactions and their individual working on the computer or using paper and pencil After completion of group work, there is whole-class discussion and the teacher serves as a facilitator.

To create mathematical culture in the classroom we emphasize four key features:

(i) Various levels of math communication.
(ii) Processes as well as results.
(iii) Leadership and authority shared with the students, and
(iv) Reflective mathematical practice with thinking mathematically.

Derive 5 is used because it is user-friendly, easy to learn and it supports most of the concepts needed for the freshman and sophomore levels of college mathematics. All computers in our computer labs are equipped with the latest features and there are enough computers available to provide each student with at least a computer.

For group work, the class is divided into small groups of two or three members each. The students are allowed to choose their group members. During the group work, the teacher observes the group interactions and individual contributions. While observing the groups, the teacher checks their work, makes corrections, answers questions and provides motivation. In the class discussion that follows group work, the teacher generally serves as a facilitator.

The class size is generally kept between 16 and 20. We use a combination of quail-tative and quantitative instruments in gathering data. The students' performance is assessed by observations of student's presentations and discussions, students' worksheets, out-of-class assignments, pop-quizzes, class tests, and midterm and final examinations.

The evaluation of the effectiveness of the teaching style is done through formal assessments such as quizzes and tests, students' and teachers' weekly logs, and informal surveys and peer evaluations.

4. EXAMPLES HIGHLIGHTING INTEGRATED APPROACH

In this section, we present a variety of our classroom activities and examples that highlight our integrated approach to teaching and learning mathematics.

Using technology for sound pedagogical reasons

Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances a student's learning; see NCTM (2000). But we must use technology for sound pedagogical reasons. Here is an example that we often use in our College Algebra course:

Example 1: Use Derive to draw the graphs of some members of the family of functions $f(x) = x^n$, where

(a) $n = 1$
(b) $n = 2, 4, 6, \ldots$
(c) $n = 3, 5, 7, \ldots$

Analyze these graphs and write down similarities and differences between them. In other words, explain how does the value of n affect the shape of the graph?

Using technology for motivation

New material may be introduced either with a class discussion or via teacher-made worksheets. Technology can be very handy in motivating students for class discussion. For example, before we give properties of exponential functions, the following group work followed by class discussion is found very useful in students' learning new material.

Example 2: Using Derive, draw graphs, study the patterns and display as much information as possible about members of the following family of exponential functions:

(a) $f(x) = a^x$, where $a = 2, 3, 4, \ldots$
(b) $f(x) = a^x$, where $a = \frac{1}{2}, \frac{1}{3}, \ldots$

Using technology via problem solving in developing concepts

It is a well-known fact that students learn math more meaningfully when they can make sense of what they are talking about and they can connect ideas or skills they learn in new situations, in real-life and in other subjects. For example, the following worksheet followed by classroom discussion helped our students in understanding exponential functions.

Example 3: Annie decides to invest \$10,000 for x years in "Mary Investments Company" and she gets 10% per annum as the rate of interest. If the interest is compounded quarterly and $A(x)$ is the amount of money that she gets after x years, then:

(a) Write the formula for function $A(x)$.
(b) Use Derive and find $A(10)$. Explain the meaning of $A(10)$.
(c) Use Derive and find out how Annie's money will grow during the first twenty-one years.
(d) Use Derive and find out the amount at the end of years 7, 14, 21, 28 and 35.
(e) Plot the graph of $A(x)$.
(f) From (c) to (e), investigate the properties of $A(x)$.

Example 4: Suppose most of the banks in a county offer an interest rate of 10 percent per annum with a condition that an investor will open a fixed deposit of \$10,000 for 10 years. However, these banks offer different compounded periods ranging from annual to daily to minutely.

a. Use Derive and construct a table to show possible compounded periods n and the corresponding amounts after 10 years.
b. Study your table and write your observations.
c. What happens to the compound interest formula when n approaches to infinity?

Using technology for discovering facts

Example 5: Use Derive and sketch the graphs of the following functions. Observe changes in graphs as you draw. How do these graphs differ? In what ways are they similar?

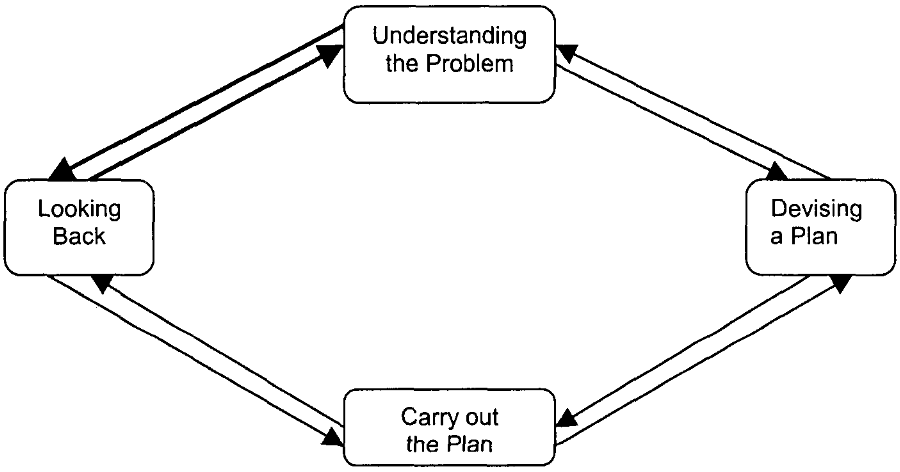
a. $y = x(x-2)(x-3)$ and $y = x^2(x-2)(x-3)$,
b. $y = x(x-2)(x-3)$ and $y = x(x-2)^2(x-3)$,
c. $y = x(x-2)(x-3)$ and $y = x^2(x-2)^2(x-3)^2$.

Using problem solving approach for sound pedagogical reasons

Problem solving is designed as a process by which an individual uses previously acquired knowledge, skills and understanding to satisfy the demands of an unfamiliar situation. The situation must synthesize what she or he has learned and apply it to new and different situations Krulik & Rudnick (1989). In our integrated approach, we generally select those problems that can;

(a) engage students in mathematical discussion,
(b) promote mathematical thinking,
(c) focus on the development of both cognitive and meta-cognitive strategies, and
(d) wherever possible, help students to learn math through problem solving.

Our main strategy of problem solving is small group work followed by class discussion. Also, we encourage students to use Polya's 4-step approach to problem solving as outlined in the above figure.



The following problem is from our Developmental Mathematics, a remedial course, which is not a credit course, but is a prerequisite for the Introductory Algebra course.

Example 6: (Handshake Problem) Steve gave a New-Year party. As his guest Linda arrives, he shakes hands with her. There was one handshake. When his second guest Andrew arrives, he shakes hands with Steve and then Linda. There were 1+2=3 handshakes.

(a) How many non-repeating handshakes can 10 guests have? Explain how you compute your answer.
(b) Draw a diagram showing relationship between number of guests and the number of handshakes.
(c) Is it easy to answer the questions in parts (a) and (b) above for 49 guests?
(d) Find a formula to compute the possible number of non-repeating handshakes if there were n guests at the party.
(e) Apply your formula in part (d) for $n = 2, 3, 10, 49$.
(f) At a New Year Party, everyone shook hands just once with everyone else. There were 510 handshakes in all. How would you figure out how many people were at

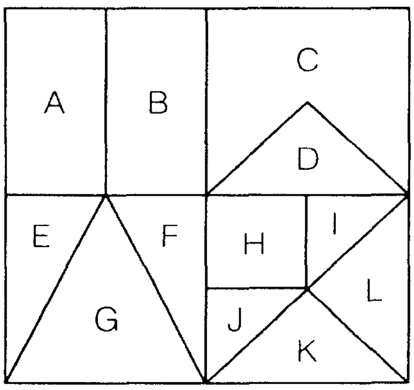
the party?

Using JAT Approach by Mathematical Connections

Mathematics makes sense and is easier to remember and apply when students can connect new knowledge to existing knowledge in meaningful ways; (see NCTM 2000). We have found that by using "Just At Time" (JAT) approach by mathematical connections helps students to meaningfully recall pre-requisite concepts and skills and connect them with new knowledge. For instance, we use the following example with several follow-up activities as 'Warm-Up Problem' (adapted from Choike (2000)) for our Introductory Algebra students.

Example 7: Consider the square given below which contains the smaller figures A through L. Suppose that the rectangles A and B are congruent, the right triangles D, K and L are congruent, and the area of A is twice the area of the square H.

a. Find the areas of figures A through L if the area of the largest square is 1.
b Find the areas of figures A through K if the area of L is 1.
c. Find the areas of figures A through L if the area of the largest square is x.
d. Make up two word problems that use the figures A through L, and solve them.



Using Open Approach

In this approach we select problems that exemplify a diversity of approaches to solving a problem or multiple correct answers. There are three aspects of the approach: open process, open-end product, and open problem formulation (see Becker & Shimada 1997). The main purpose of an open approach is to make mathematics alive and relevant and help students to develop divergent thinking.

Example 8: (a) Graph the following functions:

1) $y = x$	2) $y = x^2$	3) $y = x^3$
4) $y = -x$	5) $y = -x^2$	6) $y = -x^3$

(b) Write as many properties as you can that two or more of the functions have in common.

Example 9: (Choike 2000) List as many ways as possible to change a fifty-dollar bill into five dollar and/or twenty dollar bills.

a) Use trial and error approach and solve the problem.
b) If x is number of 5-dollar bills and y is number of twenty dollar bills, then
   (i) translate the word problem as an algebraic equation,
   (ii) draw the graph of the equation in (i) on a grid paper,
   (iii) read the solutions on the graph and write at least 10 solutions of the problem,
   (iv) how do you explain (–10, 5) as a solution? How many solutions are possible?

Example 10: Describe how to solve a quadratic equation and give a story problem to illustrate. Solve your story problem by using as many methods of solving a quadratic equation as possible.

Writing about Mathematics

The simple exercise of writing an explanation of how a problem was solved not only helps to clarify a student's thinking but also may provide other students with fresh insights gained from viewing the problem from a new perspective (NCTM 1989).

Example 11: Make up a word problem with two variables having a linear relationship Find a linear formula for these two variables. Try your formula for two hypothetical situations.

Example 12: One of your friends in College Algebra sends you an email and asks you to explain how to graph a polynomial function. Starting with quadratic polynomials, write a clear instruction to help her with her problem. Can you generalize this to cubic polynomial, and to polynomials of degree n? Consider cases when n is even or odd.

Example 13: Explain the difference between an equation and an algebraic expression.

Using puzzles and interesting examples for motivation

There are many websites and books that give variety of mathematical puzzles and games and some of them may be used as motivation prior to teaching a new concept or skills. Next example gives a well-known puzzle followed by motivation for some algebraic concepts.

Example 14: Add 5 to your age. Multiply the number by 2. Add 10. Then multiply by 5. Show me the final answer. I can find your age.

In order to help students to understand the math behind Example 14, we encourage students to make flow chart as shown in the figure below. Finally, we ask the following questions.

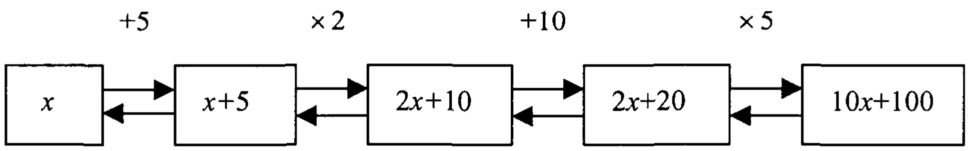
(a) If your age is x, then what is the algebraic expression for the problem above? Show the operations in the flow chart.

(b) Take away 100 from the algebraic expression and then divide by 10. Is this x, (that is your age)?

(c) Starting from the algebraic expression in (a), do backtracking: Divide it by 5, Subtract 10, Divide by 2, and then subtract 5. What do you get?

(d) What is the math behind the puzzle? Discuss this in your group and then report to the class.

Forward Tracking:



Backward Tracking

−5 /2 −10 /5

(e) Make a puzzle similar to Example 14 and explain the mathematical logic you used.

Using worksheets for understanding story problems and improving writing

The following example and accompanying worksheet from our college algebra classes show how a worksheet can serve as a medium that can permit students to follow sequence of steps in learning math.

Example 15: George Washington deposited \$1.00 in the Continental Bank in 1776. The account has paid 12% simple interest all these years. How much is in the bank account this year? He also deposited \$1.00 in the Bank of America in 1776. This account has paid him 9% interest compounded yearly for all these years. How much is in the bank account this year?

Worksheet: ***Part 1.*** Use Simple Interest Formulas: $I = \text{Prt}$ and $A = P + \text{Prt}$

Given: $P =$ ________, $r =$ ________, $t =$ ______________

Plug in the second formula and find amount: ________________________

Part 2. Write Compound Interest Formula: $A(t) = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$

Given: $P =$ $r =$ $t =$

Plug in the formula, find the equation, and simplify: ____________________

Write the answer in the language of the problem: ____________________

5. SUMMARY AND CONCLUSION

Traditionally, students learn college mathematics by going through three essential activities: attending lectures, studying textbooks, and doing exercises from their texts. In our integrated approach, we give more stress on ten points listed in Section 3 than on three essential activities in traditional method. As indicated by several examples in Section 4, our integrated approach includes not only ten points as mentioned in Section 3, but also having students perform various tasks of several types given in Section 4. Thus, we reduce lecturing in favor of students working on computers and using paper-and-pencil tasks.

The students' evaluations of teaching, feedback, and peer evaluation of the integrated approach showed that the students' attitude about the approach has been very positive. In their informal and formal feedback, the students perceived that various in-class activities and out-of class assignments helped them to gain a conceptual understanding rather than just memorizing the formulas. Comments such as: "In this class I learnt not only **how** to solve a problem, but the reason **why** I do it", "Now I think I deserve the grade that I get in this class because I understand what I am doing", "I hope my other classes will be like this class" were very encouraging and rewarding.

We also observed that students with diverse learning difficulties in math learn math by working actively in a relaxed and informal atmosphere in the classroom. They freely ask and answer questions. Many students maintained high level of interest in mathematical activities inside as well as outside the classroom. Students were also asked to express the extent of their agreement or disagreement with various statements on a scale of 1 (strongly disagree) to 5 (strongly agree). A majority of students agreed or strongly agreed with the statements: "I enjoy doing math problems", "In the long run math will help me", "I have learnt math through examples", "I can apply algebra in real-life problems", "I find the integrated approach as more useful to me than the traditional approach", etc.

We observed that the integrated teaching strategy along with daily homework and frequent quizzes and class tests played a significant effect on the students' understanding, satisfaction, as well as classroom retention. We also discovered that a relaxed and informal classroom atmosphere played an important part in students' learning. Teachers' help was readily available; students were encouraged to freely ask questions so that even a weak student could find the class meaningful and exciting.

Finally, the integration of computer technology along with the appropriate pedagogy created a significant impact on teacher and students. This approach evolved students from passive note takers to active learners. Students engaged in the classroom affairs in a meaningful and productive manner. It also helped students in developing conceptual understanding, confidence, enjoyment, and gave them problem-solving power. Since students generally enjoy the use of real-world mathematical problems along with integrated approach, it is worth experimenting such a non-traditional approach in teaching college mathematics.

There are certain limitations in using the integrated approach to teaching and learning mathematics. Firstly, the instructors need to spend a lot of time in preparing non-traditional activities and tasks. But, this limitation is not quite serious especially if we plan to prepare such activities over a period of say two to three semesters by teaching the same course. Another limitation is a lack of well-equipped computer laboratory and funds for math software. This can be overcome over a period of time by convincing administrators about the value of using technology in teaching and learning mathematics. Finally, some of the students may not like new pedagogical approaches and so they may express their dislike by giving lower scores on their evaluation. This negative reaction by some students can have serious implications for untenured and tenure-track faculty in those universities where students' evaluation is given too much weight in deciding renewals and promotions of faculty. In spite of these limitations, we have a fair amount of data, which suggest us that most students' attitudes about the new pedagogy are more positive than the students' attitudes who take courses with traditional pedagogy. In order to reduce negative reactions of students and enhance their learning, we suggest that the instructors must be sensitive to their students' concerns and convince them through persuasive arguments about the benefits they would get by using the integrated approach.

REFERENCES

Ahuja, O. P.; Lim-Teo, S. K. & Lee, P. Y. (1998): Mathematics Teachers' Perspective of Their

Students' Learning in Traditional Calculus and Its Teaching Strategies. *Journal of the Korea Society of Mathematical Education: Research in Mathematical Education* **2(2)**, 89–108. MATHDI **1999a.00058**

_____ (1999a): Teachers' Perspective on Non-traditional Approaches in Calculus Teaching. *Australian Senior Mathematics Teachers Journal* **13(2)**, 48–63.

_____ (1999b): *Challenges of Calculus Education in Singapore: Findings from the Calculus Project and Recommendations of the Calculus Conference 1998*, Division of Mathematics, National Institute of Education, Nanyang Technological University, Singapore, August, v+44 pages.

Ahuja, O. P. & Jahangiri, J. M. (2000): *How much technology "is too much?" — Integrated Approach to Teaching Mathematics*. One-hour invited talk at the 26th Annual American Mathematical Association of Two-Year Colleges Conference, Chicago (Nov 9–12).

Andrew, G. E. (1995): The irrelevance of calculus reforms: Ruminations of a sage-on-the-stage. *UME Trends* **6(6)**, pp. 17, 23.

Arcavi, A.; Kessel, C.; Meira, L. & Smith III J. P. (1998): *CBMS Issues in Mathematical Education* **7**, 1–70.

Becker, J. P. & Shimada, S. (1997): *The open-ended approach: A new proposal for teaching mathematics*, Translated from the 1977 Japanese version by Shigeru Shimada and Shigeo Yoshikawa. Reston, VA: The Council. MATHDI **1998b.01596**

Choike, J. R. (2000): Teaching strategies for "Algebra for all". *Mathematics Teachers* **93(7)**, 556–563.

Fad, K. S. & Ryser, G. R. (1993): Social/behavioral variables related to success in general education. *Remedial and Special Education* **14(1)**, 25–35.

Heid, M. K. (1989): How symbolic mathematical systems could and should affect pre college mathematics. *Mathematics Teacher* **82(6)**, 410–419. MATHDI **1990x.00034**

_____ (1998): Resequencing skills and concepts in applied calculus using the computer as a tool. *Journal for Research in Mathematics Education* **19**, 3–25. MATHDI **1988d.01773**

Hickernall, F. & Proskurowski, W. (1985): The use of microcomputers in teaching of calculus. *Collegiate Microcomputer* **3(2)**, 111–124.

Hodges, D. L. (1981): *How to teach effectively and get high student achievement and motivation*. Internal Report for Lane Community College, Eugene, OR. (ERIC Document Reproduction Service, No. ED 220151).

Judson, P. (1990): Using computer algebra systems in mathematics courses. *Journal of mathematical behavior* **9**, 153–158.

Kenelly, J. W. & Eshinger, R. C. (1988): Computer algebra systems. In: L. A. Steen (Ed.), *Calculus for a new century* (pp. 78–95). Mathematical Association of America, Washington, DC.

Krulik, S. & Rudnick, J. (1989): *Problem Solving: A handbook for senior high school teachers*.

Boston: Allyn and Bacon.

Lim-Teo, S. K; Ahuja, O. P. & Lee, P. Y. (2000): Attitude of Junior College and Tertiary students to Calculus. *PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies* **10 (2)**, 123–142. MATHDI **2000e.03545**

National Council of Teachers of Mathematics (NCTM) (1989): *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: NCTM. MATHDI **1996f.04386**

_____ (2000): *Principals and Standards for School Mathematics*. Reston, VA: NCTM. MATHDI **1999f.04754**

Palmiter, J. R. (1991): Effects of computer algebra systems on concept and skill acquisition in calculus. *Journal for research in Mathematics Education* **22**, 151–156. MATHDI **1992a.00850**

Schroeder, T. L. & Lester, F. K. (1989): Developing understanding in mathematics via problem solving. In: Paul R. Trafton & Albert P. Shuttle (Eds.), *New directions for elementary school mathematics* (pp. 31–56). Reston, VA: NCTM. MATHDI **1989k.01720**

Schuell, T. (1985): Knowledge representation, cognitive structure, and school learning: A historical perspective. In: West, L. & A. Pines (Eds.), *Cognitive structure and conceptual change* (pp. 117–129). Orlando, FL: Academic Press.

Stein, S. K. (1972): Mathematics for the captured students. *American Mathematical Monthly* **79(9)**, 1023–1032.

White, P. & Michelmore, M. (1996): Conceptual knowledge in introductory calculus. *Journal for Research in Mathematics Education* **27**, 79–95. MATHDI **1997a.00334**