796 KSME International Journal, Vol. 17 No. 6, pp. 796~804, 2003

Fatigue Crack Propagation Behavior in STS304 Under Mixed-Mode Loading

Sam-Hong Song
Department of Mechanical Engineering, Korea University, Seoul 136-701, Korea
Jeong-Moo Lee*
Department of Mechanical Engineering, Graduate School, Korea University, Seoul 136-701, Korea

The use of fracture mechanics has traditionally concentrated on crack growth under an opening mechanism. However, many service failures occur from cracks subjected to mixed-mode loading. Hence, it is necessary to evaluate the fatigue behavior under mixed-mode loading. Under mixed-mode loading, not only the fatigue crack propagation rate is of importance, but also the crack propagation direction. In modified range $0.3 \leq a/W \leq 0.5$, the stress intensity factors (SIFs) of mode I and mode II for the compact tension shear (CTS) specimen were calculated by using elastic finite element analysis. The propagation behavior of the fatigue cracks of cold rolled stainless steels (STS304) under mixed-mode conditions was evaluated by using K_I and K_{II} (SIFs of mode I and mode II). The maximum tangential stress (MTS) criterion and stress intensity factor were applied to predict the crack propagation direction and the propagation behavior of fatigue cracks.

Key Words : Mixed Mode, Fatigue Crack Propagation Behavior, Compact Tension Shear (CTS) Specimen, Loading Application Angle, Stress Intensity Factor (SIF), Finite Element Analysis (FEA), Maximum Tangential Stress (MTS) Criterion, Fatigue Life

1. Introduction

The stress conditions acting on a practical structure are complex, and thus most cracks existing in practical structures are under mixed-mode loading conditions. Therefore, the demand for research on cracks under mixed-mode loading has been increasing recently. Many researchers have been attempting to verify crack initiation and propagation criteria under mixed-mode loading conditions (Qian and Fatemi, 1996a). Most research on mixed mode has focused on the development of specimens and experimental method (Richard and Benitz, 1983), the determination of fracture toughness (Hallbäck and Nilson, 1994; Hong and Kang, 1994; Oh, 2002), the prediction of propagation direction (Erdogan and Sih, 1963; Sih, 1974) and the evaluation of crack propagation rate (Tanaka, 1974; Richard, 1986).

However, a standard examination method in research on mixed mode is not established yet. For that reason, it is difficult to compare the findings achieved by various experimental methods (Qian and Fatemi, 1996b). Since no standardized specimen geometries exist, the compact tension shear (CTS) specimen is frequently used to produce mixed-mode I/II loading under cyclic tension conditions. CTS specimen has advantages such that its shape is simple and the loading mode can be easily controlled using a loading device. However, a CTS specimen has a vulnerability in

* Corresponding Author,
E-mail : icrav215@korea.ac.kr
TEL : +82-2-3290-3353; **FAX** : +82-2-921-8532
Department of Mechanical Engineering, Korea University, 1, 5ga, Anam-dong, Sungbuk-gu, Seoul 136-701, Korea. (Manuscript **Received** August 5, 2002; **Revised** March 3, 2003)

shape at the connection with the loading device. Thereby it requires excessive notch, and we have some difficulties in high load tests, such as overloads, in specimen. Consequently, the experimental range in CTS specimens is limited comparing to the single edge notch tension (SENT) specimens. In general, the normalized crack size range from 0.44 to 0.7 has been examined using CTS specimen. Accordingly, the research on fatigue crack propagation behavior under mixed-mode loading in more general experimental conditions such as a short single edge notch is lacking. Therefore, research is needed to decrease the limitations in mixed-mode fatigue tests and to expand experimental conditions for CTS specimens.

In this study, we evaluate the fatigue crack propagation behavior under the mixed-mode I/II loading in a modified experimental range of CTS specimens. Detail contents in this study are as follows.

First, a stress intensity factor (SIF) formula is determined and its evaluation is executed for $0.3 \leq a/W \leq 0.5$ of CTS specimens.

Second, the relationship between mixed-mode loading conditions of crack tip and crack branch propagation angle was investigated. To predict the branch angle for propagating fatigue cracks, we examined the application of prediction criterion and its adequacy.

Third, we examined variation of SIF for mixed-mode fatigue cracks in the initiation and propagation stages of crack behavior. The effect of mode II component was confirmed on fatigue crack propagation behavior. In order to calculate SIF on arbitrary mixed-mode fatigue cracks, the application of the results of the crack evaluation method was examined for deflected propagating cracks.

2. Experiment and Finite Element Analysis

2.1 Experimental materials

The material used in the experiments was cold rolled stainless steel plates (STS304) ruled by KS D 3698. This material is mainly used for pipes of structures, connecting gaskets and reinforcement of structures. The chemical composition and mechanical properties of experimental material are given in Tables 1 and 2.

Table 1 Chemical composition of STS304

Composition (weight percent, %)				
C	Cr	Ni	Mn	Si
0.08	18.10	7.98	1.42	0.23

Table 2 Mechanical properties of STS304

Yield strength	MPa	286
Tensile strength	MPa	618
Hardness	Hv	198

2.2 Specimen and loading device

To execute the fatigue crack propagation experiment under mixed-mode loading, CTS specimens and loading device (Richard and Benitz, 1983) was used. The CTS specimens used in this experiment were revised in order to expand the experimental range in its machined notch length. The shape and dimension of specimens are shown in Fig. 1.

The modified loading device was based on the device designed by Richard and Benitz. A loading device which was able to make up various mixed-mode loadings in the specimen with uni-axial tensile load of fatigue tester by changing degrees of pin-hole was manufactured. Varying experimental conditions could be changed from mode I

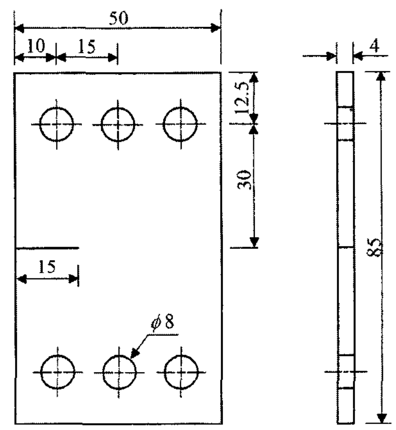


Fig. 1 Configuration of test specimen (unit : mm)

Table 3 Load conditions of fatigue tests

Load type	Tension-tension, constant amplitude		
σ_{max}	44 MPa	Stress ratio	0.1
Wave type	Sine curve	Frequency	10 Hz

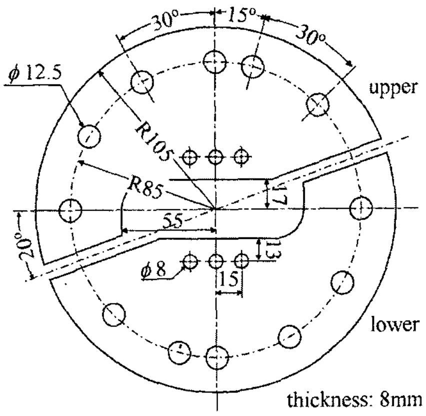
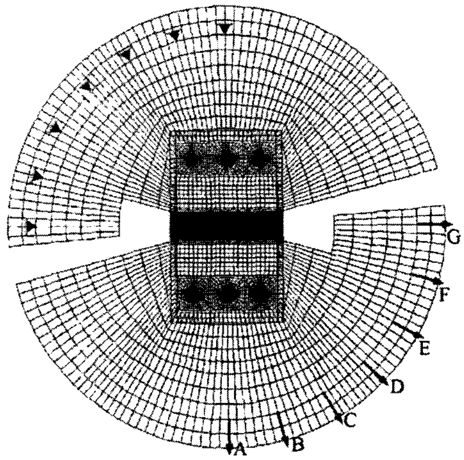


Fig. 2 Configuration of loading device (unit : mm)

to mode II using the loading device. The configuration and dimension of loading device are shown in Fig. 2.

2.3 Experimental method

The test machine used the present experiment was the INSTRON 1331, with electrical hydraulic tension-compression fatigue tester. The conditions of fatigue tests are shown in Table 3. The load states of the crack tip got into the phase of mode I and mixed mode I/II by changing of the loading application angle (ϕ). In the case that ϕ is 0°, the load state of crack tip is mode I, while in the cases that ϕ is equal to 30°, 45°, or 60°, the load states of the crack tip are under mixed-mode I/II loading conditions. The length of a fatigue crack was measured using a traveling microscope with a magnifying power of 100. The crack propagation positions were measured from the original point in an established absolute system of coordinates. The length of a fatigue crack was calculated by measuring x and y coordinates values. The propagation angle of a branched fatigue crack was defined as the angle between the measured crack position and the x-axis of a coordinate system.



Loading point : A, B, C, D, E, F, G
Mode I : A Mode II : G
Mixed Mode (I and II): B, C, D, E, F

Fig. 3 Finite element analysis model

2.4 Elastic finite element analysis

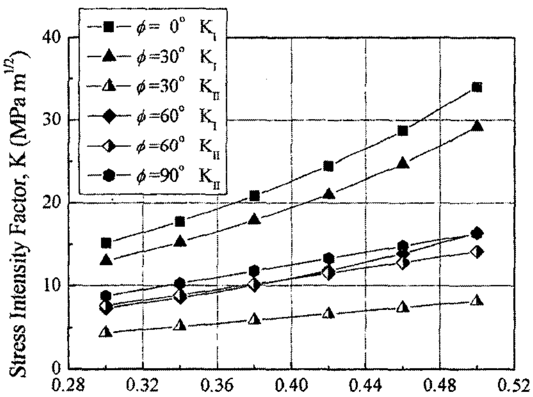
The SIF was calculated by 2-dimensional elastic finite element analysis using NISA II and ENDURE of EMRC. The stress state of specimen and loading device were supposed in 2-dimensional plane stress conditions. The modeling of the loading device was partly simplified for convenience in analysis. The finite element model consisted of 2-dimensional 8-node isoparametric elements and singular elements at the crack tip. The K_I and K_{II} were calculated by the crack tip opening displacement(CTOD) method (NG and Lau, 1997). The model and constraint conditions are represented in Fig 3. The appropriateness of modeling and accuracy of calculated results of SIF were examined by comparison between theoretical (Tada et al, 1973 ; Murakami, 1987) and FEM solutions of SENT specimen on K_I. Their results showed an error range of about -3% (Ong et al, 1989 ; Han, 1999).

The K_I and K_{II} for mixed-mode I/II loading were calculated according to the following procedure. The SIF was firstly calculated when increment of crack length in the same crack plane was equal to 2 mm for $0.3 \leq a/W \leq 0.5$. Then it was calculated at each a/W as ϕ was changed at 15° difference from 0° to 90°.

3. Test Result and Discussion

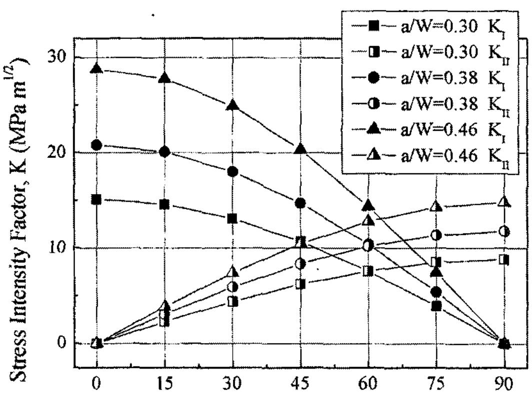
3.1 Determination of stress intensity factor equation for a modified range $0.3 \leq a/W \leq 0.5$

The SIF formula on CTS specimens was mainly investigated in the range of normalized crack size (a/W) from 0.44 to 0.70 (Richard and Benitz, 1983; Hallbäck and Nilson, 1994; Hong and Kang, 1996). The investigated range in this study is that normalized crack size a/W is the range from 0.3 to 0.5. The formula of SIF on mixed mode in this range was determined by elastic finite element and numerical analysis.



ϕ : loading application angle

(a) K variation with various ϕ



a/W : normalized crack size

(b) K variation with various a/W

Fig. 4 K variation in CTS specimen by FEA

The variations of calculated K_I and K_{II} are shown in Fig. 4. Since the variation of K shows consistent aspect about ϕ and a/W, the SIF formula on mixed mode can be obtained by a common function with two factors. It was reported that the SIF on the CTS specimen is an independent function of ϕ (Hallbäck and Nilson, 1994). The basic type is the same as Eq. (1), below,

$$K_I = \frac{P\sqrt{\pi a}}{Wt} \cos\phi \, f_I(a/W)$$
$$K_{II} = \frac{P\sqrt{\pi a}}{Wt} \sin\phi \, f_{II}(a/W) \qquad (1)$$

where P is applied load, a is crack length, and W and t are width and thickness of specimen, respectively. ϕ shows loading application angle. The basic type of Eq. (1) can be find in the SIF formula for $0.5 \leq a/W \leq 0.7$ (Richard, 1986; Hong and Kang, 1996).

The SIF formula for a modified test range of $0.3 \leq a/W \leq 0.5$ in this study could be arranged to a basic type that is the same as Eq. (2), below, where basic type of K_I is different from Eq. (1) for $0.44 < a/W \leq 0.7$:

$$K_I = \frac{P\sqrt{\pi a}}{Wt} \cos\phi \sqrt{\cos\frac{\phi}{3}} \, f_I(a/W)$$
$$K_{II} = \frac{P\sqrt{\pi a}}{Wt} \sin\phi \, f_{II}(a/W) \qquad (2)$$

Then $f_I(a/W)$ and $f_{II}(a/W)$ at Eq. (2) were obtained by numerical analysis as follows.

$$f_I(a/W) = 2.32158 - 14.36777(a/W) + 66.85752(a/W)^2 - 117.66921(a/W)^3 + 89.72502(a/W)^4$$

$$f_{II}(a/W) = -0.05741 + 4.36076(a/W) - 4.46168(a/W)^2 + 2.48807(a/W)^3$$

The deviation on K_I and K_{II} obtained from Eq. (2) was estimated by comparing with the results of finite element analysis. K_I showed difference below 4.2 % for $\phi \leq 60°$, for $60° < \phi < 90°$; when a/W is 0.3 the error was 19 % and when a/W is 0.4 the deviation was 9 %. The convergence was improved with increase of a/W. On the other side, K_{II} showed only deviation below 0.46 % for $0° < \phi < 90°$. This result was estimated that as loading application angle and shear load increased to short initial crack, K_I under mixed-mode loading was influenced by size effect owing to the edge of specimen. However, K_{II} was not influenced at $0.3 \leq a/W \leq 0.5$. The SIF for cracks under arbitrary mixed-mode loading could be calculated by using the SIF formula (2) with ease.

3.2 Propagation direction of fatigue cracks

3.2.1 Branch propagation angle of fatigue cracks

In this section, propagation aspect of cracks was examined by comparing the relationship between mixed-mode loading condition and crack branch propagation angle with variation of ϕ. Mixed-mode loading conditions at initial crack tip with variation of ϕ are represented as $K_I/(K_I+K_{II})$ by using K_I and K_{II} obtained from Eq. (2). These are shown in Fig. 5. From Fig. 5, it was confirmed that mixed-mode loading condition at each initial crack tip changes constantly with variation of ϕ. The branch propagation angle of fatigue cracks (θ_p) in this experiment was defined as the angle between propagated fatigue crack and initial crack when crack propagates up to 0.2 mm. As ϕ changes, the measured θ_p at a/W=0.3 are as follows. In the case of $\phi=30°$, θ_p were measured as 30°~32° which show differences of about −6 % for ϕ. In the case of $\phi=45°$, θ_p are 40°~43°. These show differences from about −7 % to −10 % for ϕ. In the case of $\phi=60°$, θ_p were measured to be 48°~54°, which show the differences from −10 % to −20 % for ϕ.

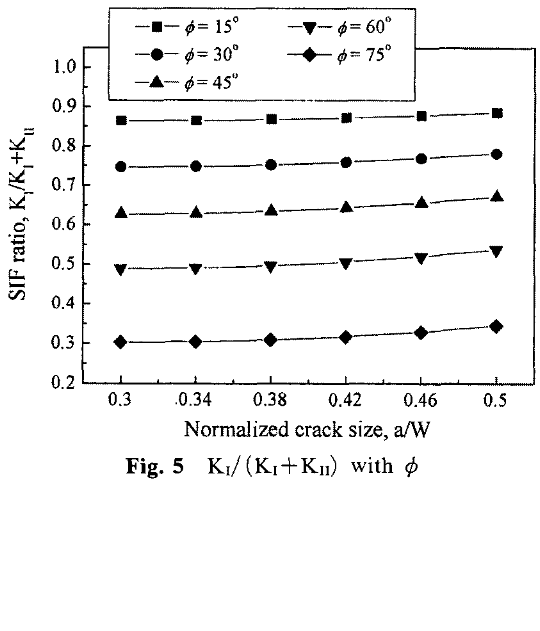


Fig. 5 $K_I/(K_I+K_{II})$ with ϕ

θ_p did not agree with ϕ. As mode II component increased, the difference between θ_p and ϕ increased. As θ_p have values of constant extent with variation of ϕ, it was known that there is a constant relationship between θ_p and mixed-mode loading state.

It is estimated that the experimental deviation of branch propagation angle increases as mode II component increases due to state of crack tip, material condition, deviation of loading transmission in experiment and complexity in mechanism of crack initiation. As a result, it was found that the fatigue crack under mixed-mode loading propagated toward direction which has constant range in branch propagation angle with stress state of initial crack tip.

3.2.2 The application of the prediction criterion

It was confirmed that θ_p can be predicted by using constant relationship between $K_I/(K_I+K_{II})$ and the crack propagation angle as described in the previous section. In this section, θ_p under mixed-mode loading were predicted by using prediction criterion based on stress. The comparison between prediction and experimental results gave information for properness in applying the prediction criterion. The applied prediction criterion used in this study is the maximum tangential stress (MTS) criterion (Erdogan and Sih, 1963).

MTS criterion assumes that fatigue crack propagation occurs in perpendicular to maximum principal stress σ_θ according to the stress components in the polar coordinate system as shown in Fig. 6. Hence θ_p subjected to mixed-mode loading can be calculated through the following formula :

$$\left(\tan\frac{\theta_p}{2}\right)_{1,2}=\frac{K_I}{4K_{II}}\pm\frac{1}{4}\sqrt{\left(\frac{K_I}{K_{II}}\right)^2+8} \tag{3}$$

The properness in applying the MTS criterion on prediction of propagation angle was investigated by comparing prediction with experiment. This was represented in Fig. 7. In the cases of

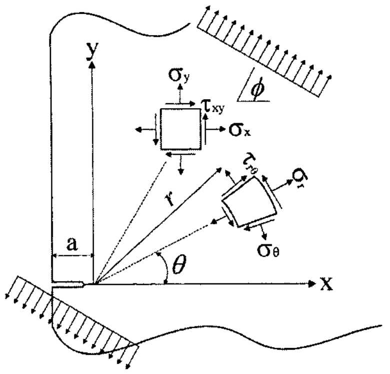


Fig. 6 Stress components near the crack tip

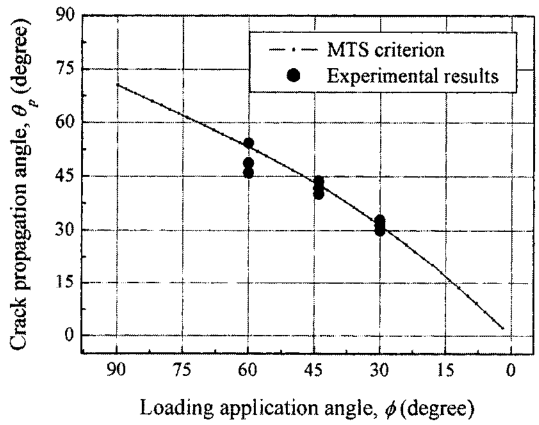


Fig. 7 Comparison with the MTS criterion and experimental crack propagation angle

$\phi=30°$ and $\phi=45°$, two results showed good agreement. However, when ϕ is 60° with a large mixed-mode ratio, the predicted result showed that it has wide deviation relatively. From this fact, it was reconfirmed that SIF ratio (K_I/K_{II}) with variation of ϕ is a governing factor for crack propagation angle. It was also clarified that the crack branch propagation angle of fatigue crack under mixed-mode loading could be predicted by using the MTS criterion.

3.3 The propagation behavior of fatigue cracks

3.3.1 The variation of stress intensity factor on propagating cracks

To investigate the propagation behavior of mixed-mode fatigue cracks, the variation of SIF

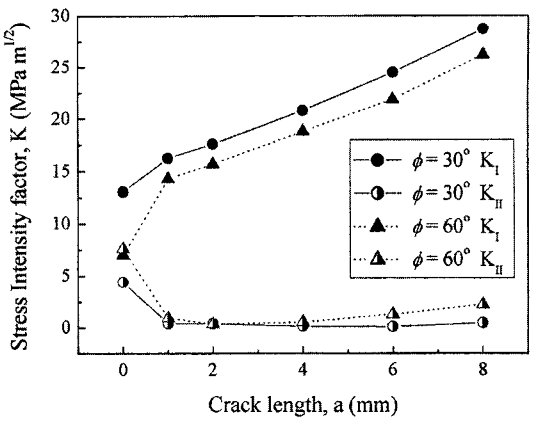


Fig. 8 K variation on crack propagation by FEA

in the crack propagation process was investigated in this section. To calculate K with propagation of cracks, the propagation paths of mixed-mode fatigue crack were reflected to finite element models. The analyses were performed for both models of $\phi=30°$(MC30) and $\phi=60°$(MC60). The reflected propagation path was modeled with the assumption that crack length was a straight line per 1 mm unit. Then K_I and K_{II} were calculated for the crack length propagating up to 1, 2, 4, 6 and 8 mm respectively. The comparison of calculated results was shown in Fig. 8.

The variation of K appeared in propagation process is as follows. In the case of $\phi=30°$, when the fatigue crack propagated up to 1mm from the initial crack tip, K_{II}/K_I decreased greatly from 0.34 to 0.025. When the crack propagated up to 8 mm K_{II}/K_I were determined to be 0.02. In the case of $\phi=60°$, K_{II}/K_I decreased rapidly from 1.09 to 0.065 and when the crack propagated up to 8 mm, K_{II}/K_I recovered to 0.08.

In the crack initiation process, K_I increased and K_{II} decreased suddenly. From this fact, it was found that the crack initiation occurred toward K_I dominant condition. K_I increased continuously as the crack length increased in the crack propagation process. K_{II} decreased rapidly but does not converge to 0 entirely, so K_{II} recovered gradually as the crack propagated. The larger the loading components of mode II are, the faster the recovery of K_{II} showed. Also, the effect ratio in propagation process increased gradually.

From the above, fatigue cracks under mixed-mode loading occurred toward K_I dominant conditions (Song et al, 2001 ; Tong et al, 1997). Also, the effect of K_I was kept consistent in the propagation process. K_{II} decreased rapidly in the crack initiation process and recovered gradually with propagation of cracks. K_{II} affected in the crack propagation process continuously. The effect of K_{II} was largely recovered in a relative manner under loading conditions of a large mode II component. Therefore, the effect of K_{II} in propagation process of fatigue cracks has to be reflected to evaluate of mixed-mode fatigue cracks continuously.

3.3.2 Fatigue life and evaluation method of mixed-mode fatigue cracks

In this section, the behavior aspect evidenced by the K_{II} effect was confirmed in crack initiation and propagation processes. In the previous section, the variation of K_I and K_{II} examined by finite element analysis reflected the propagation path. In this section, the calculation method of K for mixed-mode cracks is investigated, where the crack propagates toward an arbitrary position from the initial crack plane. To calculate K, the evaluation method of the crack that was propagating toward deflection direction from the initial crack plane was examined.

K_{II} effect was confirmed by comparing of initiation and propagation life shown as ϕ changes. The initiation life and propagation life were defined as a number of cycles when the crack propagates up to 0.1 mm and 10 mm. The above definitions were made by considering the observed ease of crack and deformation degree of specimen. The initiation and propagation life of cracks are shown as in Fig. 9. Fatigue cracks under mixed-mode loading initiated about 10~30 % faster than the cracks under the mode I conditions. Also, the propagation life of mixed mode decreased compared to mode I condition ($\phi=0°$).

In the cases of $\phi=30°$, 45° and 60°, the propagation life showed about 20 %, 50 %, and 60 % reduction in regular order, respectively. The initiation and propagation lives of mixed-mode fatigue cracks decreased compared to life in mode I condition. As mode II loading component increased, the reduction ratio of propagation life became larger. It was confirmed that mode II component induced increase of propagation rate in propagation behavior of mixed-mode fatigue cracks.

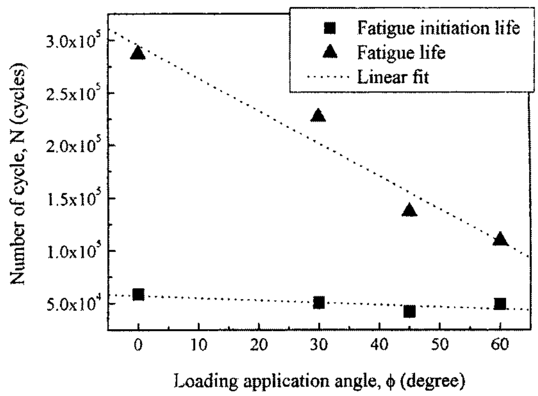
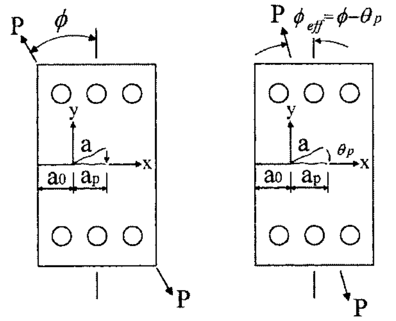


Fig. 9 The fatigue life with the variation of loading application angle



(a) projected length (b) rotating length

a_0 : machined notch length or pre-crack length

a_p : projected crack length

θ_p : crack propagation angle

θ_{eff} : effective loading application angle

Fig. 10 K evaluation on crack propagation

The propagated fatigue crack under mixed-mode loading deflected from the initial crack plane and was situated in an arbitrary position. The crack evaluation method has to be considered in calculation of SIF on a mixed-mode fatigue crack that deviates from the initial crack plane due to a/W factor. There are crack evaluation methods on mixed-mode fatigue cracks represented in Fig. 10. The method applied in

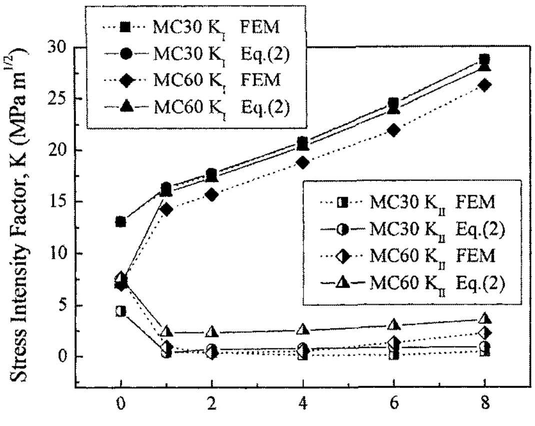


Fig. 11 Comparison of K variation

this study is method (b) shown in Fig. 10. The method (b) applies actual crack length, which rotates to the x axis of the initial crack plane and effective loading application angle (ϕ_{eff}).

The validity of this evaluation method was verified by comparing two SIFs calculated from the evaluation method and finite element analysis. The comparison of two results is shown in Fig. 11. In the case of $\phi=30°$, K_I agreed with each other, while in the case of $\phi=60°$, the difference of K_I showed within 10 %. However, K_{II} showed great difference in both cases. The crack evaluation method that applies actual crack length and ϕ_{eff} caused overestimated results of K_{II}. When K_{II} compared with K_I, K_{II} occupied about 2 % and 6 % in proportion of SIF in cases of $\phi=30°$ and 60°. The improvement of evaluation methods for mixed-mode fatigue crack is necessary because K_{II} consistently has an influence on propagation behavior of crack.

From above results, we could see that the initiation and propagation lives of cracks are decreased by increasing mode II loading component as ϕ increases. It was confirmed that the crack evaluation method has to be considered in calculation of K for mixed-mode fatigue cracks which were propagating toward an arbitrary position. Because the method that applies actual crack length and effective load application angle produces over estimation of K_{II}, we could know that the method has to be improved.

4. Conclusions

In this study, the mixed-mode fatigue test and elastic finite element analysis for the revised experimental range in CTS specimen were executed. The conclusions obtained from the facts for propagation behavior of mixed-mode fatigue crack are as follows.

(1) To remove limitations in mixed-mode fatigue tests and expand experimental conditions for CTS specimens, the investigation was executed in the modified experiment range of $0.3 \leq a/W \leq 0.5$. The SIF formula on mixed-mode loading in $0.3 \leq a/W \leq 0.5$ could be expressed as follows.

$$K_I = \frac{P\sqrt{\pi a}}{Wt} \cos\phi \sqrt{\cos\frac{\phi}{3}}\, f_I(a/W)$$

$$K_{II} = \frac{P\sqrt{\pi a}}{Wt} \sin\phi\, f_{II}(a/W)$$

$$f_I(a/W) = 2.32158 - 14.36777(a/W) + 66.85752(a/W)^2 - 117.66921(a/W)^3 + 89.72502(a/W)^4$$

$$f_{II}(a/W) = -0.05741 + 4.36076(a/W) - 4.46168(a/W)^2 + 2.48807(a/W)^3$$

(2) Fatigue cracks under mixed-mode loading propagated toward deflected direction from initial crack plane depending on the stress state that is governing the crack tip. The propagation angle of fatigue crack branching did not agree with the loading application angle. The branch propagation angle of mixed-mode fatigue cracks could be predicted by using the maximum tangential stress criterion. The prediction results have wide error scattering as mode II loading component increases. Hence, when mode II components increase, it is found that the importance of the prediction for crack propagation direction become larger.

(3) Fatigue cracks under mixed-mode I/II loading propagated in the direction that K_I increases and K_{II} decreases suddenly from initial crack tip. Then it becomes a condition that mode I is dominant but K_{II} does not converge to 0, so K_{II} was recovered as crack propagates gradually. K_{II} has a continuously influence on the propaga-

tion process of fatigue cracks.

From this fact, increase of mode II component induced the reduction in crack initiation and propagation life. It was shown that the crack evaluation method has to be considered in calculation of K for mixed-mode fatigue cracks which were propagating toward an arbitrary position. Because the method that applies actual crack length and effective load application angle produces overestimation of results, we confirmed that the method has to be improved.

References

Erdogan, F. and Sih, G. C., 1963, "On the Crack Extension in Plates Under Plane Loading and Transverse Shear," *Journal of Basic Engineering*, Vol. 85, pp. 519～525.

Hallbäck, N. and Nilson, F., 1994, "Mixed Mode I/II Fracture Behavior of an Aluminum Alloy," *Journal of the Mechanics and Physics of Solids*, Vol. 42, pp. 1345～1374.

Han Moon Sik, 1999, "A Numeric Modeling Technique for the Shape Development of Fatigue Crack," *Transaction of the KSPE*, Vol. 16, No. 2, pp. 225～233.

Hong, K. J. and Kang, K. J., 1996, "Method to Measure K_I, K_{II} and J-Integral for CTS Specimen under Mixed Mode Loading," *Transaction of the KSME*, Vol. 20, No. 1, pp. 3498～3506.

Murakami, Y., 1987, "Stress Intensity Factors Hand book," Pergamon Books, pp. 9～10.

NG, S. W. and Lau, K. J., 1997, "Determination of Mixed-mode Stress Intensity Factors Using a Semi-Infinite Crack Model," *Engineering Fracture Mechanics*, Vol. 58, No. 1/2, pp. 43～59.

Oh Dong-Joon, 2002, "Ductile Fracture Behavior of AS4P Under Mixed Mode (I/II) Loading," *KSME International Journal*, Vol. 16, No. 4, pp. 476～484.

Ong Jang Woo, Jin Keun Chan, Lee Sung Geun and Kim Jong Bae, 1989, "Mixed-Mode Fatigue Crack Growth Behaviors in 5083-H115 Aluminum Alloy," *Transaction of the KSME*, Vol. 13, No. 3, pp. 461～471.

Qian, J. and Fatemi, A., 1996, "Mixed Mode Fatigue Crack Growth : A Literature Survey," *Engineering Fracture Mechanics*, Vol. 55, pp. 969～990.

Qian J. and Fatemi A., 1996, "Fatigue Crack Growth under Mixed Mode I and II Loading," *Fatigue & Fracture of Engineering Materials & Structures*, Vol. 19, pp. 1277～1284.

Richard, H. A. and Benitz, K., 1983, "A Loading Device for the Criterion of Mode in Fracture Mechanics," *International Journal of Fracture*, Vol. 22, pp. R55～R58.

Richard, H. A., 1986, "Some Theoretical and Experimental Aspects of Mixed Mode Fracture," *Advances Fracture Research*, Vol. 5, pp. 3337～3344.

Sih, G. C., 1974, "Strain Energy Density Factor Applied to Mixed Mode Crack Problems," *International Journal of Fracture*, Vol. 10, pp. 305～321.

Song Sam-Hong, Bae Jun-Soo and Choi Byoung-Ho, 2001, "An Experimental Study on the Fatigue Behavior and Stress Interaction of Arbitrarily Located Defects (II) (For Variable Loads and Distances between Defects)," *Transaction of the KSPE*, Vol. 18, No. 1, pp. 201～212.

Tada, H., Paris, P. and Irwin, G., 1973, "The Stress Analysis of Cracks Handbook," Del Research Corporation, pp. 2～11.

Tanaka Keisuke, 1974, "Fatigue Crack Propagation from a Crack Inclined to the Cyclic Tensile Axis," *Engineering Fracture Mechanics*, Vol. 6, pp. 493～507.

Tong, J., Yates, J. R. and Brown, M. W., 1997, "The Formation and Propagation of Mode I Branch Cracks in Mixed Mode Fatigue Failure," *Engineering Fracture Mechanics*, Vol. 56, No. 2, pp. 213～231.