FINITE ELEMENT MODELING FOR HYDRODYNAMIC AND SEDIMENT TRANSPORT ANALYSIS (II) : SEDIMENT TRANSPORT STUDY

Joon-Woo Noh

Department of Civil Engineering, Colorado State University, Ft. Collins, Colorado, 80523 USA

Abstract: Since bed elevation changes are mainly dependent on the flow velocity and corresponding shear stress, it is possible to predict bed elevation numerically using velocity components. For the scour analysis due to channel contraction, a bed load transport model is developed and applied to estimate scour depth around coffer dam in the Mississippi River. During Phase I of the Lock & Dam No. 26 replacement project, a coffer dam was constructed to reduce the flow area approximately by 50 %. Flow velocity increases due to the flow area reduction yields significant lowering (erosion) of the channel bed elevation. The proposed numerical model solves the sediment continuity equation using the finite element method to evaluate scour process in the vicinity of the coffer dam

---

Keywords: finite element method, sediment transport, scour, channel contraction

1. INTRODUCTION

Channel contractions result in the reduction of flow area. In channel contraction zones, due to the local increase in velocities, shear stresses, vortices, and turbulence, bed topographies are subject to significant changes. The combination of these factors leads to the removal of channel bed material from the foundations of these hydraulic structures and the development of local scour holes. Flow area reduction may occur due to the natural river geometry or due to the presence of manmade hydraulic structures. Therefore, the determination of flow properties in channel contractions is very important in the design and construction of various hydraulic structures.

Past studies on the scour problem were focused on the high shear zones around bridge abutments and groins. Tingsanchali and Maheswaran (1990) incorporated streamline curvature for the shear stress computations near the tip of groins. Their computational results were verified using the experimental data provided by Rajaratnam and Nwachukwu (1983). Ouillon and Dartus (1997) investigated flow and shear stress distributions around groins using three-dimensional k-ε model adopting porosity method to method to track the free surface. Molinas et al. (1998) investigated shear stress on the channel bottom around short abutments and derived an equation for shear stress amplification factor.

Flow velocity increase due to the flow area reduction yields significant lowering (erosion) of the channel bed elevation around the coffer dam. The proposed numerical model in this paper solves the sediment continuity equation using the finite element method. In the algorithm, the concept of equilibrium scour is taken into account by incorporating the flow continuity equation into the shear stress computation. Also, lumped matrix formulation was incorporated to the model in an effort to achieve the benefit of matrix diagonalization. Through the comparison with measured data, the model is verified and its applicability is evaluated.

2. GOVERNING EQUATIONS

Research on the prediction of bed forms involves both theoretical and empirical approaches. This study is based on recent work that mainly covers theoretical approaches. From the sediment continuity equation, theoretical approach can be explained from the flow over dunes, which is shown in Fig. 1.

The governing equation is given as

$$\gamma_s \frac{\partial z}{\partial t} + \frac{\partial q_s}{\partial x} = 0 \tag{1}$$

where γ_s = specific weight of sediment, z = bed elevation, and q_s = sediment discharge per unit channel width

Tanguy et al. (1989) predicted bed elevation distributions for a channel with a dike. Solving the 2D shallow water equation along with sediment continuity equation, they developed a numerical model using the finite element method. The governing equation except the unsteady term is diagonalized for easy matrix manipulation using modified shape function. Brørs (1999) conducted a numerical simulation of scour development under the submerged pipelines. The flow distributions are computed using finite element rigid-lid model by solving Navier-Stokes turbulent flow equations with k-ε closure and using the finite difference scheme, the following sediment continuity equation is solved for 1 and 2D as

$$\frac{\partial z}{\partial t} = \frac{1}{1-\lambda}\left[-\frac{\partial q_i}{\partial x_i} + D - E \right] \qquad i = 1, 2 \tag{2}$$

where λ = porosity, D = the rate of which sediment is deposited, and E = the rate of sediment

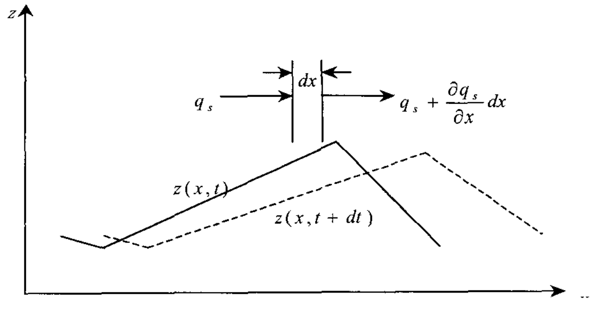


Fig. 1 Typical bed form transition to the downstream direction

erosion.

The bed load transport equation is given in terms of critical shear stress, shear stress, and particle diameter. Also, there is a close relationship between scour process and flow depth. The flow depth increase as a result of scour reduces corresponding shear stresses. Then the scour process gradually decreased until the equilibrium state is met in which scour process discontinues. Conversely, in the deposition zone, due to decrease in total flow depth, a minor scour occurs until sediment transport reaches an equilibrium state. The changes in bed elevation can be mathematically described by solving the sediment continuity equation. In this paper, the sediment continuity equation is solved by using the finite element method and the results of quantitative analysis are presented.

3. MODEL DESCRIPTIONS

The bed elevation change can be described by two-dimensional sediment continuity equation given as follow

$$(1-\lambda)\frac{\partial z}{\partial t}+\frac{\partial q_{sx}}{\partial x}+\frac{\partial q_{sy}}{\partial y}=0.0 \tag{3}$$

where, $\lambda =$ porosity, $Z =$ bed elevation, $q_{sx} =$ the amount of sediment due to the x-component of the flow velocity, and $q_{sy} =$ the amount of sediment due to the y-component of the flow velocity.

The finite element equation is

$$\delta J_e = \int_{Ve}\lfloor \delta z \rfloor\{N\}\lfloor N \rfloor\{\dot{z}\}dV + \int_{Ve}\lfloor \delta z \rfloor\{N\}\left[\frac{\partial N}{\partial x}\right]\{q_{sx}\}dV + \int_{Ve}\lfloor \delta z \rfloor\{N\}\left[\frac{\partial N}{\partial y}\right]\{q_{sy}\}dV \tag{4}$$

where, N = shape function, δz= weighting function, and V = the volume with unit height

In order to obtain $\{\dot{z}\}$ from Eq. (4), it is necessary to invert the capacitance matrix $[C]$. Eq. (4) for a specific time step t is given in abbreviated matrix form as

$$[C]\{\dot{z}\}_t = -\{Q'\}_t \tag{5}$$

As it can be seen from the above equation, if the capacitance matrix $[C]$ is diagonal, $\{\dot{z}\}$ can be easily obtained by back substitution. In an effort to achieve this convenience, several methods are available for capacitance matrix diagonalization.

This method diagonalizes the matrix by replacing each diagonal with the sum of the terms on its row. Thus,

$$\begin{aligned} \mathrm{C(I,I)} &= \sum_{J=1}^{NNPE}\int_{Ve} N_I \mu N_J\, dV \\ \mathrm{C(I,J)} &= 0, \qquad \mathrm{I} \neq \mathrm{J} \end{aligned} \tag{6}$$

where μ = an arbitrary coefficient of the capacitance matrix

Because the sum of the shape functions always add to be unity, the above gives

$$\mathrm{C(I,I)} = \int_{Ve} N_I \mu\, dV \tag{7}$$

Hence summing rows results in lumping the distributed capacitance at the nodes in exactly the same manner that we have lumped distributed sources at the nodes.

4. MODEL VERIFICATION

Using the experimental results given by Tanguy et al. (1989), the bed load transport model is verified. The parameters and experimental configurations for model verification are given as follows. The channel width is 1.85 m, channel length is 20 m, height of the groin is 0.6m, flow discharge is 60 l/s, depth of the flow is 0.2 m, bed layer thickness is given as 0.2 m, respectively. From the geometrical condition of the flume, the contraction ratio of the flow area is approximately 30 %. The computational result for the flow distribution and the zone of analysis is shown in Fig. 2. In this case, the groin is located at 3m away from the channel inlet.

The mean diameter of the sand covering the bed is given as 0.32 mm. Using van Rijn's bed load transport equation, the amount of sediment that can be transported is determined from

$$q_b = 0.053\left[\left(\frac{\rho_s}{\rho} - 1\right)g\right]^{0.5} d_{50}^{1.5} d_*^{-0.3} T^{2.1} \tag{8}$$

where $d_* = d_{50}\left[(s-1)\frac{g}{\nu_m^2}\right]^{\frac{1}{3}}$, $T = \frac{\tau - \tau_c}{\tau_c}$, and $\nu_m =$ viscosity of the mixture of fluid and sediment

The critical shear stress τ_c can be computed from

$$\tau_c = 0.047(\rho_s - \rho)gd_{50} \tag{9}$$

The shear stress corresponding to the flow velocity and the flow depth is given as

$$\tau = \rho g \frac{n^2|V|V}{(H+h)^{1/3}}\left(\frac{H}{h}\right)^2 \tag{10}$$

where n is the Manning roughness coefficient and can be given from $n = d_{90}/26$ following Strickler's formula; H is mean flow depth; and h is total flow depth.

The channel bed elevation is assumed as 70 m. The plan-view contour with 3-dimensional wire frame is shown in Fig. 3 at the end of numerical simulation. The maximum scour depth at the nose region is computed as 8.9 cm, while the maximum scour depth measured in the experiment is 11 cm according to Tanguy et al. (1989). Considering the complexity of flows at the nose region (truly 3D) and the fact that the present model is not a truly 3-dimensional model, the discrepancy of 20% is encouraging. Away from the local maximum scour region, the agreement was much closer to the measured values. Since the focus of this study is not the nose region, the results are acceptable.

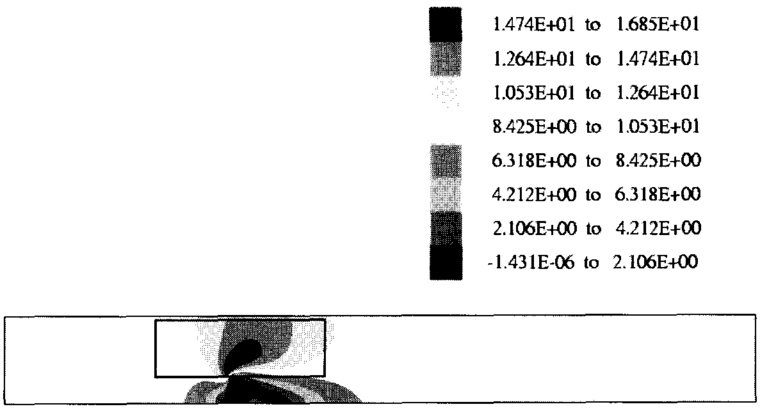


Fig. 2 The zone of analysis and velocity distributions

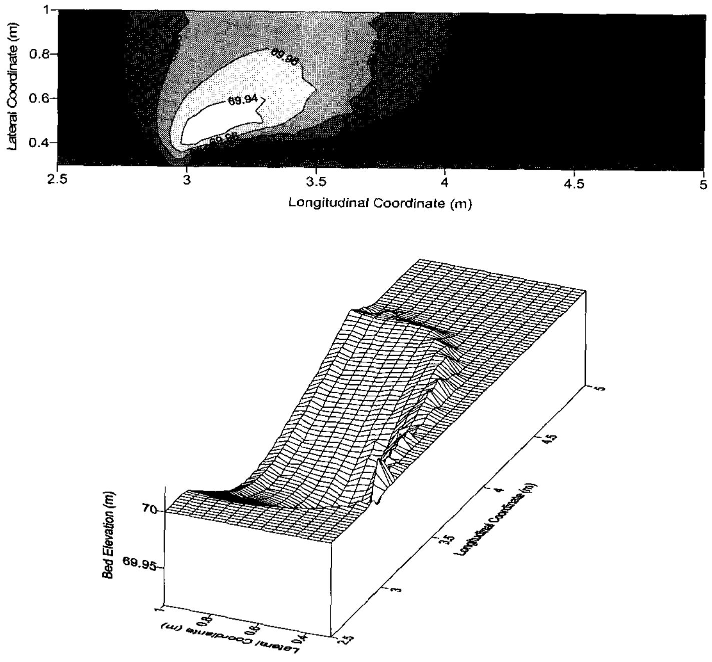


Fig. 3 Channel bed topographies in the contracted reach

5. MODEL APPLICATION

Applying the bed load transport model to a natural river case, numerical simulation on the bed load transport is conducted. In this example, the model application is done for a reach in the Mississippi River located near St. Louis, Missouri and Hartford, Illinois (shown in Fig. 4). The length of study reach is approximately 6520 m and the average channel width is about 460 m. Fig. 5 shows finite element mesh provided by the mesh generation program. The mesh domain is composed of 3794 elements and 3980 nodal points. The boundary condition of the modeling case is when the flow discharge is 10,000 cms: The approach average velocity is 1.93 m/sec, shear velocity is given as 0.11 m/sec, and the average flow depth is approximately 12.5m.

Fig. 6 shows the velocity distribution of the study reach. The coffer dam reduces the flow area by almost 50% of the average channel width. Even for this severe contraction, the present hydrodynamic model successfully solves the motions of flow. The maximum total velocity through the entire reach is computed as 5.40 m/sec; that is more than 2.5 times to the initial approach flow velocity.

The contraction ratio of the study reach due to the coffer dam is approximately 50% and most of scour process occurs in the contracted zone. The computational time step is chosen as 100 seconds and the results are plotted for 1 day (24 hours), 3 days (72 hours), and 10 days from the initial run. Since the present analysis is focused on channel contractions, the result is shown in Fig. 7(a), (b), and (c) respectively pertains to contracted zone. The initial bed elevation is assumed to be 361 feet and after 24 hours, the maximum scour depth is computed as 5.13 ft as

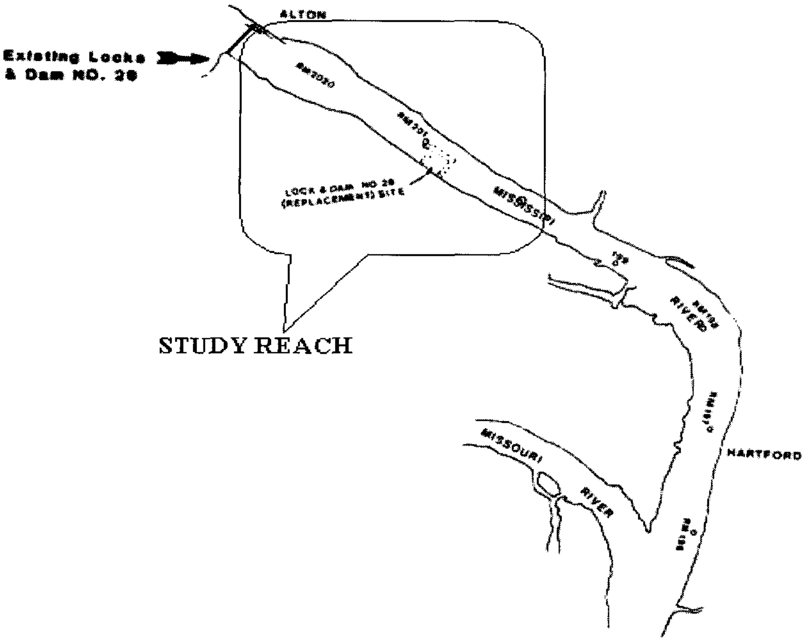


Fig. 4 Location and plan view of the study reach

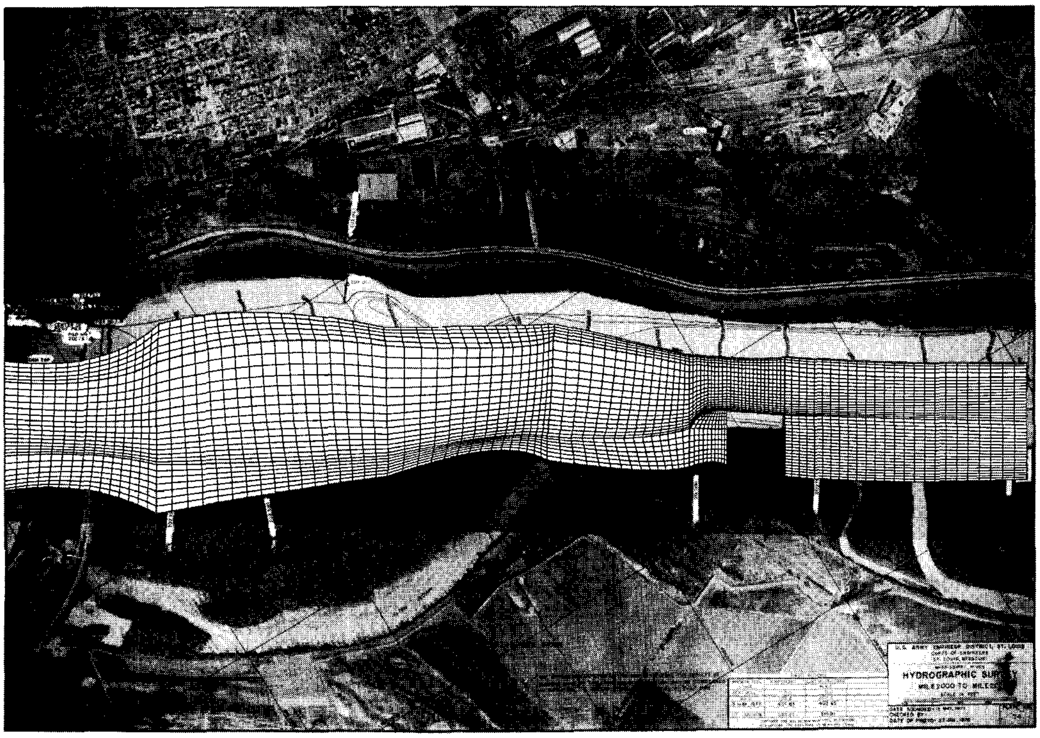


Fig. 5 Finite element mesh for the study reach in the Mississippi River

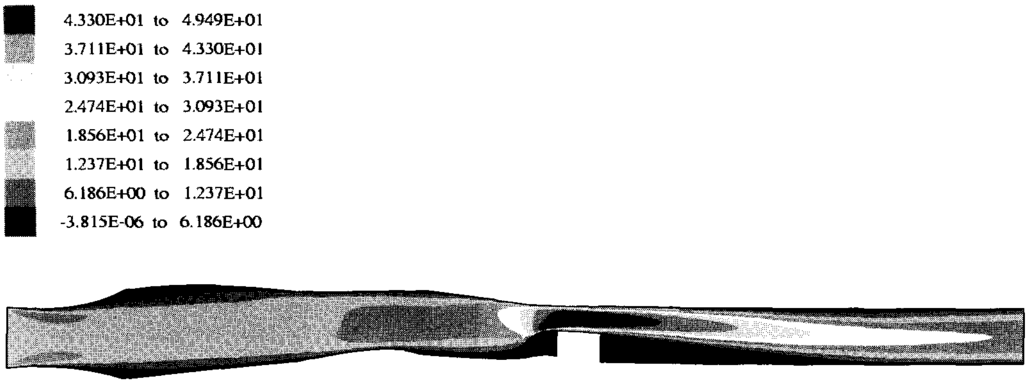
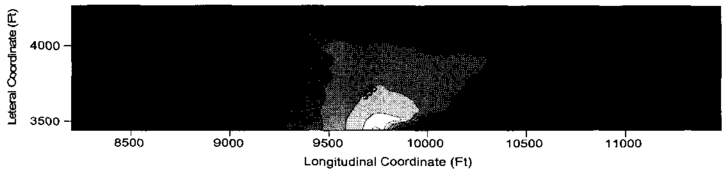


Fig. 6 Flow velocity distributions of the study reach



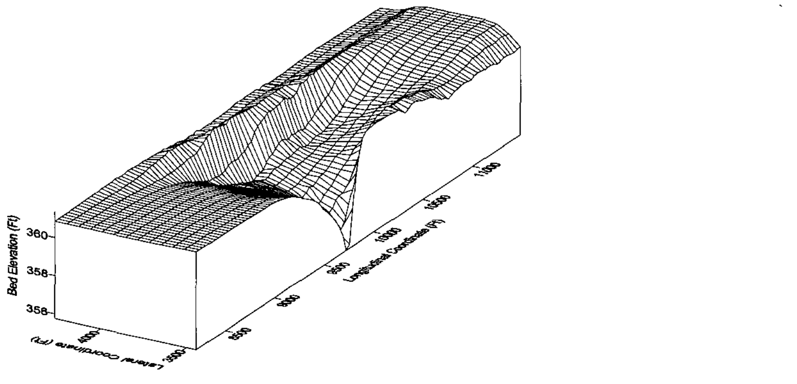


Fig. 7(a) Bed elevation distributions in contracted reach after 1 day (24 hours)

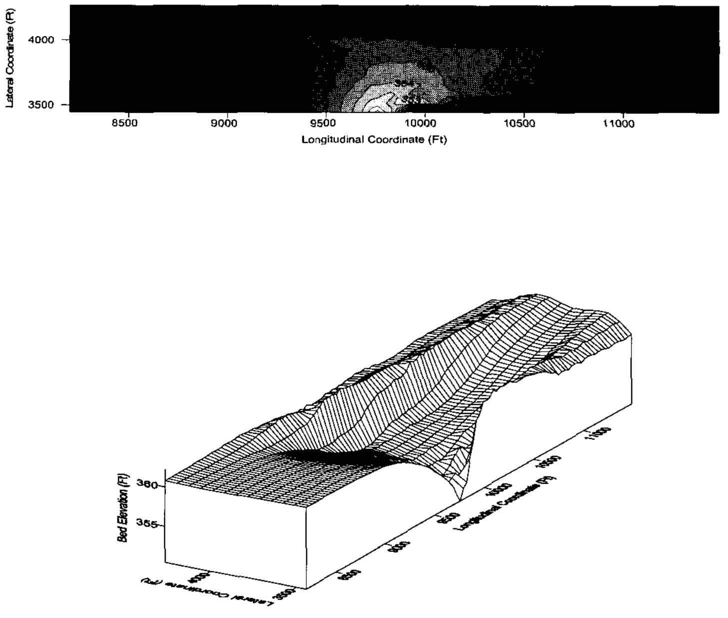


Fig. 7(b) Bed elevation distributions in contracted reach after 3 days (72 hours)

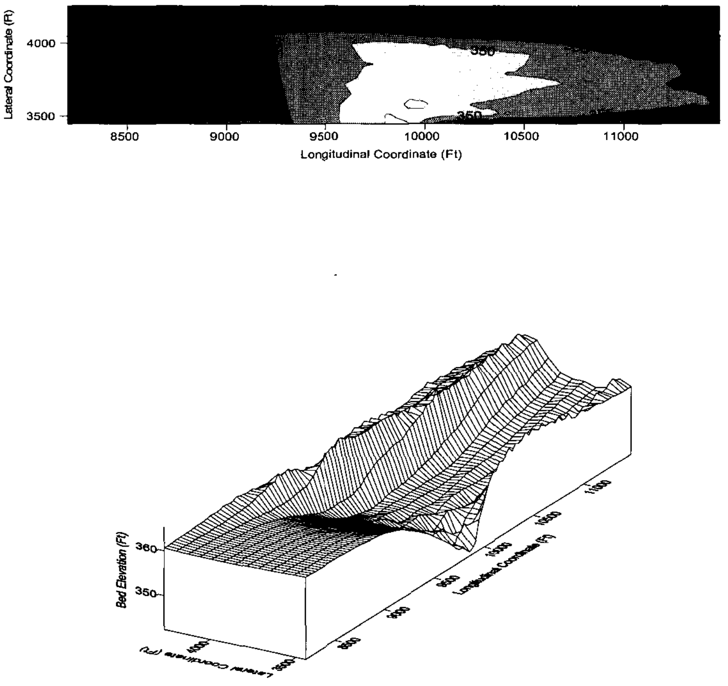


Fig. 7(c) Bed elevation distributions in contracted reach after 10 days

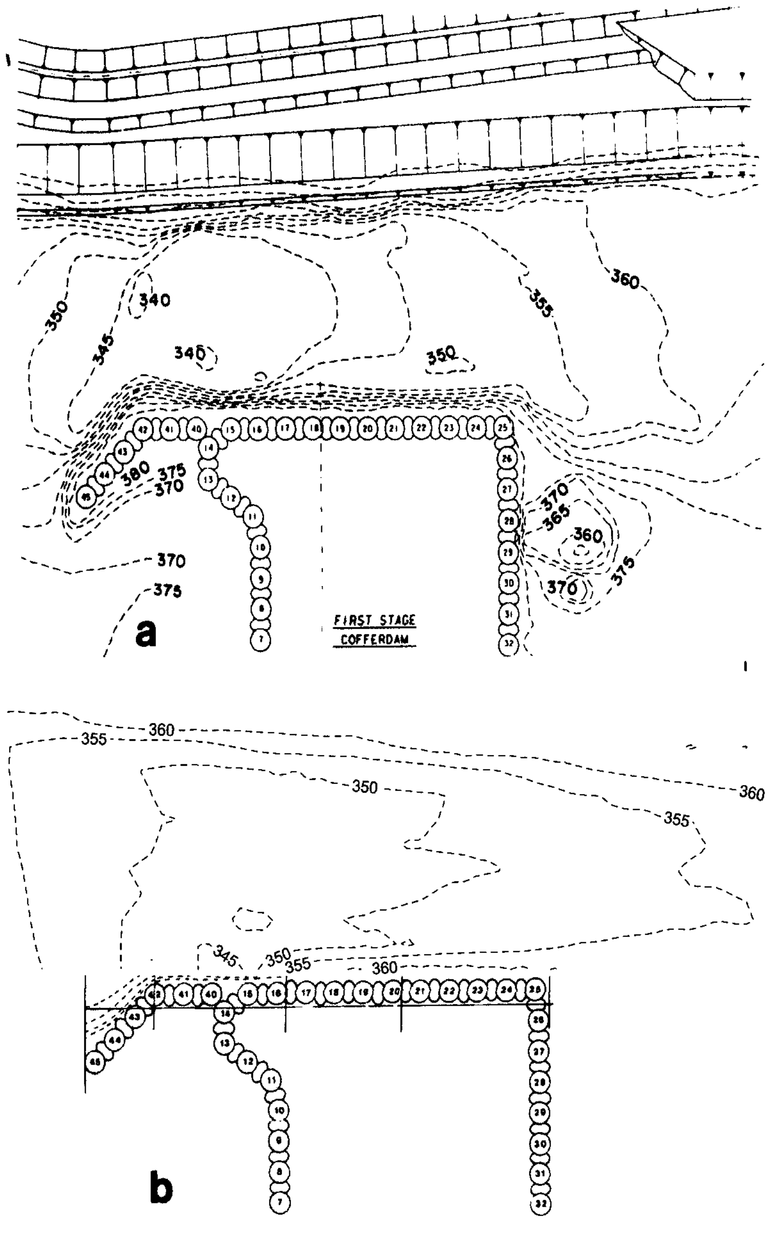


Fig. 8 Comparison between (a) measured and (b) computed bed elevation in ft

shown in Fig. 7(a); and after 3 days (72 hours) the maximum computed scour depth is 10.4 ft as shown in Fig. 7(b); and after 10 days of simulation, scour is computed to be 18.45 ft as shown in Fig. 7(c).

In these simulations, the van Rijn bed load equation is employed for bed load transport with assumed porosity ratio of 0.6. Comparing Fig. 7(a), 7(b), and 7(c), it can be seen that the scour process gradually propagates toward downstream of the contracted zone until it approaches equilibrium condition. In order to take account for the reduction of shear stress with the progress of scouring, the continuity equation was incorporated into shear stress computations by reducing velocities through deepening of flows.

6. SUMMARY AND CONCLUSIONS

In this study, a sediment transport model was developed and applied to the Mississippi River for Lock & Dam No. 26 (replacement) site. The contraction ratio in this reach is approximately 50 % and approach flow velocities are increased about 2.5 times due to severe channel contraction. The flow velocity distribution computed from the hydrodynamic model is directly incorporated into the bottom shear stress computation. From the shear stress distributions, it is possible to evaluate the bed elevation changes by solving the sediment continuity equation for bed load transport.

For the treatment of unsteady term, the lumped matrix concept was employed to provide the diagonalization of the mass matrix. The fundamental assumption of the modeling condition is that the flow remains in steady state, while the variation of channel bed elevation is unsteady. Through the comparison between measured data and numerical results, the model proved to be able to predict the channel bed elevation changes successfully. Measured and computed bed elevation contours are shown in Fig. 8 in order to facilitate evaluation of numerical results These results show very close agreement between computed values and measured data.

Since the fundamental assumption of this model is solely dependent on 2D configuration, fully 3D process is not taken into account. However, by incorporating the continuity equation into the shear stress formulation of the numerical model, the scour depth development is realistically simulated. As a result, the numerical result from the model shows good agreement with measured data. The model is also capable to determine the foundation depth in the process of hydraulic structure design to find the ultimate scour depth under equilibrium conditions. Additionally, the model provides valuable information in identifying zone of severe bed elevation changes through time under various flow conditions.

REFERENCES

Brørs, B. (1999). "Numerical modeling of flow and scour at pipelines." *Journal of Hydraulic Engineering*, ASCE, Vol. 125 (5), pp. 511-523.

Heinrich, J.C., Huyakorn, P.S., Zienkiwiewicz, O.C., and Mitchell, A.R., (1977). "An Upwind Finite Element Scheme for Two Dimensional Convective Transport Equation." *International Journal for Numerical Methods in Engineering,* Vol. (11) pp. 627-643.

Molinas, A., Khiereldin. K., and Wu (1998) "Shear stress around vertical wall abutments." *Journal of hydraulic engineering,* ASCE, Vol. 124 (8), pp. 822-830.

Ouillon, S., and Dartus, D. (1997). "Three Dimensional Computation of Flow Around Groyne" *Journal of Hydraulic Engineering*, ASCE, Vol. 123 (1), pp. 962-970.

Rajaratnam, N., and Nwachukwu, B. (1983). "Flow Near Groyne-Like Structures." *Journal of Hydraulics Division,* ASCE, Vol. 109 (3), pp. 463-480.

Tanguy, J. M., Dhatt, G., Frenette, M., and Monadier, P. (1989). "Modelisation of solid bed-load transport with the finite element method." *La Houille Blanche,* Paris, France, Vol. 3-4, pp. 263-267 (in French).

Tingsanchali, T., and Maheswaran, S. (1990). "2-D Depth Averaged Flow Computation near Groyne." *Journal of Hydraulic Engineering,* 116 (1), pp. 71-86.

Zienkiewicz, O. C. (1989). "The Finite Element Method." (4th ed. Vol. 2) McGraw-Hill

---

Department of Civil Engineering, Colorado State University, Ft. Collins, Colorado, 80523 USA
(E-mail : hijnoh@empal.com)