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ON TESTING THE EQUALITY OF THE COEFFICIENTS OF VARIATION IN TWO INVERSE GAUSSIAN POPULATIONS

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ABSTRACT

This paper deals with testing the equality of the coefficients of variation in two inverse Gaussian populations. The likelihood ratio, Lagrange-multiplier and Wald tests are presented. Monte-Carlo simulations are performed to compare the powers of these tests. In a simulation study, the likelihood ratio test appears to be consistently more powerful than the Lagrange-multiplier and Wald tests when sample size is small. The powers of all the tests tend to be similar when sample size increases.

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1. INTRODUCTION

The inverse Gaussian (IG) distribution with a location parameter μ and a scale parameter λ, abbreviated $IG(\mu, \lambda)$, has potentially useful applications in a wide variety of fields such as biology, ecology, environmental studies, engineering, management science, reliability, *etc.*, because of the versatility and flexibility in modeling skewed data. For a comprehensive discussion on the IG distribution, see Chhikara and Folks (1989) and references therein. Theoretically, the IG distribution is well-known as a first passage time distribution in Brownian motion with positive drift. Tweedie (1957a, 1957b) established many important statistical properties of the IG distribution, similar to those of the normal distribution. The probability density of the IG distribution is of the form

$$f(x;\mu,\lambda) = \left(\frac{\lambda}{2\pi x^3}\right)^{\frac{1}{2}} \exp\left\{-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right\}, \quad x>0,\ \mu>0,\ \lambda>0. \tag{1.1}$$

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The mean, variance and coefficient of variation of the IG distribution are given by μ, μ^3/λ and $(\mu/\lambda)^{1/2}$, respectively. By denoting $\gamma = (\mu/\lambda)^{1/2}$, the coefficients of skewness and kurtosis of the distribution are 3γ and $15\gamma^2 + 3$.

The coefficient of variation which does not depend on the units of measurements is an important parameter of describing populations widely used in many fields because its usefulness in examining the consistency or uniformity of results from different experiments involving the same character. A lot of work has been done on testing hypotheses concerning location parameters and scale parameters by Chhikara (1975), Chhikara and Folks (1989) and Davis (1980), whereas there seems to be little literature on deriving tests for the coefficients of variation. In this paper, we are interested in testing the equality of the coefficients of variation in two inverse Gaussian populations.

In single IG population, Hsieh (1990) considered testing $H_0 : \phi \geq \phi_0$ against $H_a : \phi < \phi_0$, where $\phi = \lambda/\mu$ is the shape parameter and ϕ_0 is a fixed constant. The derived likelihood ratio test is based on the statistic $W = 1/(\overline{X}V)$, where $\overline{X}$ and V are maximum likelihood estimators of μ and $1/\lambda$. Since the coefficient of variation is a monotone function of ϕ, Hsieh (1990) indicated that the test can be applied to the coefficient of variation. To develop a likelihood ratio test for the equality of two coefficients of variation, one may use the approach considered by Hsieh (1990). However, in this case, the derived test involves algebraically unsolvable equations and thus the standard numerical routine such as the Newton-Raphson method is required to solve the non-linear likelihood equations. Such a routine is in want of the complex Hessian matrix that contains the second derivatives of the log-likelihood. Thus, instead of using Hsieh's approach to derive tests for the coefficients of variation, we consider a different approach, which provides a more efficient method of solving the equations numerically.

In Section 2, we present the likelihood ratio (LR), Lagrange-multiplier (LM) and Wald tests for the equality of the coefficients of variation in two inverse Gaussian populations. To compare the powers of the tests, a simulation study is performed for selected sample sizes and alternatives in Section 3. Finally, brief conclusions are provided in Section 4.

2. Tests for the Equality of Two Coefficients of Variation

Assume that there are two independent populations where the i^{th} population follows an inverse Gaussian distribution, $IG(\mu_i, \mu_i/\gamma_i^2)$, $\mu_i > 0$, $\gamma_i > 0$, $i = 1, 2$, where μ_i and γ_i is unknown mean and coefficient of variation. Let

$X_{i1}, X_{i2}, \dots, X_{in}$ represent a random sample of size n drawn from the i^{th} inverse Gaussian population. One inference that we wish to consider is to test the null hypothesis $H_0 : \gamma_1 = \gamma_2 = \gamma$, γ unspecified, against $H_a : \gamma_1 \neq \gamma_2$. Now, we derive the likelihood ratio, Lagrange-multiplier and Wald tests for the null hypothesis in this section.

2.1. Likelihood ratio test

The likelihood function for the restricted situation of equal γ_i's based on random samples is

$$L_0(\mu_1, \mu_2, \gamma) = c\gamma^{-2n}(\mu_1\mu_2)^{\frac{n}{2}} \exp\left\{ -\frac{1}{2\gamma^2}\sum_{i=1}^{2}\frac{1}{\mu_i}\sum_{j=1}^{n}\frac{(X_{ij}-\mu_i)^2}{X_{ij}}\right\}, \tag{2.1}$$

where c is a constant, $(2\pi)^{-n}\prod_{i=1}^{2}\prod_{j=1}^{n} X_{ij}^{-3/2}$. Routine calculations lead to the following set of likelihood equations for μ_i and γ:

$$\frac{\partial \log L_0}{\partial \gamma} = -\frac{2n}{\gamma} + \frac{1}{\gamma^3}\sum_{i=1}^{2}\sum_{j=1}^{n}\frac{(X_{ij}-\mu_i)^2}{\mu_i X_{ij}} = 0, \tag{2.2}$$

$$\frac{\partial \log L_0}{\partial \mu_i} = \frac{n}{2\mu_i} + \frac{1}{2\gamma^2\mu_i^2}\sum_{j=1}^{n}\frac{(X_{ij}-\mu_i)^2}{X_{ij}} + \frac{1}{\gamma^2\mu_i}\sum_{j=1}^{n}\frac{X_{ij}-\mu_i}{X_{ij}} = 0, \quad i = 1, 2. \tag{2.3}$$

Simplifying these equations gives

$$\mu_i^2 - \gamma^2\overline{X}_{H,i}\,\mu_i - \overline{X}_{H,i}\overline{X}_i = 0, \quad i = 1, 2 \tag{2.4}$$

and

$$\gamma^2 = \frac{1}{2n}\sum_{i=1}^{2}\frac{1}{\mu_i}\sum_{j=1}^{n}\frac{(X_{ij}-\mu_i)^2}{X_{ij}}, \tag{2.5}$$

where $\overline{X}_i = \sum_{j=1}^{n} X_{ij}/n$ and $\overline{X}_{H,i} = n/\sum_{j=1}^{n} X_{ij}^{-1}$. Solving (2.4) and (2.5) simultaneously gives the restricted maximum likelihood estimators of μ_1, μ_2 and γ. By the theorem of Choi and Kim (2001), it can be found that the largest root of $f(\mu_1) = 2(C_1 + 1)\mu_1^3 - (C_1 + C_2 - 2)\overline{X}_1\mu_1^2 - 6\overline{X}_1^2\mu_1 + 2\overline{X}_1^3 = 0$ becomes the restricted maximum likelihood estimator, $\widetilde{\mu}_1$, of μ_1, where $C_i = \overline{X}_i V_i$ and $V_i = 1/\overline{X}_{H,i} - 1/\overline{X}_i$, $i = 1, 2$. The restricted maximum likelihood estimators for μ_2 and γ^2 are given by $\widetilde{\mu}_2 = \widetilde{\mu}_1\overline{X}_2/(2\widetilde{\mu}_1 - \overline{X}_1)$ and $\widetilde{\gamma}^2 = \widetilde{\mu}_1/\overline{X}_{H,1} - \overline{X}_1/\widetilde{\mu}_1$. On the other hand, the unrestricted maximum likelihood estimators, $\widehat{\mu}_1$, $\widehat{\mu}_2$, $\widehat{\gamma}_1$ and

$\widehat{\gamma}_2$, are easily obtained as $\widehat{\mu}_1 = \overline{X}_1$, $\widehat{\mu}_2 = \overline{X}_2$, $\widehat{\gamma}_1 = (\overline{X}_1 V_1)^{1/2}$ and $\widehat{\gamma}_2 = (\overline{X}_2 V_2)^{1/2}$, respectively.

The maximum likelihood under H_0, denoted by $\mathcal{L}_0$, is given by

$$\mathcal{L}_0(\widetilde{\mu}_1, \widetilde{\mu}_2, \widetilde{\gamma}) = c\left(\frac{1}{\widetilde{\gamma}}\right)^{2n} (\widetilde{\mu}_1 \widetilde{\mu}_2)^{\frac{n}{2}} e^{-n} \tag{2.6}$$

and the maximum likelihood under H_a, denoted by $\mathcal{L}_a$, is given by

$$\mathcal{L}_a(\widehat{\mu}_1, \widehat{\mu}_2, \widehat{\gamma}_1, \widehat{\gamma}_2) = c\left(\frac{1}{V_1 V_2}\right)^{\frac{n}{2}} e^{-n}. \tag{2.7}$$

Thus, letting Λ be a likelihood ratio, we obtain the following LR test statistic

$$T_{LR} = -2\log\Lambda = n\left\{\log\left(\frac{\widetilde{\gamma}^2}{\widetilde{\mu}_1 V_1}\right) + \log\left(\frac{\widetilde{\gamma}^2}{\widetilde{\mu}_2 V_2}\right)\right\}. \tag{2.8}$$

Since T_{LR} is the chi-square distributed with one degree of freedom under the null hypothesis, H_0 is rejected in favor of H_a if $T_{LR} > \chi_1^2(\alpha)$ for a significance level α, where $\chi_1^2(\alpha)$ is the upper 100α-percentile of the chi-square with one degree of freedom.

2.2. Lagrange-multiplier test

The Lagrange-multiplier test, which is also known as the score test, is based on score functions evaluated at the restricted maximum likelihood estimators. Let $\boldsymbol{\theta}$ be the parameter vector, $\widetilde{\boldsymbol{\theta}}$ be the restricted maximum likelihood estimator of $\boldsymbol{\theta}$, $\boldsymbol{g}(\boldsymbol{\theta})$ be the score vector and $I(\boldsymbol{\theta})$ be the information matrix. Then the LM test is based on the statistic $T_{LM} = \boldsymbol{g}^t(\widetilde{\boldsymbol{\theta}}) I^{-1}(\widetilde{\boldsymbol{\theta}}) \boldsymbol{g}(\widetilde{\boldsymbol{\theta}})$, where $\boldsymbol{g}(\widetilde{\boldsymbol{\theta}})$ and $I(\widetilde{\boldsymbol{\theta}})$ are the values of $\boldsymbol{g}(\boldsymbol{\theta})$ and $I(\boldsymbol{\theta})$ evaluated at $\widetilde{\boldsymbol{\theta}}$. It is well-known that if the null hypothesis is true, T_{LM} is asymptotically distributed as the chi-square with r degrees of freedom, where r is the number of restrictions (Rao, 1973).

In our case, the parameters and the corresponding restricted maximum likelihood estimators are given by $\boldsymbol{\theta} = (\gamma_1, \gamma_2, \mu_1, \mu_2)^t$ and $\widetilde{\boldsymbol{\theta}} = (\widetilde{\gamma}, \widetilde{\gamma}, \widetilde{\mu}_1, \widetilde{\mu}_2)^t$, respectively. The score vector $\boldsymbol{g}(\boldsymbol{\theta}) = (\partial \log L/\partial\gamma_1,\ \partial \log L/\partial\gamma_2,\ \partial \log L/\partial\mu_1,\ \partial \log L/\partial\mu_2)^t$ takes elements such that

$$\frac{\partial \log L}{\partial \gamma_i} = -\frac{n}{\gamma_i} + \frac{1}{\gamma_i^3 \mu_i}\sum_{j=1}^{n}\frac{(X_{ij}-\mu_i)^2}{X_{ij}}, \quad i = 1, 2 \tag{2.9}$$

and

$$\frac{\partial \log L}{\partial \mu_i} = \frac{n}{2\mu_i} + \frac{1}{2\gamma_i^2\mu_i^2}\sum_{j=1}^{n}\frac{(X_{ij}-\mu_i)^2}{X_{ij}} + \frac{1}{\gamma_i^2\mu_i}\sum_{j=1}^{n}\frac{X_{ij}-\mu_i}{X_{ij}}, \quad i = 1, 2, \tag{2.10}$$

where L is the unrestricted likelihood function. The information matrix $I(\boldsymbol{\theta})$ is of the form

$$I(\boldsymbol{\theta}) = \begin{pmatrix} \frac{2n}{\gamma_1^2} & 0 & -\frac{n}{\gamma_1\mu_1} & 0 \\ 0 & \frac{2n}{\gamma_2^2} & 0 & -\frac{n}{\gamma_2\mu_2} \\ -\frac{n}{\gamma_1\mu_1} & 0 & \frac{n(\gamma_1^2+2)}{2\gamma_1^2\mu_1^2} & 0 \\ 0 & -\frac{n}{\gamma_2\mu_2} & 0 & \frac{n(\gamma_2^2+2)}{2\gamma_2^2\mu_2^2} \end{pmatrix}. \tag{2.11}$$

Replacing μ_i by $\widetilde{\mu}_i$, γ_i by $\widetilde{\gamma}$, in (2.9) and (2.10), we obtain $\boldsymbol{g}(\widetilde{\boldsymbol{\theta}})$ with elements

$$\left.\frac{\partial \log L}{\partial \gamma_i}\right|_{\substack{\gamma_i=\widetilde{\gamma}\\ \mu_i=\widetilde{\mu}_i}} = -\frac{n}{\widetilde{\gamma}} + \frac{1}{\widetilde{\gamma}^3\widetilde{\mu}_i}\sum_{j=1}^{n}\frac{(X_{ij}-\widetilde{\mu}_i)^2}{X_{ij}}, \quad i=1,2, \tag{2.12}$$

$$\begin{aligned}\left.\frac{\partial \log L}{\partial \mu_i}\right|_{\substack{\gamma_i=\widetilde{\gamma}\\ \mu_i=\widetilde{\mu}_i}} &= \frac{n}{2\widetilde{\mu}_i} + \frac{1}{2\widetilde{\gamma}^2\widetilde{\mu}_i^2}\sum_{j=1}^{n}\frac{(X_{ij}-\widetilde{\mu}_i)^2}{X_{ij}} + \frac{1}{\widetilde{\gamma}^2\widetilde{\mu}_i}\sum_{j=1}^{n}\frac{X_{ij}-\widetilde{\mu}_i}{X_{ij}} \\ &= 0, \quad i=1,2. \end{aligned} \tag{2.13}$$

By using $I(\widetilde{\boldsymbol{\theta}})$ evaluated at $\widetilde{\boldsymbol{\theta}}$ and letting $s_i = (\partial \log L/\partial\gamma_i)|_{\gamma_i=\widetilde{\gamma},\ \mu_i=\widetilde{\mu}_i}$, $i=1,2$, the LM test statistic, after algebraic calculations, is simplified as

$$T_{LM} = \frac{\widetilde{\gamma}^2(\widetilde{\gamma}^2+2)(s_1^2+s_2^2)}{4n}. \tag{2.14}$$

If the null hypothesis is true, then T_{LM} is asymptotically distributed as the chi-square with one degree of freedom. Thus the null hypothesis is rejected if $T_{LM} > \chi_1^2(\alpha)$, where $\chi_1^2(\alpha)$ is the upper 100α-percentile of the chi-square with one degree of freedom.

2.3. Wald test

The Wald test is based on asking whether the vector of restrictions, evaluated at the unrestricted maximum likelihood estimators, is close enough to a zero vector when the restrictions hold. Suppose that the null hypothesis is of the form $H_0 : \boldsymbol{h}(\boldsymbol{\theta}) = (h_1(\boldsymbol{\theta}),\ldots,h_r(\boldsymbol{\theta}))^t = \mathbf{0}$, where $\boldsymbol{\theta}$ is the parameter vector with size p. Let $H(\boldsymbol{\theta})$ be the $p\times r$ matrix with elements $\partial h_j(\boldsymbol{\theta})/\partial\theta_i$, $i=1,\ldots,p$, $j=1,\ldots,r$, and $I(\boldsymbol{\theta})$ be the $p\times p$ information matrix. Then the Wald test is based on the statistic $T_W = \boldsymbol{h}^t(\widehat{\boldsymbol{\theta}})[H^t(\widehat{\boldsymbol{\theta}})I^{-1}(\widehat{\boldsymbol{\theta}})H(\widehat{\boldsymbol{\theta}})]^{-1}\boldsymbol{h}(\widehat{\boldsymbol{\theta}})$, where $\boldsymbol{h}(\widehat{\boldsymbol{\theta}})$, $I^{-1}(\widehat{\boldsymbol{\theta}})$ and $H(\widehat{\boldsymbol{\theta}})$

are the values of $\boldsymbol{h}(\boldsymbol{\theta})$, $I^{-1}(\boldsymbol{\theta})$ and $H(\boldsymbol{\theta})$, evaluated at the unrestricted maximum likelihood estimator $\widehat{\boldsymbol{\theta}}$. Asymptotically, T_W is the chi-square distributed with r degrees of freedom under the null hypothesis (Wald, 1943).

In our case, the parameters and the corresponding unrestricted maximum likelihood estimators are given by $\boldsymbol{\theta} = (\mu_1, \lambda_1, \mu_2, \lambda_2)^t$ and $\widehat{\boldsymbol{\theta}} = (\overline{X}_1, 1/V_1, \overline{X}_2, 1/V_2)^t$, respectively. The null hypothesis $H_0 : \gamma_1 = \gamma_2$ can be expressed as the form of $h(\boldsymbol{\theta}) = (\mu_1/\lambda_1)^{1/2} - (\mu_2/\lambda_2)^{1/2}$. The information matrix $I(\boldsymbol{\theta})$ takes the form of

$$I(\boldsymbol{\theta}) = \begin{pmatrix} \frac{n\lambda_1}{\mu_1^3} & 0 & 0 & 0 \\ 0 & \frac{n}{2\lambda_1^2} & 0 & 0 \\ 0 & 0 & \frac{n\lambda_2}{\mu_2^3} & 0 \\ 0 & 0 & 0 & \frac{n}{2\lambda_2^2} \end{pmatrix} \tag{2.15}$$

and $H(\boldsymbol{\theta}) = (1/\sqrt{4\mu_1\lambda_1},\ -\mu_1/\sqrt{4\lambda_1^3},\ -1/\sqrt{4\mu_2\lambda_2},\ \sqrt{\mu_2}/\sqrt{4\lambda_2^3})^t$. By using $h(\widehat{\boldsymbol{\theta}})$, $H(\widehat{\boldsymbol{\theta}})$ and $I^{-1}(\widehat{\boldsymbol{\theta}})$ evaluated at $\widehat{\theta}$, we yield the Wald test statistic as

$$T_W = \frac{4n\left\{\overline{X}_1V_1 + \overline{X}_2V_2 - 2(\overline{X}_1V_1\overline{X}_2V_2)^{\frac{1}{2}}\right\}}{\overline{X}_1V_1(2 + \overline{X}_1V_1) + \overline{X}_2V_2(2 + \overline{X}_2V_2)}. \tag{2.16}$$

If the null hypothesis is true, then T_W is also asymptotically distributed as the chi-square with one degree of freedom. The null hypothesis is rejected if $T_W > \chi_1^2(\alpha)$, where $\chi_1^2(\alpha)$ is the upper 100α-percentile of the chi-square with one degree of freedom.

3. Power Comparison of the Tests

To compare the powers of the tests presented in Section 2, Monte-Carlo simulations were conducted by generating 10,000 random samples of size n = 30, 50, 70, 100 from $IG(\mu_1, \mu_1/\gamma_1^2)$ and $IG(\mu_2, \mu_2/\gamma_2^2)$ using the algorithm of Michael *et al.* (1976). Selected values of μ_1, μ_2, γ_1 and γ_2 are $\mu_1 = 5$, $\mu_2 = 10$, $\gamma_1 = 1$ and γ_2 = 0.5, 0.75, 1.25, 1.5, 2.0, 2.5. Table 3.1 displays the simulation results for the likelihood ratio, Lagrange-multiplier and Wald tests. The entries appeared in the table are simulated powers for significance level 5% measured by counting the number of falling into the rejection region out of 10,000 random samples.

The test T_{LR}, as shown in Table 3.1, is consistently more powerful than T_{LM} and T_W. The power of T_{LM} is observed to be lower than T_{LR} and higher than T_W.

TABLE 3.1 *Simulated powers of the likelihood ratio, Lagrange-multiplier and Wald tests for significance level 5%, based on 10,000 random samples*

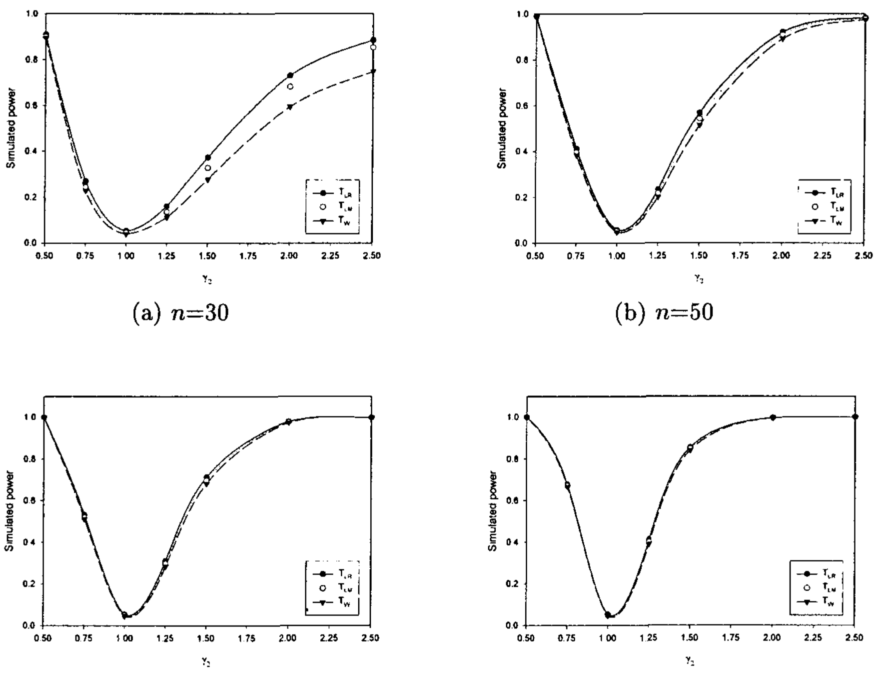
		$n = 30$			$n = 50$		
γ_1	γ_2	T_{LR}	T_{LM}	T_W	T_{LR}	T_{LM}	T_W
1	0.50	0.911	0.900	0.895	0.989	0.988	0.988
1	0.75	0.269	0.243	0.227	0.411	0.395	0.383
1	1.25	0.161	0.135	0.112	0.233	0.217	0.201
1	1.50	0.373	0.328	0.277	0.569	0.543	0.514
1	2.00	0.731	0.682	0.595	0.921	0.910	0.892
1	2.50	0.886	0.853	0.748	0.983	0.979	0.975
		$n = 70$			$n = 100$		
γ_1	γ_2	T_{LR}	T_{LM}	T_W	T_{LR}	T_{LM}	T_W
1	0.50	0.999	0.999	0.999	1.000	1.000	1.000
1	0.75	0.532	0.518	0.511	0.679	0.672	0.668
1	1.25	0.311	0.297	0.282	0.412	0.401	0.390
1	1.50	0.713	0.698	0.682	0.856	0.849	0.842
1	2.00	0.979	0.977	0.973	0.997	0.996	0.996
1	2.50	0.998	0.998	0.998	1.000	1.000	1.000

TABLE 3.2 *Type I error rates of the likelihood ratio, Lagrange-multiplier and Wald tests*

	$n = 30$			$n = 50$			$n = 70$			$n = 100$		
γ	T_{LR}	T_{LM}	T_W	T_{LR}	T_{LM}	T_W	T_{LR}	T_{LM}	T_W	T_{LR}	T_{LM}	T_W
0.25	0.059	0.050	0.054	0.050	0.047	0.048	0.052	0.049	0.051	0.051	0.049	0.050
0.50	0.056	0.047	0.047	0.057	0.052	0.053	0.050	0.047	0.048	0.052	0.050	0.050
1.00	0.056	0.048	0.040	0.056	0.050	0.045	0.052	0.048	0.046	0.052	0.050	0.048
1.50	0.056	0.040	0.027	0.053	0.043	0.034	0.052	0.046	0.040	0.053	0.047	0.042

T_W appears to be least powerful in most cases. All the tests have low rejection rates less than 50% for $(\gamma_1, \gamma_2) = (1,\ 0.75),\ (1,\ 1.25),\ (1,\ 1.5)$ when $n = 30$, for $(\gamma_1, \gamma_2) = (1,\ 0.75),\ (1,\ 1.25)$ when $n = 50$, and for $(\gamma_1, \gamma_2) = (1,\ 1.25)$ when $n = 70, 100$. However, their rejection rates are more than 50% except for these cases. As the value of γ_2 is larger or smaller than that of γ_1, the powers of the tests show a tendency to increase rapidly for all sample sizes as displayed in Figure 3.1. Also, with increasing sample size, the rejection rates of all the tests increase for a fixed value of (γ_1, γ_2). In conclusion, when $n = 30, 50$, T_{LR} performs better than T_{LM} and T_W, and T_{LM} works better than T_W. However, there is no difference in the powers of T_{LR}, T_{LM} and T_W when sample size is large ($n = 70, 100$).

To compute the type I error rates of T_{LR}, T_{LM} and T_W, we simulate 10,000 random samples of size $n = 30, 50, 70, 100$ from $IG\left(1, 1/\gamma^2\right)$ and $IG\left(2, 2/\gamma^2\right)$ for $\gamma = 0.25, 0.5, 1.0, 1.5$. The nominal error rate is $\alpha = 0.05$ and the type I error rate of each test is obtained by counting the number of times that the computed value of the statistic corresponding to each test is greater than $\chi_1^2(0.05) = 3.84$ out 10,000 random samples. Table 3.2 shows the summarized simulation results. Although T_W has somewhat smaller type I error than the nominal 0.05, especially for $n \leq 50$ and $\gamma = 1.5$, the error rates of all the tests show considerably close to the nominal 0.05.



(a) n=30 (b) n=50 (c) n=70 (d) n=100

FIGURE 3.1 *Plot of simulated powers of the likelihood ratio, Lagrange-multiplier and Wald tests for significance level 5%*

4. CONCLUSION

In this paper, we presented the likelihood ratio, Lagrange-multiplier and Wald tests for the equality of the coefficients of variation in two inverse Gaussian populations. The simulation results report that the likelihood ratio test has the highest rejection rate among all the tests, especially when sample size is small ($n \leq 50$). However, the powers of all the tests tend to be similar as sample size increases. Thus, in practice, the use of the likelihood ratio test is recommended to achieve the high power gain over the Lagrange-multiplier and Wald tests when sample size is small.

In this study, a situation where sample size is unequal could not be considered because it was difficult to reveal the existence of the unique maximum likelihood estimator of μ_1 satisfying a cubic equation for μ_1. We leave it as a future research of interest.

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