Journal of the Optical Society of Korea, Vol. 7, No. 2, June 2003, pp. 59-63

Theoretical Study of the Effect of Pulse Chirping on Polarization Mode Dispersion and Polarization-Dependent Loss

Il Yong Yoon, Yong Wook Lee, and Byoungho Lee*

School of Electrical Engineering, Seoul National University, Seoul 151-744, KOREA

(Received March 27, 2003)

We accomplished numerical simulations for two-uniform-fiber concatenation with both polarization mode dispersion (PMD) and polarization-dependent loss (PDL). The effective overall PMD is increased with the chirp parameter and the effective overall PDL is decreased with the chirp parameter. For PDL, chirping just makes the signal bandwidth wider, so makes the pulse be more depolarized than a chirp-free pulse. We showed that PDL increases the frequency dependence of the principal states of polarization, and the combination of this dependency and the bandwidth broadening by chirping can affect the effective PDL.

OCIS codes : 060.0060, 060.2340.

I. INTRODUCTION

Polarization mode dispersion (PMD) is one of the serious problems in high speed optical communication. Differential group delay (DGD) between the two orthogonal states of polarization called the principal states of polarization (PSP's) causes PMD. [1] If an optical pulse is not launched along the PSP's of the whole system of optical components, the output pulse will be the sum of the pulses along the fast PSP and slow PSP and as the result, the pulse will be broadened. Moreover, most optical components have more or less polarization-dependent loss (PDL). If PDL is added to optical communication systems, more complex phenomena arise. These phenomena have not yet been fully investigated. Besides, in such a case as a pulse suffering from chirping that comes from modulation, chromatic dispersion or a nonlinear effect, a relation among PMD, PDL and chirping has not yet been investigated. In this paper, we investigate effective PMD and PDL which chirped optical pulses suffer from in concatenated optical fiber sections. And we accomplish the mathematical analysis and interpretation of the simulation result.

II. THEORY

The fiber with PMD and PDL can be modeled in a 2 × 2 complex transmission matrix, and the output pulse can be calculated on the basis of it. If the fiber consists of concatenation of N uniform trunks, the total transmission matrix is given by multiplying each transmission matrix of each trunk. [2] For a given transmission matrix, the 2-dimensional output Jones vector can be expressed as

$$\Psi_{out}(\omega) = T_N(\omega) \cdot \hat{\varphi}_{in} \quad (1)$$

where $T_N(\omega)$ is the Jones matrix of the fiber, and $\hat{\varphi}_{in}$ is the unit vector which represents the input state of polarization (SOP). Because we launch a polarized optical pulse, we can assume that $\hat{\varphi}_{in}$ is independent of optical frequency.

The PSP's are defined as the polarization states such that the output polarization is independent of the optical frequency in first order. [3] Therefore, for such states, output polarization states satisfy the following: [1]

$$\frac{\partial \Psi_{out}(\omega)}{\partial \omega} = -i\frac{\chi}{2}\Psi_{out}(\omega) \quad (2)$$

where χ is real and equal to DGD. From [3], output PSP can be expressed as $\Psi_{out}(\omega) = \sigma(\omega) \cdot \exp(i\phi(\omega))\hat{\varphi}_{out}$, where $\hat{\varphi}_{out}$ is the unit vector which represents output PSP, $\sigma(\omega)$ is a magnitude, and $\phi(\omega)$ is a phase. By definition of PSP, $\hat{\varphi}_{out}(\omega)$ is frequency-invariant to first order. In other words, because $\partial\hat{\varphi}_{out}(\omega)/\partial\omega = 0$ in PSP's, we can remove common delay and get half of the difference of two delays in two PSP's, $\partial\phi(\omega)/\partial\omega$, through differentiation in (2). If PSP's were not defined, the notion of group velocity itself would be very confused.

Because $\partial\Psi_{out}(\omega)/\partial\omega = (\partial T_N(\omega)/\partial\omega)\hat{\varphi}_{in} = (\partial T_N(\omega)/\partial\omega)T_N^{-1}\Psi_{out}(\omega)$, we can calculate PSP's and DGD from the eigenstates and the eigenvalue of $(\partial T_N(\omega)/\partial\omega)T_N^{-1}$, respectively.

There exists a recursion relation between the Nth and the $(N-1)$th trunks as follows

$$\frac{\partial T_N(\omega)}{\partial\omega}T_N^{-1} = \frac{-i}{2}\beta_N\hat{e}_N\vec{\sigma} + e^{b_N\hat{e}_N\cdot\vec{\sigma}/2}\cdot\frac{\partial T_{N-1}(\omega)}{\partial\omega}T_{N-1}^{-1}\cdot e^{-b_N\hat{e}_N\cdot\vec{\sigma}/2} \tag{3}$$

where $\vec{\beta}_N$ is the birefringent vector, $\hat{e}_N$ is the unit vector describing the direction of polarization and attenuation of the fiber section as $-i\vec{\beta}_j\omega + \vec{\alpha}_j \equiv b_j\hat{e}_j$, $\vec{\alpha}_j$ is the attenuation vector, and $\vec{\sigma}$ is the Pauli matrix vector with

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{4}$$

The directions of the vectors $\vec{\alpha}_j$ and $\vec{\beta}_j$ define the two polarization modes of the jth trunk, and their magnitudes define loss coefficient and DGD between two modes relatively.

Using Pauli matrices, we can also represent PMD in the 3-dimensional principal state vector as [1]

$$\frac{\partial T_N(\omega)}{\partial\omega}T_N^{-1} = -\frac{i}{2}\vec{\Omega}_N\cdot\vec{\sigma}. \tag{5}$$

However, when PDL is added, χ is no longer real and PSP's are not orthogonal generally. Because $\vec{\Omega}_N\cdot\vec{\sigma}\Psi_{out}(\omega) = \chi\Psi_{out}(\omega)$, the relation of $\chi^2 = \vec{\Omega}_N\cdot\vec{\Omega}_N$ is satisfied. [1] For χ, if χ is represented as $\chi = \delta\tau + i\eta$, $\delta\tau$ is DGD and η is the frequency derivative of the differential attenuation of the two PSP's. We can rewrite the recursion in the vector form as

$$\vec{\Omega}_N = (\beta_N + \hat{e}_N\cdot\vec{\Omega}_{N-1})\cdot\hat{e}_N + \cosh(b_N)(\vec{\Omega}_{N-1} - (\vec{\Omega}_{N-1}\cdot\hat{e}_N)\hat{e}_N) - i\sinh(b_N)\vec{\Omega}_{N-1}\wedge\hat{e}_N \tag{6}$$

where the symbol of $\wedge$ represents the wedge product and becomes the cross product in the case of operating on real vectors. If we set $\vec{\Omega}_{N-1} = 0$, the whole principal state vector up to the Nth fiber section is $\vec{\Omega}_N = \beta_N\cdot\hat{e}_N$ irrespective of PDL of the Nth fiber section. [1]

When the principal state vector is complex, the PSP cannot be expressed as the direction of the principal state vector, and therefore, in this situation PSP's, $\vec{m}_\pm$, are given by

$$\vec{m}_\pm = \frac{\pm\vec{W}_r + \vec{W}_r\wedge\vec{W}_i}{W_r^2} \tag{7}$$

where $\vec{W}_r$ and $\vec{W}_i$ are the components of normalized principal state vector, $\hat{W} \equiv \vec{\Omega}/\chi = \vec{W}_r + i\vec{W}_i$. [1]

And then, as explained in [4], the output optical power can be expressed as

$$P_{out} = \int \Psi_{out}^+(\omega)\cdot\Psi^*(\omega)\cdot\Psi_{out}(\omega)\cdot\Psi(\omega)\,d\omega \tag{8}$$
$$= \hat{\varphi}_{in}^+\cdot\int T^+(\omega)T(\omega)|\Psi(\omega)|^2\,d\omega\cdot\hat{\varphi}_{in} = \hat{\varphi}_{in}^+\cdot D\cdot\hat{\varphi}_{in}$$

where $\Psi(\omega)$ is the Fourier transform of input optical pulse, and $D = \int T^+(\omega)T(\omega)|\Psi(\omega)|^2\,d\omega$ is a 2×2 Hermitian matrix.

Thus, the PDL value is obtained from $\max(P_{out})$-$\min(P_{out})$. Then PDL for a pulse is easily obtained from two eigenvalues of D matrix through linear algebra. [2]

Lu *et al.* showed that when PDL exists, the average time delay of a pulse, roughly the position of a pulse, can be expressed as [2]

$$<t>= \left(\int_{-\infty}^{\infty}\Psi_{out}^+(t)\cdot t\cdot\Psi_{out}(t)\,dt\right)/P_{out} = \left(\int_{-\infty}^{\infty}\Psi_{out}^+(\omega)\cdot i\frac{\partial}{\partial\omega}\Psi_{out}(\omega)\,d\omega\right)/P_{out} = (\hat{\varphi}_{in}^+\cdot P_1\cdot\hat{\varphi}_{in})/P_{out} \tag{9}$$

where $\Psi_{out}(t)$ is the output electrical field vector as a function of time t and

$$P_1 = i\int_{-\infty}^{\infty}\left[T^+(\omega)T'(\omega)|\Psi(\omega)|^2 + T^+(\omega)T(\omega)\Psi'(\omega)\Psi^*(\omega)\right]d\omega. \tag{10}$$

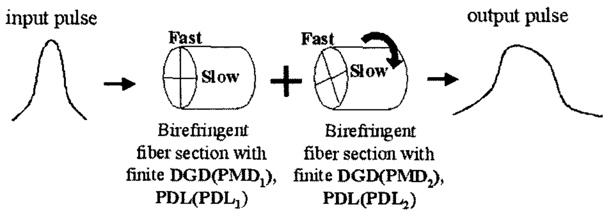

FIG. 1. The configuration of simulation.

Here, P_1 is a 2 × 2 Hermitian matrix also and $T'(\omega)$, $\Psi'(\omega)$ represent the derivatives of $T(\omega)$, $\Psi(\omega)$ with respect to frequency. In the time domain, the effective PMD can be expressed as max($< t >$)-min($< t >$). However, contrary to the case of PDL, an analytic solution to $< t >$ is not yet known since P_{out} is also dependent on input SOP's. [2]

III. RESULTS AND DISCUSSIONS

In this calculation, we consider two concatenated uniform fibers with both PMD and PDL as in Fig. 1. The input pulses are assumed to be chirped-Gaussian. We assume that input pulse-widths are equal and the value is 100 ps but chirp parameters of the input pulses are different and their values are 0, 1, 2 and 3. The chirp parameter C for Gaussian pulses is defined from $A(0,t) = A_0 \exp[-(1+iC)(t/T_0)^2/2]$. For example, the value of C is typically -6 at 1.55 μm for directly modulated semiconductor lasers. [5]

Then in the time domain effective overall PMD and PDL, including the effects of all frequency components, of the concatenated fiber are calculated while changing the relative angle between birefringent axes of the two fiber sections, i.e. while rotating one fiber about the other fiber axis. As mentioned above, since an analytical solution to $< t >$ in (9) is not yet known, we launch 62000 SOP's into the system in the simulation, and calculate the maximum and minimum of $< t >$ from the results. We designate PMD and PDL of the first fiber section as PMD$_1$ and PDL$_1$ and likewise for the second fiber section.

Fig. 2(a) shows the dependence of the effective overall PMD on chirping. The effective PMD increases with the chirp parameter. But, it is generally known that in the case of a chirp-free pulse, as the bandwidth of the pulse becomes broader, depolarization becomes higher, and effective PMD, therefore, becomes lower. [2] As shown in the figure, a chirp-free $100/\sqrt{10}$ ps pulse with almost the same frequency bandwidth as a chirped pulse with chirp parameter C=3 shows lower PMD than a chirp-free original 100 ps pulse.

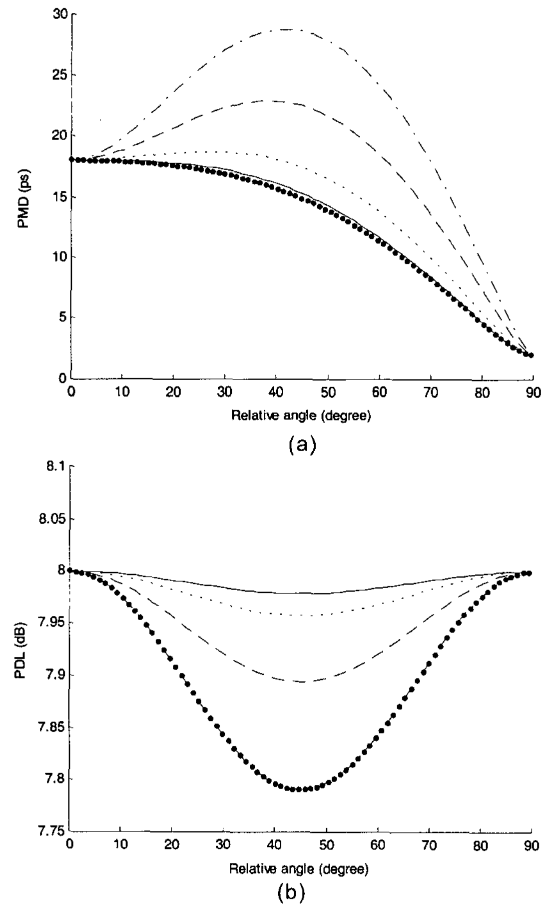

FIG. 2. (a) The dependence of the effective overall PMD on chirping when PDL$_1$ = 0 dB, PDL$_2$ = 8 dB, PMD$_1$ = 8 ps, PMD$_2$ = 10 ps. Solid curve: chirp-free, dotted curve: chirp =1, dashed curve: chirp=2, dash-dot curve: chirp=3, filled symbol: chirp-free pulse with pulse width of $100/\sqrt{10}$ ps. (b) The dependence of the effective overall PDL on chirping when the parameters are the same as in (a).

However, for a chirped pulse, a bandwidth is not the only effective factor. Though the first term of P_1 in (10), $i\int_{-\infty}^{\infty}[T^+(\omega)T'(\omega)|\Psi(\omega)|^2 d\omega$, is not affected by chirp except the effect of bandwidth broadening, the second term of P_1, $\hat{\varphi}_{in}^{+} \cdot i\int T^+T\Psi'\Psi^* d\omega \cdot \hat{\varphi}_{in}\left(= i\int P_{out}(\hat{\varphi}_{in},\omega)(\partial \ln\Psi/\partial\omega)\, d\omega\right)$, can be affected by chirp. When the phase term of the frequency component of the input chirped-Gaussian pulse, arg($\Psi(\omega)$), and the output optical power P_{out} depend on the frequency and SOP, the second term of P_1 can not be neglected. [6] As in the Fig. 2(a), in some configurations effective PMD of a chirped pulse can increase greatly. In the light of the results of simulation, the PMD is influenced by the second term of P_1 considerably, and the input polarization state which leads to maximum PMD is chosen so that the sum of two effects of the first and the second term of P_1 is maximized.

Fig. 2(b) shows the dependence of the effective overall PDL on chirping. As can be seen from the figure, the effective PDL decreases with chirp parameter. This is purely due to the frequency dependence of PSP's. [7] Therefore a chirp-free $100/\sqrt{10}$ pulse has the same PDL profile as a chirped pulse with chirp parameter C=3. In this case, the matrix representing output optical power P_{out}, $\int T^{+}(\omega)T(\omega)|\Psi(\omega)|^2\,d\omega$, is the same regardless of chirping. Because we select the PDL combination of PDL_1 and PDL_2 to make the frequency dependence of the PSP higher, the effect of the bandwidth of the pulse is higher.

The reason why PDL increases the frequency dependence of PSP's is as follows. Gisin and Huttner derived a dynamical equation for the evolution of the principal state vector ($\vec{\Omega}$). [8] From this equation, we can derive the 3-component complex vector $\vec{\Omega}$ for two-uniform-fiber concatenation using principal state vectors of each fiber trunk and it is shown in Fig. 3.

$$\begin{aligned}\vec{\Omega} &= (\beta_2 + \hat{e}_2\cdot\vec{\Omega}_1)\cdot\hat{e}_2 + \cosh(\alpha_2)[\cos(\beta_2\omega)(\vec{\Omega}_1 - (\vec{\Omega}_1\cdot\hat{e}_2)\hat{e}_2)\\ &- \sin(\beta_2\omega)(\vec{\Omega}_1\times\hat{e}_2)] + i\sinh(\alpha_2)[\cos(\beta_2\omega+\frac{\pi}{2})(\vec{\Omega}_1 - (\vec{\Omega}_1\cdot\hat{e}_2)\hat{e}_2)\\ &- \sin(\beta_2\omega+\frac{\pi}{2})(\vec{\Omega}_1\times\hat{e}_2)]\end{aligned} \tag{11}$$

where α_n and β_n represent PDL and DGD of each trunk, respectively, $\vec{\Omega}_1$ is the principal state vector of the first fiber section and is defined by $\vec{\Omega}_1 = \beta_1\cdot\hat{e}_1$. We assume the directions of the vectors $\vec{\alpha}$ and $\vec{\beta}$ are same. Since $\vec{\Omega}_1$ and $\hat{e}_2$ are real vectors, the wedge product in (6) becomes the cross product in Eq. (11).

As we know from the above expression, even if each fiber section has a frequency-independent principal state vector, the whole principal state vector of the concatenated fibers is dependent on frequency. In other words, the second-order PMD is created. The second-order PMD is defined as the frequency derivative of the principal state vector.

When PDL exists, the principal state vector becomes a complex vector, and can be written as $\vec{\Omega} = \vec{\Omega}_r + i\vec{\Omega}_i$. As shown in Fig. 3, PDL_1 does not affect the final vector $\vec{\Omega}$.

$\vec{\Omega}_{1\perp}$, the component of $\vec{\Omega}_1$ orthogonal to $\hat{e}_2$, is rotated by $\beta_2\omega$ in the plane orthogonal to $\hat{e}_2$, and weighted by $\cosh(\alpha_2)$. The imaginary term orthogonal to the rotated $\vec{\Omega}_{1\perp}$ is created in the same plane being weighted by $\sinh(\alpha)$. Because $\hat{e}_2$, $\vec{\Omega}_{1\perp}$, and imaginary

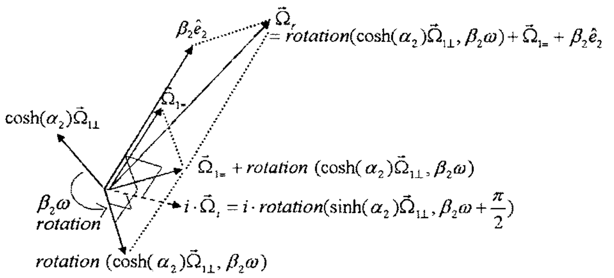

FIG. 3. The diagram of $\vec{\Omega}$ concatenation. ($\hat{e}_2$: the unit vector describing the direction of polarization and attenuation of the second fiber section, $\vec{\Omega}_{1\perp}$: the component of $\vec{\Omega}_1$ orthogonal to $\hat{e}_2$, $\vec{\Omega}_{1=}$: component of $\vec{\Omega}_1$ parallel to $\hat{e}_2$).

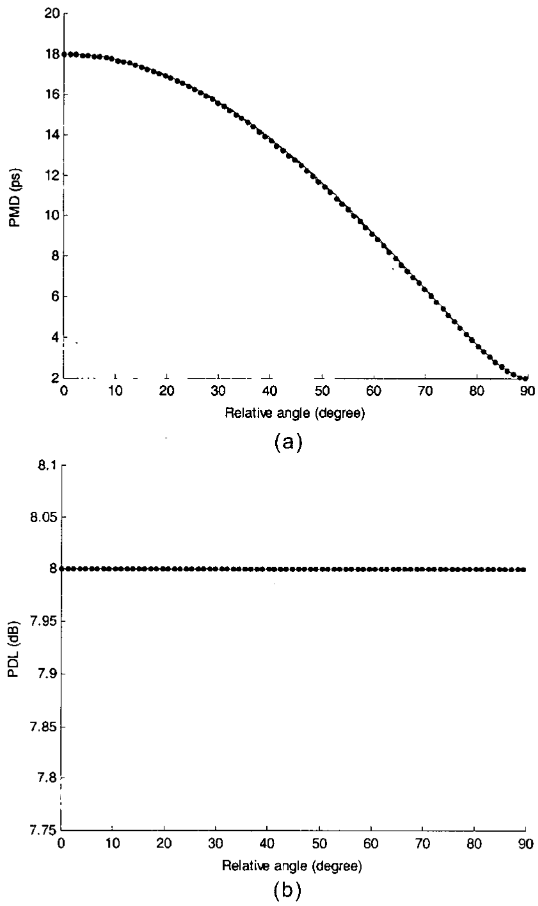

FIG. 4. (a) The dependence of the effective overall PMD on chirping when PDL_1 = 8 dB, PDL_2 = 0 dB, PMD_1 = 8 ps, PMD_2 = 10 ps. Solid curve: chirp-free, dotted curve: chirp =1, dashed curve: chirp=2, dash-dot curve: chirp=3, filled symbol: chirp-free pulse with pulse width of $100/\sqrt{10}$ ps. Since the differences among the lines are slight, the lines are overlapped. (b) The dependence of the effective overall PDL on chirping when the parameters are the same as in (a).

term $\vec{\Omega}_i$ are orthogonal to each other, real and imaginary terms of the concatenated principal state vector $\vec{\Omega}$ are orthogonal. Because PDL_2 increases the component of $\vec{\Omega}_1$ orthogonal to rotational axis, the frequency dependency of the direction of $\vec{\Omega}$ vector is increased. PMD_1 also decreases the frequency dependence of $\vec{\Omega}$ vector. Moreover, when the relative angle is 45°, i.e. $\vec{\Omega}_{1=} = 0$, the frequency dependence of the $\vec{\Omega}$ vector is maximized. Because the direction of PSP's on the Poincare sphere is different from the direction of $\vec{\Omega}$ generally [1], we cannot conclude simply that the frequency dependency of $\vec{\Omega}$ leads to the frequency dependency of PSP's. However, fortunately in this given configuration we can know $\vec{\Omega}_r \cdot \vec{\Omega}_i = 0$ as shown in (11) and Fig. 3. Therefore, by straightforward calculation we can know in this case, χ, the eigenvalue of $T'T^{-1}$, is real and equal to DGD in PDL-free case and therefore, the magnitude of $\chi = \delta\tau$ is frequency-invariant, moreover, the direction of $\vec{\Omega}$ is the same as the direction of the normalized principal state vector in (7). This fact guarantees that the frequency dependency of $\vec{\Omega}$ leads to the frequency dependency of PSP's.

We select the PDL configuration which minimizes frequency dependency of PSP's, i.e. PDL_1 is much higher than PDL_2 , $PDL_2 = 0$ dB, while the sum of PDL_1 and PDL_2 is maintained. Figs. 4(a) and (b) show PMD and PDL in the case of $PDL_1 = 8$ dB and $PDL_2 = 0$ dB. Therefore, the frequency dependency of PSP's is minimized while PMD is maintained, and we can check from the figure that chirp and pulse width do not particularly affect the effective overall PMD and PDL. In simulations, this tendency is maintained as we change the combination of PDL_1 and PDL_2 of the fiber concatenation.

IV. CONCLUSION

We have investigated a relation among PMD, PDL and chirping in a fiber concatenation. We have considered two concatenated uniform fibers with both PMD and PDL. We have assumed the input pulse to be chirped-Gaussian. Then effective PMD and PDL of the concatenated fibers have been calculated while changing the relative angle. We find that the effective PMD increases with the chirp parameter from the numerical simulation. The reason is not simply that the bandwidth of the pulse is broadened but that a term of $< t >$ is affected by chirp. And PDL decreases with the chirp parameter like a chirp-free short pulse with the same bandwidth as the chirped pulse. For PDL, chirping just makes signal bandwidth broader, so makes the pulse more depolarized than a chirp-free pulse. Hence the more depolarized pulse suffers less from PDL. Therefore, if frequency dependency of PSP's is increased, the pulse will be more depolarized and suffer less PDL. As shown in the calculation of the principal state vector of the concatenated fibers, PDL_2 increases the frequency dependency of the direction of the principal state vector. In other words, PDL_2 increases the second-order PMD. To verify this conclusion we have accomplished the simulation in which PDL_2 is minimized. If frequency dependency of PSP's is minimized while total PMD is maintained, the effect of chirp and pulse width on PDL is also minimized.

*Corresponding author : byoungho@snu.ac.kr.

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