Input Constrained Receding Horizon H_∞ Control: Quadratic Programming Approach

Young Il Lee

Abstract: This work is a modified version of an earlier work that was based on ellipsoidal type feasible sets. Unlike the earlier work, polyhedral types of invariant and feasible sets are adopted to deal with input constraints. The use of polyhedral sets enables the formulation of on-line algorithm in terms of QP (Quadratic Programming), which can be solved more efficiently than semi-definite algorithms. A simple numerical example shows that the proposed method yields larger stabilizable sets with greater bounds on disturbances than is the case in the earlier approach.

Keywords: Disturbance rejection, input constraints, polyhedral sets, invariance and feasibility.

1. INTRODUCTION

A constrained receding horizon H_∞ predictive control (RHHC) is derived in the form of Quadratic-Programming (QP), that not only guarantees stability but also provides an induced l_2 norm bound from disturbance to state. This norm bounding property will be referred to as the "disturbance boundedness property" and the induced l_2 norm as "disturbance H_∞ norm." Such use will be made of a min-max formulation, which is known to be an effective way of synthesizing robust controllers using H_∞ concepts [1, 4-6]. Input constraint was taken into account in [1] based on ellipsoidal invariant sets and the problem is solved via semi-definite programming. The concern here is to modify this approach using polyhedral invariant sets, so that it is formulated in the form of QP which can be solved more efficiently than the semi-definite programming.

A closed-loop prediction strategy [1-3] will be deployed to reduce the effect of disturbances in state predictions. First the worst case disturbances are computed as a function of current state and future control inputs and then they are substituted in the min-max formulation, to yield a quadratic cost that can be minimized using QP. It is shown that for controllable plants, there exist terminal weights which achieve closed-loop stability while keeping the effect of disturbances within prescribed bounds. In the case of input saturation, stability of the system is determined by the existence of feasible inputs, which steer the initial state into a feasible and invariant set in a finite number of time steps.

2. RECEDING HORIZON H_∞ PREDICTIVE CONTROL

Consider a linear time invariant system described by

$$x_{k+1} = A x_k + Bu_k + D\omega_k \tag{1}$$

and a constant state feedback gain F, where

$$x \in R^n, \quad u_k \in R^m, \quad \omega_k \in R^q,$$
$$|u_k| \le u_{\lim} \tag{2}$$

u_k and y_k are the system input and output at time k, and ω_k is a disturbance on the system. In the sequel, modulus of a vector/matrix and inequalities between vectors are defined as

$$|M| = \left|\begin{bmatrix} m_{1,1} & m_{1,2} & \cdots & m_{1,p} \\ m_{2,1} & m_{2,2} & \cdots & m_{2,p} \\ \vdots & & \cdots & \vdots \\ m_{q,1} & m_{q,2} & \cdots & m_{q,p} \end{bmatrix}\right| := \begin{bmatrix} |m_{1,1}| & |m_{1,2}| & \cdots & |m_{1,p}| \\ |m_{2,1}| & |m_{2,2}| & \cdots & |m_{2,p}| \\ \vdots & & \cdots & \vdots \\ |m_{q,1}| & |m_{q,2}| & \cdots & |m_{q,p}| \end{bmatrix},$$

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_p \end{bmatrix} \le \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_p \end{bmatrix} \text{ implies } \begin{bmatrix} \alpha_1 \le \beta_1 \\ \alpha_2 \le \beta_2 \\ \vdots \\ \alpha_p \le \beta_p \end{bmatrix}.$$

Our aim here is to establish a strategy for computing

Manuscript received April 10, 2002; accepted August 2, 2002. This work was supported by a research fund from the Seoul National University of Technology.

Young Il Lee is with the Department of Control and Instrumentation, Seoul National University of Technology, Gongreung 2-dong, Nowon-gu, Seoul 139-743, Korea. (e-mail: yilee@snut.ac.kr).

perturbations, c_k, on the state feedback control Fx_k i.e.

$$u_k = Fx_k + c_k, \quad (3)$$

so that the H_∞-norm of the transfer function $T_{x\varpi}$ from ω to x_k is bounded by a prescribed value γ^2. System (1) with control (3) can be rewritten as

$$x_{k+1} = A_c x_k + B c_k + D \omega_k, \quad A_c := A + BF \quad (4)$$

with constraints

$$| Fx_k + c_k | \le u_{\lim}. \quad (5)$$

At the present time x_k, the future states $x_{k+j}, j = 1, 2, \cdots, N$ can be predicted as follows:

$$\bar{x}_k = \bar{A} x_k + \bar{B} \bar{c}_k + \bar{D} \bar{\omega}_k, \quad (6)$$

where

$$\bar{x}_k := [x'_{k+1|k} \quad x'_{k+2|k} \quad \cdots \quad x'_{k+N|k}]',$$

$$\bar{A} := [A_c \quad A_c^2 \quad \cdots \quad A_c^N]',$$

$$\bar{c}_k := [c'_{k|k} \quad c'_{k+1|k} \quad \cdots \quad c'_{k+N-1|k}]',$$

$$\bar{\omega}_k := [\omega'_{k|k} \quad \omega'_{k+1|k} \quad \cdots \quad \omega'_{k+N-1|k}]',$$

$$\bar{B} := \begin{bmatrix} B & 0 & \cdots & 0 \\ A_c B & B & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ A_c^{N-1} B & A_c^{N-2} B & \cdots & B \end{bmatrix},$$

$$\bar{D} := \begin{bmatrix} D & 0 & \cdots & 0 \\ A_c D & D & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ A_c^{N-1} D & A_c^{N-2} D & \cdots & D \end{bmatrix},$$

and $x_{k+i|k}$ is the predicted value of x_{k+i} based on data available at time k. Using the vector representation (6), a finite horizon cost index with positive definite weights Q, R and Ψ can be written in the following one-shot form [1]:

$$J_k(\bar{c}_k, \bar{\omega}_k) = \left\| \bar{A} x_k + \bar{B} \bar{c}_k + \bar{D} \bar{\omega}_k \right\|_{\bar{Q}} + \left\| \bar{c}_k \right\|_{\bar{R}} - \gamma^2 \left\| \bar{\omega}_k \right\|_I \quad (7)$$

where

$$\|x\|_Q := x'Qx,$$

$$\bar{Q} := diag(Q, \cdots, Q, \Psi), \ \bar{R} := diag(R, \cdots, R),$$

and $diag(Q, \cdots, Q)$ is a block diagonal matrix with $Q, \cdots, Q$ as diagonal block matrices.

We define the discrete game as the following problem:

$$\min_{\bar{c}_k} \max_{\bar{\omega}_k} J_k(\bar{c}_k, \bar{\omega}_k). \quad (8)$$

The maximizing disturbance $\bar{\omega}_k^*(\bar{c}_k)$ is obtained by $\frac{\partial J_k}{\partial \bar{\omega}_k} = 0$ as

$$\bar{\omega}_k^*(\bar{c}_k) = (\gamma^2 I - \bar{D}'\bar{Q}\bar{D})^{-1} \bar{D}'\bar{Q}(\bar{A} x_k + \bar{B} \bar{c}_k), \quad (9)$$

$$\frac{\partial^2 J_k}{\partial \bar{\omega}_k^2} = 2(\bar{D}'\bar{Q}\bar{D} - \gamma^2 I) \prec 0, \quad (10)$$

where $M \prec 0$ denotes that M is negative definite. The cost of (7), with $\bar{\omega}_k^*(\bar{c}_k) = [\omega_{k|k}^*(\bar{c}_k)' \omega_{k+1|k}^*(\bar{c}_k)' \cdots, \omega_{k+n-1|k}^*(\bar{c}_k)]$ of (9), can be written as

$$J_k(\bar{c}_k, \bar{\omega}_k^*(\bar{c}_k)) = \| \bar{A} x_k + \bar{B} \bar{c}_k \|_{\bar{Q}_{off}} + \| \bar{c}_k \|_{\bar{R}}, \quad (11)$$

where $\bar{Q}_{off} := (\bar{Q} + \bar{Q}\bar{D}\Omega^{-1}\bar{D}'\bar{Q}), \Omega := \gamma^2 - \bar{D}'\bar{Q}\bar{D}$, and (8) is converted to

$$\bar{c}_k^* = \arg\{\min_{\bar{c}_k} J_k(\bar{c}_k, \bar{\omega}_k^*(\bar{c}_k))\}, \quad (12)$$

$$u_{\lim} \le Fx_{k+i|k}(\bar{\omega}_k^*) + c_{k+i|k} \le u_{\lim},$$

for $i = 0, 1, \cdots, N-1$, where $x_{k+i|k}(\bar{\omega}_k^*)$ is the state at $k+i$ when disturbances $\bar{\omega}_k^*(\bar{c}_k)$ are applied to the system for a given x_k and perturbations $c_{k+j}, j = 0, 1, \cdots, i-1$. Note that the cost index (11) is quadratic in terms of $\bar{c}_k$ and the minimization problem of (12) can be solved via well-known QP methods.

Remark 1: The use of F in (3) makes the values of predicted control inputs depend on the predicted states, which means that the predictions are made in closed-loop sense. The relations (6) and (10) show that the predictions of states and the lower bound on γ depend on the choice of F. Thus, the choice optimal F in the sense of disturbance H_∞ norm would be an interesting problem although not pursued in this work. □

The Receding Horizon H_∞ Predictive Control (RHHC) strategy is to compute the optimal $\bar{c}_k^*$ by solving the QP problem of (12) and apply only the first element of $\bar{c}_k^*$ at time k, at the time $k+1$, $\bar{c}_{k+1}^*$ will be obtained for the receded future horizon and the same procedure will be repeated thereafter. In order to guarantee the closed-loop stability of RHHC, it is required to have additional constraints on the problem (12) that the terminal predicted state belongs to a feasible and invariant target set [1]. Unlike the earlier work [1], here we adopt a polyhedral type feasible and invariant set as described in the next section.

3. STABILITY AND H_∞ NORM BOUND WITH INPUT SATURATION

Here we will follow the approach used in [2]: First define a polyhedral set of states, $\Re_F^W$, which is robustly invariant with respect to state feedback law $u = Fx$ in the presence of bounded disturbances. Then, for a measured state x_k, compute perturbations $c_{k+j|k}, j = 0,1,\cdots,N-1$ which minimizes $J_k(\overline{c}_k, \overline{\omega}_k^*(\overline{c}_k))$ while guaranteeing that $Fx_{k+i|k}(\overline{\omega}_k) + c_{k+i|k}$ $(i = 0,1,\cdots,N-1)$ are feasible and $x_{k+N|k}(\overline{c}_k, \overline{\omega}_k) \in \Re_F^W$ despite disturbances. Note that, because of the invariance property of $\Re_F^W$ with respect to the feedback gain F, $\overline{c}_k^*$ satisfying the above mentioned feasibility and membership conditions guarantees the feasibility of $\hat{c}_{k+1} := [c_{1|k} c_{2|k} \cdots c_{N-1|k} 0]$ for state x_{k+1} provided that ω_k is bounded properly.
We assume that the disturbance is bounded as:

$$|\omega_k| \le \omega_{\lim} \tag{13}$$

In order to define a feasible and invariant set, we consider a state transformation matrix W, which is a design parameter to make the sets non-empty. Using the state transformation $z = Wx$, the closed-loop state equation (5) can be transformed into:

$$z_{k+1} = A_c^W z_k + WB c_k + WD \omega_k, \tag{14}$$

where $A_c^W = WA_cW^{-1}$. Based on the relation (14), feasible and invariant set, $\Re_F^W$, can be defined as per the following lemma.

Lemma 1: Consider the system (1) with transformed state $z = Wx$ and a stabilizing state feedback gain F. A set of states defined as:

$$\Re_F^W(\alpha) = \{x \,||\, z| \le \alpha\}, \tag{15}$$

where α is a $n \times 1$ vector with positive elements, is feasible and invariant with respect to F despite the bounded disturbance *i.e.* for any $x \in \Re_F^W(\alpha)$ and ω satisfying (13), $u = Fx$ satisfies the input constraint (2) and makes the state remain in the set if and only if:

$$|A_c^W|\alpha + |WD|\omega_{\lim} \le \alpha \tag{16}$$

$$|FW^{-1}|\alpha \le u_{\lim}, \tag{17}$$

are met.

Proof: Assume that $x_k \in \Re_F^W(\alpha)$ from (16) and (17), use of $u_k = Fx_k$ yields:

$$\begin{aligned} |z_{k+1}| &= |A_c^W z_k + WD\omega_k| \\ &\le |A_c^W z_k| + |WD\omega_k| \\ &\le |A_c^W|\alpha + |WD|\omega_{\lim} \\ &\le \alpha, \end{aligned} \tag{18}$$

and from (17) we have:

$$|Fx_k| = |FW^{-1}z_k| \le |FW^{-1}|\alpha \le \omega_{\lim}. \tag{19}$$

Relation (18) implies $x_{k+1} \in \Re_F^W(\alpha)$ and (19) guarantees that $u = Fx$ satisfies the input constraint (2) for any x in the set. This proves the sufficiency part of the theorem. The observation that we can choose $u = Fx$ and ω_k among the states of $\Re_F^W(\alpha)$ and bounded disturbances, respectively, so that the relations (18-19) hold with equality (elementwise) for any given A_c^W, F, W and D proves that conditions (16-17) are necessary for the invariance and feasibility of the set. □

Predictions of future states can be made as follows based on (14):

$$z_{k+i|k} = (A_c^W)^i z_k + \sum_{j=1}^{i}(A_c^W)^{i-j} WBc_{k+j-1} + \sum_{j=1}^{i}(A_c^W)^{i-j} WD\omega_{k+j-1}. \tag{20}$$

It is possible to compute elementwise maximum/minimum values of $z_{k+i|k}$ based on (13) and (20). The following lemma summarizes conditions under which x_k is steered into $\Re_F^W(\alpha)$, *i.e.* $x_{k+N|k} \in \Re_F^W(\alpha)$ using feasible perturbations $c_{k|k} c_{k+1|k} \cdots c_{k+N-1|k}$.

Lemma 2: Consider system (1) with transformed state $z = Wx$ and set $\Re_F^W(\alpha)$ defined as (15) with respect to a state feedback gain F. A state x_k is guaranteed to be steered into set $\Re_F^W(\alpha)$ in N control steps, *i.e.* $x_{k+N|k} \in \Re_F^W(\alpha)$ despite bounded disturbances (13) if:

$$|F^{W+} z_{k+i-1|k}^{\max} - F^{W-} z_{k+i-1|k}^{\min} + c_{k+i-1|k}| \le \omega_{\lim} \tag{21}$$

$$|F^{W+} z_{k+i-1|k}^{\min} - F^{W-} z_{k+i-1|k}^{\max} + c_{k+i-1|k}| \le \omega_{\lim} \tag{22}$$

$$|z^{\max}_{k+N|k}| \le \alpha, \qquad |z^{\min}_{k+N|k}| \le \alpha, \tag{23}$$

are met, where

$$z^{\max}_{k+i|k} := (A_c^W)^i z_k + \sum_{j=1}^{i} (A_c^W)^{i-j} WBc_{k+j-1|k} + \sum_{j=1}^{i} |(A_c^W)^{i-j} WD| \omega_{\lim} \tag{24}$$

$$z^{\min}_{k+i|k} := (A_c^W)^i z_k + \sum_{j=1}^{i} (A_c^W)^{i-j} WBc_{k+j-1|k} - \sum_{j=1}^{i} |(A_c^W)^{i-j} WD| \omega_{\lim}, \tag{25}$$

for $i = 1,2,\cdots,N$ with $z^{\max}_{k|k} = z^{\min}_{k|k} = z_k$ and $F^W := FW^{-1}$, $M^+ := \max(M,0)$, and $M^- := \max(-M,0)$. (the choice of maximum value is done elementwise)

Proof: The proof is based on the fact that the min/max value of Mz for $z_{\min} \le z \le z_{\max}$ can be represented as:

$$\max_{z_{\min} \le z \le z_{\max}} Mz = M^+ z_{\max} - M^- z_{\min} \tag{26}$$

$$\min_{z_{\min} \le z \le z_{\max}} Mz = M^+ z_{\min} - M^- z_{\max}. \tag{27}$$

Applying the above facts to (20), we obtain $z^{\max}_{\cdot|k}, z^{\min}_{\cdot|k}$ in (24), (25), which are maximum and minimum possible values of $z_{\cdot|k}$, respectively, subject to disturbances satisfying (13). For the given bounds $z^{\max}_{\cdot|k}, z^{\min}_{\cdot|k}$ on $z_{\cdot|k}$ bounds on $z_{\cdot|k}$ also can be obtained in the same way to yield the feasibility condition (21-22). It is easy to see that (23) ensures $x_{k+N|k} \in \Re^W_F(\alpha)$, since $z^{\max}_{\cdot|k}, z^{\min}_{\cdot|k}$ are maximum and minimum possible values of the transformed terminal state. □

Note that relations (16-17) and (21-23) are linear inequalities with respect to $\alpha, c_{\cdot|k}, z^{\max}_{\cdot|k}$ and $z^{\min}_{\cdot|k}$. Now, we are ready to summarize the constrained receding horizon algorithm as follows:

Algorithm RHHC

Step 1: At time instant k and for the measured state x_k solve the QP problem of minimizing cost index (11) subject to constraints (16-17) and (21-23) with variables to $\alpha, c_{\cdot|k}, z^{\max}_{\cdot|k}$ and $z^{\min}_{\cdot|k}$.

Step 2: Apply $u_k = Fx_k + c_{k|k}$ to the system.

Step 3: At the next time $k+1$, repeat Step 1 and 2.

Allowing α searched on-line enables us to use the union of $\Re^W_F(\alpha)$ as our target set *i.e.* Step1 of Algorithm RHHC ensures

$$x_{k+N|k} \in \Re^W_F = \bigcup_{\alpha \in S_\alpha} \Re^W_F(\alpha), \tag{28}$$

where S_α denotes the set of α satisfying (16-17). The stability of Algorithm RHHC can be summarized as per the following theorem.

Theorem 1: Algorithm RHHC is guaranteed to be feasible and keeps the state bounded while the truncated induced H_∞ norm is bounded as γ:

$$\frac{\sum_{i=1}^{L} \| x_{k+i} \|_Q}{\sum_{i=0}^{L-1} \| \omega_{k+i} \|_I} \le \gamma^2 \tag{29}$$

provided that

1) ω_{k+i} is bounded as (13) for all $i \ge 0$.\
2) F, γ, ρ and Ψ satisfy

$$\gamma^2 I - \bar{D}'\bar{Q}\bar{D} \succ 0 \tag{30}$$

$$\Psi \succ (1 + (\rho - 1)^{-1}) A_c{}'\Psi A_c + Q \tag{31}$$

$$\gamma^2 I - \rho D'\Psi D \succ 0 \tag{32}$$

3) an initial feasible solution are obtained in Step 1, where Ψ is a terminal weight used in $\bar{Q}$.

Proof: If feasible perturbation $\bar{c}_k$ are obtained at time k then the existence of feasible perturbations $\bar{c}_{k+1}$ at time $k+1$ is guaranteed, since perturbations $\hat{c}_{k+1} := [c_{1|k} c_{2|k} \cdots c_{N-1|k} 0]$ will provide one feasible set of perturbations at time $k+1$. This argument can be applied recursively to yield the guaranteed feasibility of the algorithm. Using the procedure used in [1], we have:

$$J_k(\bar{c}^*_k, \bar{\omega}^*_k(\bar{c}^*_k)) \ge \sum_{i=1}^{L} (\| x_{k+i} \|_Q + \| c^*_{k+i-1|k+i-1} \|_R) - \gamma^2 \sum_{i=0}^{L-1} \|\omega_{k+i}\|_I, \tag{33}$$

Provided that (31) is satisfied. Since we are interested in the induced l_2 norm from disturbance to state, we take x_k to be zero to remove the effect of initial state on x_{k+i} $(i \ge 1)$. By (11), $J_k(\bar{c}^*_k, \bar{\omega}^*_k(\bar{c}^*_k)) = 0$ for zero x_k. Thus, we obtain condition (29).

Now for the boundedness of the state consider a set of states $\Re^W_F(N)$ such that for any state $x \in \Re^W_F(N)$, there exist a bounded input sequence $[c_0(x) c_1(x) \cdots c_{N-1}(x)]$ which steers x into $\Re^W_F$. It is easy to see that the set $\Re_F^W(N)$ is bounded because any state of it can be steered into a bounded set of states $\Re_F^W$ in a finite number of steps using bounded inputs. Due to the guaranteed feasibility of Algorithm RHHC, it is obvious that the actual state of the system must lie in $\Re_F^W(N)$ and therefore will be bounded.

In the case when the disturbance is an l_2 signal, we can establish additional asymptotic stability result.

Corollary 1: If the disturbance ω_k is bounded as (13) for every $k \geq 0$ and defines a sequence in l_2, then the closed loop system is asymptotically stable with guaranteed disturbance H_∞ norm bound *i.e.*

$$\frac{\sum_{i=1}^{\infty} \| x_{k+i} \|_Q}{\sum_{i=0}^{\infty} \| \omega_{k+i} \|_I} \leq \gamma^2 \tag{34}$$

under the conditions of Theorem 3.

Proof: The procedure of [1] and the guarantee of feasibility, gives:

$$\sum_{i=1}^{\infty} (\| x_{k+i} \|_Q + \| c^*_{k+i-1|k+i-1} \|_R) \tag{35}$$

$$\leq \gamma^2 \sum_{i=0}^{\infty} \| \omega_{k+i} \|_I - J_k(\bar{c}_k^*, \bar{\omega}_k^*(\bar{c}_k^*))$$

$$< \infty.$$

This suggests that x_{k+i} goes to zero asymptotically for $Q \succ 0$, since its sum is bounded from above. Similarly c_{k+i} goes to zero asymptotically for $R \succ 0$ and this proves asymptotic stability. For $x_k = 0$, $J_k(\bar{c}_k^*, \bar{\omega}_k^*(\bar{c}_k^*)) = 0$ and this in turn implies the disturbance H_∞ norm bound (34). □

The result above takes into account the input limits and therefore requires that $x_{k+N|k} \in \Re_F^W(\alpha)$, $|\omega_{k+i}| \leq \omega_{\lim}$ for $i \geq 0$ as well as that the matrices F, Ψ should satisfy the relations (30-32) for some $|\omega_k| \leq \omega_{\lim}$. The value of ρ can be regarded as a design factor. As it approaches 1, condition (31) is satisfied by only those stabilizing controllers F that result in closed loop poles that are close to the origin. Such controllers however, would result in dead-beat like predicted trajectories for which, in the presence of input saturation, the feasibility region would be rather limited. Conversely, for arbitrary large ρ, condition (31) can be satisfied for almost the entire class of stabilizing F, thereby yielding an enlarged feasibility region. By (32) however, large ρ imply large γ and this suggests that there exists a trade off between feasibility, which requires large ρ and disturbance rejection making small γ desirable.

4. NUMERICAL EXAMPLE

Consider the trivial scalar dynamics [1, 4]:

$$x_k + 1 = x_k + u_k + \omega_k . \tag{36}$$

For $F = -0.8$, the values $N = 3$, $\Psi = 1.15$ $Q = R = 1$ satisfy the stability conditions (30-32), with $K = 0, \rho = 1.6348$ and $\gamma^2 = 1.9$, which is less than the norm bound obtained in [4]. When $u_{\lim} = 1$, we obtain a feasible and invariant set $\Re_F^W = \{x \mid -1.25 \leq x \leq 1.25\}$ with allowable bound on disturbance $\omega_{\lim} = 1$ which is larger than the case of [1] given as $\{x \mid -1.118 \leq x \leq 1.118\}$ and $\omega_{\lim} = 0.44$, respectively.

5. CONCLUSIONS

The RHHC method was modified by replacing the ellipsoidal type invariant sets with polyhedral type invariant sets. Use of polyhedral invariant sets enables the formulation of RHHC in terms of Quadratic Programming which can be solved more efficiently than the semi-definite programming adopted in the earlier work. Furthermore, it was shown that the resulting feasible invariant set is larger than that of ellipsoidal approach with larger disturbance bound for an example .

REFERENCES

[1] Y. I. Lee and B. Kouvaritakis, "Receding horizon H_∞ predictive control for systems with input saturation," *IEE Proc.-D*, vol. 147, no. 2, pp.153-158, 2000.

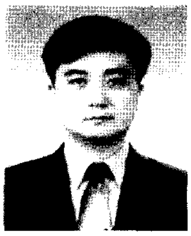
[2] Y. I. Lee and B. Kouvaritakis, "Constrained receding horizon predictive control for systems with disturbances," *Int. J. of Control*, vol. 72, no. 11, pp. 1027-1032, 1999.

[3] Y. I. Lee and B. Kouvaritakis, "A linear programming approach to constrained robust predictive control," *IEEE Trans. on AC*, vol. 45, no. 9, pp. 1765-1770, 2000.

[4] T. Basar, "A dynamic games approach to controller design: Disturbance rejection in discrete-time," *IEEE Trans. on AC*, vol. 36, no. 8, pp. 936-951, 1991.

[5] J. W. Lee, W. H. Kwon, and J. H. Lee, "Receding Horizon H_∞ control for time-varying discrete linear systems," *Int. J. of Control*, vol. 68, no. 2, pp. 385-399, 1997.

[6] I. Yaesh and U. Shaked, "Minimum H_∞ norm regulation of linear discrete-time systems and its relation to linear quadratic discrete games," *IEEE Trans on AC*, vol. 35, pp. 1061-1064, 1990.



Young Il Lee received the B.S. (1986), M.S. (1988) and Ph.D. (1993) degrees in control and instrumentation engineering from Seoul National University. He was a visiting research fellow of the Department of Engineering Science, Oxford University from February 1998 to July 1999. He was with the Department of Control and Instrumentation Engineering at the Gyeongsang National University from 1994 to 2001. Since 2001, he has been with Seoul National University of Technology, where he is currently Associate Professor. His research interests are in the areas of receding horizon control, automatic tuning of PID controller, and Home-networking system.