A Suboptimal Algorithm of the Optimal Bayesian Filter Based on the Receding Horizon Strategy

Yong-Shik Kim and Keum-Shik Hong

Abstract: The optimal Bayesian filter for a single target is known to provide the best tracking performance in a cluttered environment. However, its main drawback is the increase in memory size and computation quantity over time. In this paper, the inevitable predicament of the optimal Bayesian filter is resolved in a suboptimal fashion through the use of a receding horizon strategy. As a result, the problems of memory and computational requirements are diminished. As *a priori* information, the horizon initial state is estimated from the validated measurements on the receding horizon. Consequently, the suboptimal algorithm proposed allows for real time implementation.

Keywords: State estimation, target-tracking, optimal Bayesian filter, clutter, receding horizon.

1. INTRODUCTION

In a cluttered environment, the target-tracking problem naturally involves uncertainty associated with the measurements as well as modeling inaccuracy. This uncertainty is related to revealing the origin of measurements, since the measurements may not have originated from the target of interest [1]. This problem was not recognized until the publication in 1964 of Sittler's first paper [14]. The pioneering work of Sittler was motivated by the need to find a reasonable way of incorporating the measurements of uncertain origin into existing tracks. However, since his method was based on a non-Bayesian approach, the resulting state estimate and covariance do not account for the possibility that the determined decisions are incorrect.

The Bayesian procedures use the "nearest neighbor" of the predicted measurement wherein the Kalman filter is modified to account for the *a priori* probability that the measurement might be spurious. This filter utilizes only the sensor reports that are statistically close to the predicted track measurement for track updating and calculates its data association performance parameters based on averaging over *a priori* statistics. Singer and Sea [12] extended the Bayesian approach to develop an optimal tracking filter within the class of nearest-neighbor filters that utilize *a priori* statistics for estimating correlation performance.

The need to incorporate all the observations lying in the neighborhood of the predicted measurement was pointed out by Bar-Shalom and Jaffer [2], where a suboptimal algorithm using *a posteriori* probabilities was presented. In [2], it was suggested that *a posteriori* correlation statistics, calculated on-line and based on all reports in the vicinity of a track (i.e., all-neighbors approach) should be used to obtain the best possible tracking performance based on all available data provided by the surveillance sensor.

In [13], the theoretical formulation of an optimal filter using the *a posteriori* probability and all-neighbors class was completely carried out. This filter requires an expending memory and utilizes the data located around the vicinity of the track, accounting accurately for the possibility that any particular report among these data may either be extraneous or have originated from the track. However, this filter is quite unsuitable for real-time application in dense multi-target environments.

Several approaches [1, 3, 8] for limiting memory growth and computation requirements, while still providing a reasonable approximation to the performance of the optimal filter, were proposed. In [8], the optimal *a posteriori* filter of Singer et al. [13] was combined with an adaptive filter. The resulting filter requires an expanding memory. A (M,N) scan approximation [8], as opposed to an N scan approximation, was used by Singer et al. [13]. This proposal was presented in order to obtain an algorithm with stable memory requirements. In this approximation those measurement histories which were identical for

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the most recent N scans and those input histories which were identical for the most recent M scans were combined together into new histories.

The memory growth and computation problem for real-time implementation has also been a critical issue in Kalman filtering. Hence, finite memory filters [6, 10] were suggested as a solution to overcome the poor performance or divergence due to the modeling errors of the standard Kalman filter. Finite memory filters are also useful in situations in which a system model is valid over a finite interval. In [9], a receding horizon Kalman FIR filter that combines the Kalman filter and the receding horizon strategy was presented. In their work it was shown that the suggested filter processes the unbiased property and the deadbeat property irrespective of any initial horizon condition. In [5], a receding horizon Kalman FIR filter including the estimation of the horizon initial state was investigated. Furthermore, an estimation and detection technique for the unknown inputs using an optimal FIR filter was presented [11].

The optimal Bayesian filter in a cluttered environment, though it has demonstrated enhanced performance to other filters, has drawn little attention due to exponentially increasing memory and computation requirements. The main contributions of this paper are: first to derive a suboptimal approach using only the measurements in a receding horizon. As a result, the increasing memory and computation requirements are diminished. Second, the horizon initial state is estimated from only validated measurements on the receding horizon. Third, the suboptimal algorithm solves the real-time implementation problem of the optimal Bayesian filter.

This paper is organized as follows: In Section 2, a suboptimal algorithm for the optimal Bayesian filter is derived, and the horizon initial state estimate and its covariance are obtained. *A posterior* probability of the validated measurements on the receding horizon is provided in Section 3. In Section 4, conclusions are stated.

2. A NEW SUBOPTIMAL ALGORITHM

Consider the following state-space representation of the target motion and observation

$$x_{k+1} = F_k x_k + \omega_k , \quad (1)$$

$$y_k = H_k x_k + v_k , \quad (2)$$

with ω_k and v_k being zero-mean mutually independent white Gaussian noises with covariances Q_k and R_k, respectively. The suboptimal algorithm in the sequel does not use all measurements observed from the initial time up to the present time k, but uses only a set of measurements observed in some interval with fixed window-size N, i.e., on the receding horizon interval $[k-N,\ k]$.

Assumption 1: The possibility of a false track initiation is not considered in this paper. Hence, the horizon initial estimate and its covariance, as *a priori* information which will be estimated in the suboptimal algorithm, are assumed to be in a correct track.

Let the set of validated measurements obtained at time k be

$$Y_k = \{Y_{k,i}\}_{i=1}^{m_k},$$

where m_k is the number of measurements in the validation region and $Y_{k,i}$ is the set of validated measurements obtained at time k when the number of validated measurements are i. Let the set of measurements on the receding horizon $[k-N,k]$ at time k be denoted as

$$Y^k = \{Y_j\}_{j=k-N}^{k},$$

where a superscript k is used, while a subscript k was used in the set of validated measurements. A combination of measurements on the receding horizon $[k-N,\ k]$ at the k-th scan can be denoted as $Y^{k,l}$. Then, $Y^{k,l}$ is defined as follows:

$$Y^{k,l} \triangleq \{y_{k-N,i_l}, \cdots, y_{k,i_l}\} = \{Y^{k-1,s}, y_{k,i_l}\},$$

where $Y^{k-1,s}$ stands for the combination of measurements up to time $k-1$ at the ($k-1$)-th scan and y_{k,i_l} is the i_l-th measurement at time k. Denoting the event that the l-th history at time k is the correct sequence of measurements by $\theta^{k,l}$, its *a posteriori* probability, conditioned on Y^k, is given by

$$\beta^{k,l} = P\{\theta^{k,l} \mid Y^k\}. \quad (3)$$

Now, the following theorem is stated:

Theorem 1: When the measurements used are restricted within a receding horizon, the state estimate and error covariance equations of the optimal Bayesian filter take the following forms:

$$\hat{x}_{k_N+j+1|k} = \sum_{l=1}^{L_k} F(I + P^s_{k_N+j|k} H' R^{-1} H)^{-1} \beta^{k,l} \times (\hat{x}^s_{k_N+j|k} + P^s_{k_N+j|k} H' R^{-1} y_{k_N+j,i_l}),$$

$$P_{k_N+j+1|k} = \sum_{l=1}^{L_k} \beta^{k,l} F (I + P^s_{k_N+j|k} H' R^{-1} H)^{-1} P^s_{k_N+j|k} F' + \sum_{l=1}^{L_k} \beta^{k,l} \hat{x}^l_{k_N+j+1|k} \hat{x}'^l_{k_N+j+1|k} - \hat{x}_{k_N+j+1|k} \hat{x}'_{k_N+j+1|k}.$$

Proof: The conditional mean of the state at time k can be expressed as

$$\hat{x}_{k|k} \triangleq E[x_k \mid Y^k] = \sum_{l=1}^{L_k} E[x_k \mid \theta^{k,l}, Y^k] P\{\theta^{k,l} \mid Y^k\} = \sum_{l=1}^{L_k} \hat{x}_{k|k}^l \beta^{k,l} \tag{4}$$

where $\hat{x}_{k|k}^l = E[x_k \mid \theta^{k,l}, Y^k]$ is the history-conditioned estimate. L_k is the total number of measurement histories at time k as

$$L_k = \prod_{j=1}^{k} (1 + m_j), \tag{5}$$

where m_j is the number of measurements at time j. For each history, the state estimate conditioned upon the measurement history, $Y^{k,l}$, being correct is

$$\hat{x}_{k|k}^l = \hat{x}_{k|k-1}^s + K_k^l (y_{k,i_l} - \hat{y}_{k|k-1}^s) \tag{6}$$

where y_{k,i_l} is the measurement at time k in sequence l and $\hat{y}_{k|k-1}^s$ is the predicted measurement corresponding to history $Y^{k-1,s}$, with covariance S_k^s. The gain is

$$K_k^l = P_{k|k-1}^s H' [S_k^s]^{-1}, \tag{7}$$

and the covariance of the history-conditioned updated state is

$$P_{k|k}^l = E[(x_k - \hat{x}_{k|k}^l)(x_k - \hat{x}_{k|k}^l)' \mid \theta^{k,l}, Y^k] = [I - K_k^l H] P_{k|k-1}^s. \tag{8}$$

The conditional mean of the state at time k of (4) can be expressed as

$$\hat{x}_{k|k} = \sum_{l=1}^{L_k} \hat{x}_{k|k-1}^s \beta^{k,l} + \sum_{l=1}^{L_k} P_{k|k-1}^s H' (S_k^s)^{-1} (y_{k,i_l} - \hat{y}_{k|k-1}^s) \beta^{k,l} \tag{9}$$

And again, using $S_k^s = HP_{k-1|k-1}^s H' + R$ and the matrix inversion lemma, (9) is rewritten as follows:

$$\begin{aligned}\hat{x}_{k|k} &= \sum_{l=1}^{L_k} F\hat{x}_{k-1|k-1}^s \beta^{k,l} + \sum_{l=1}^{L_k} FP_{k-1|k-1}^s H' (HP_{k-1|k-1}^s H' + R)^{-1} \times (y_{k,i_l} - H\hat{x}_{k-1|k-1}^s) \beta^{k,l} \\
&= \sum_{l=1}^{L_k} \{F\beta^{k,l} - FP_{k-1|k-1}^s H' (HP_{k-1|k-1}^s H' + R)^{-1} H\beta^{k,l}\} \hat{x}_{k-1|k-1}^s \\
&\quad + \sum_{l=1}^{L_k} FP_{k-1|k-1}^s H' (HP_{k-1|k-1}^s H' + R)^{-1} y_{k,i_l} \beta^{k,l} \\
&= \sum_{l=1}^{L_k} F\{I - P_{k-1|k-1}^s H' (HP_{k-1|k-1}^s H' + R)^{-1} H\} \beta^{k,l} \hat{x}_{k-1|k-1}^s \\
&\quad + \sum_{l=1}^{L_k} F(P_{k-1|k-1}^{-s} + H'R^{-1}H)^{-1} H'R^{-1} \beta^{k,l} y_{k,i_l} \\
&= \sum_{l=1}^{L_k} F(I + P_{k-1|k-1}^s H'R^{-1}H)^{-1} \beta^{k,l} \hat{x}_{k-1|k-1}^s \\
&\quad + \sum_{l=1}^{L_k} F(P_{k-1|k-1}^{-s} + H'R^{-1}H)^{-1} H'R^{-1} \beta^{k,l} y_{k,i_l}.\end{aligned}$$

Combining terms, the final form is

$$\hat{x}_{k|k} = \sum_{l=1}^{L_k} F(I + P_{k-1|k-1}^s H'R^{-1}H)^{-1} \beta^{k,l} \times (\hat{x}_{k-1|k-1}^s + P_{k-1|k-1}^s H'R^{-1} y_{k,i_l}).$$

The covariance associated with the combined estimate is

$$\begin{aligned}P_{k|k} &= \sum_{l=1}^{L_k} \beta^{k,l} P_{k|k}^l + \sum_{l=1}^{L_k} \beta^{k,l} \hat{x}_{k|k}^l \hat{x}_{k|k}'^l - \hat{x}_{k|k}\hat{x}_{k|k}' \\
&= \sum_{l=1}^{L_k} \beta^{k,l} (I - K_k^l H) P_{k|k-1}^s + \sum_{l=1}^{L_k} \beta^{k,l} \hat{x}_{k|k}^l \hat{x}_{k|k}'^l - \hat{x}_{k|k}\hat{x}_{k|k}' \\
&= \sum_{l=1}^{L_k} \beta^{k,l} (F - K_k^l H) P_{k-1|k-1}^s F' + \sum_{l=1}^{L_k} \beta^{k,l} \hat{x}_{k|k}^l \hat{x}_{k|k}'^l - \hat{x}_{k|k}\hat{x}_{k|k}' \\
&= \sum_{l=1}^{L_k} \beta^{k,l} F\{P_{k-1|k-1}^s - P_{k-1|k-1}^s H' (HP_{k-1|k-1}^s H' + R)^{-1} \\
&\quad \times HP_{k-1|k-1}^s\} F' + \sum_{l=1}^{L_k} \beta^{k,l} \hat{x}_{k|k}^l \hat{x}_{k|k}'^l - \hat{x}_{k|k}\hat{x}_{k|k}' \\
&= \sum_{l=1}^{L_k} \beta^{k,l} F(P_{k-1|k-1}^{-s} + H'R^{-1}H)^{-1} F' \\
&\quad + \sum_{l=1}^{L_k} \beta^{k,l} \hat{x}_{k|k}^l \hat{x}_{k|k}'^l - \hat{x}_{k|k}\hat{x}_{k|k}'.\end{aligned} \tag{10}$$

(10) can then be represented by

$$P_{k|k} = \sum_{l=1}^{L_k} \beta^{k,l} F(I + P_{k-1|k-1}^s H'R^{-1}H)^{-1} P_{k-1|k-1}^s F' + \sum_{l=1}^{L_k} \beta^{k,l} \hat{x}_{k|k}^l \hat{x}_{k|k}'^l - \hat{x}_{k|k}\hat{x}_{k|k}'. \tag{11}$$

The filter at time $k_N + i$ on the horizon $[k - N = k_N,\ k]$ is denoted as $\hat{x}_{k_N+i|k}$ for $0 \le i \le N-1$. The suboptimal algorithm on the receding horizon $[k_N, k]$ then takes on the following form:

$$\hat{x}_{k_N+j+1|k} = \sum_{l=1}^{L_k} F(I + P_{k_N+j|k}^s H'R^{-1}H)^{-1} \beta^{k,l} \times (\hat{x}_{k_N+j|k}^s + P_{k_N+j|k}^s H'R^{-1} y_{k_N+j,i_l}), \tag{12}$$

where the error covariance is obtained from (11) as follows:

$$P_{k_N+j+1|k} = \sum_{l=1}^{L_k} \beta^{k,l} F(I + P^s_{k_N+j|k} H'R^{-1}H)^{-1} P^s_{k_N+j|k} F' + \sum_{l=1}^{L_k} \beta^{k,l} \hat{x}^l_{k_N+j+1|k} \hat{x}'^l_{k_N+j+1|k} - \hat{x}_{k_N+j+1|k} \hat{x}'_{k_N+j+1|k}. \quad (13)$$

Remark 1: It is noted that $\hat{x}_{k_N+j|k}$ in (12), for $k_N + i < k$, is an intermediate variable to compute $\hat{x}_{k|k}$ and cannot be used as a real estimate. Only the state estimate $\hat{x}_{k|k}$ is used as a real estimate of the real target x_k.

Remark 2: In this paper, the suboptimal algorithm based on the receding horizon will be called a suboptimal receding horizon Bayesian filter. From (12) and (13), the state estimate is obtained from the horizon initial state estimate and the covariance and measurement on the receding horizon $[k_N, k]$. However, since the past measurements outside the horizon are discarded in this algorithm, it is necessary to estimate the horizon initial condition without past information. Therefore, the horizon initial state estimate and covariance are derived from the measurements on the receding horizon $[k_N, k]$. In addition, in accordance with Assumption 1, the horizon initial track is assumed to be correct.

On the receding horizon $[k_N, k]$, to express the finite number of measurements in terms of the horizon initial state x_{k_N} the following equations are needed:

$$x_{k_N+1} = Fx_{k_N} + \omega_{k_N}, \quad (14)$$

$$y_{k_N+1} = HFx_{k_N} + H\omega_{k_N} + v_{k_N+1},$$

and

$$y_{k_N} = Hx_{k_N} + v_{k_N}. \quad (15)$$

Consequently, the substitution of (14) into (15) yields:

$$Y^{k-1} = \bar{H}x_{k_N} + \bar{G}W^{k-1} + V^{k-1}, \quad (16)$$

where

$$Y^{k_N+j} \triangleq [y'_{k_N} \ \ y'_{k_N+1} \ \cdots \ y'_{k_N+j}]',$$

$$W^{k_N+j} \triangleq [\omega'_{k_N} \ \ \omega'_{k_N+1} \cdots \omega'_{k_N+j}]',$$

$$V^{k_N+j} \triangleq [v'_{k_N} \ \ v'_{k_N+1} \cdots v'_{k_N+j}]', \ \ 0 \le j \le N-1,$$

and $\bar{H}$ and $\bar{G}$ are as follows:

$$\bar{H} \triangleq \begin{bmatrix} H \\ HF \\ HF^2 \\ \vdots \\ HF^{N-1} \end{bmatrix}, \ \bar{G} \triangleq \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ H & 0 & \cdots & 0 & 0 \\ HF & H & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ HF^{N-2} & HF^{N-3} & \cdots & H & 0 \end{bmatrix}.$$

Theorem 2 [5]**:** Denote the horizon initial condition $\hat{x}_{k_N|k}$ of (12) as $\hat{x}_{k_N|z}$ and horizon initial covariance $P_{k_N|k}$ in (13) as $P_{k_N|z}$. The receding horizon initial state estimate and its error covariance are then expressed as follows:

$$\hat{x}_{k_N|z} \triangleq ZY^{k-1}, \quad (17)$$

$$P_{k_N|z} = Z\bar{\Theta}_N Z', \quad (18)$$

where $Z = (\bar{H}'\bar{\Theta}_N^{-1}\bar{H})^{-1}\bar{H}'\bar{\Theta}_N^{-1}$ is a gain matrix of the horizon initial state estimator [7] and

$$\bar{\Theta}_N \triangleq \bar{G}[\mathrm{diag}(Q\ Q \cdots Q)]\bar{G}' + [\mathrm{diag}(R\ R \cdots R)].$$

Proof: See [5].

Remark 3: Using (17) and (18), the state estimate of the suboptimal receding horizon Bayesian filter is given by

$$\hat{x}_{k|k} = \hat{x}_{k_N+j+1|k} \,|_{j=N-1},$$

where the intermediate variable $\hat{x}_{k_N+j|k}$ is derived from the following iterative form:

$$\hat{x}_{k_N+j+1|k} = \sum_{l=1}^{L_k} F(I + P^s_{k_N+j|k} H'R^{-1}H)^{-1} \beta^{k,l} \times (\hat{x}^s_{k_N+j|k} + P^s_{k_N+j|k} H'R^{-1} y_{k_N+j,i_l}).$$

3. A POSTERIORI PROBABILITY OF THE FILTER

The *a posteriori* probability of (3) can now be dealt with. The following assumptions are needed:

Assumption 2: The number of false validated measurements is described by a diffuse prior model.

Assumption 3: The false measurements are uniformly distributed in the gate.

First, the vector at each time on the receding horizon $[k_N, k]$ is denoted as

$$\mathbf{m}^k = [m_{k-N} \cdots m_k]. \quad (19)$$

Theorem 3: Let Assumptions 1-3 hold. Then, the *a posterior* probability of the validated measurements on the receding horizon $[k_N, k]$ is given as follows:

$$\beta^{k,l} = \begin{bmatrix} \dfrac{e^s}{b^s + \sum_{j_l=1}^{m_k} e_{j_l}^s}, & i_l = 1,\cdots,m_k \\ \dfrac{b^s}{b^s + \sum_{j_l=1}^{m_k} e_{j_l}^s}, & i_l = 0 \end{bmatrix} \times \beta^{k-1,s},$$

where

$$e_{j_l}^s \stackrel{\Delta}{=} \exp\{-\frac{1}{2}(y_{k,j_l} - \hat{y}_{k|k-1}^s)' S_k^{-s}(y_{k,j_l} - \hat{y}_{k|k-1}^s)\}$$

and $b^s \stackrel{\Delta}{=} (1-P_D P_G) | 2\pi S_k^s |^{1/2} \frac{m_k}{P_D V_k}$. P_G is the probability that the true measurement will fall in the gate and P_D is the target detection probability. V_k is the volume of the validation region at time k.

Proof: First, the computation of the probability $\beta^{k,l}$ conditioned on $\mathbf{m}^k$ can be expressed as

$$\begin{aligned} \beta^{k,l} &= P[\theta^{k,l} \mid Y^k, \mathbf{m}^k] = P[\theta_{k,i}, \theta^{k-1,s} \mid Y_k, m_k, Y^{k-1}, \mathbf{m}^{k-1}] \\ &= \frac{p[Y_k \mid \theta_{k,i_l}, m_k, \theta^{k-1,s}, m^{k-1}, Y^{k-1}]}{p[Y_k \mid m_k, Y^{k-1}, \mathbf{m}^{k-1}]} \\ &\quad \times P[\theta_{k,i_l} \mid \theta^{k-1,s}, Y^{k-1}, m_k, \mathbf{m}^{k-1}] \\ &\quad \times P[\theta^{k-1,s} \mid Y^{k-1}, m_k, \mathbf{m}^{k-1}] \\ &= \frac{1}{c} p[Y_k \mid \theta_{k,i_l}, m_k, \theta^{k-1,s}, \mathrm{m}^{k-1}, Y^{k-1}] \\ &\quad \times P[\theta_{k,i_l} \mid \theta^{k-1,s}, Y^{k-1}, m_k, \mathrm{m}^{k-1}] \\ &\quad \times P[\theta^{k-1,s} \mid Y^{k-1}, m_k, \mathrm{m}^{k-1}] \end{aligned} \tag{20}$$

where $c = p[Y_k \mid m_k, Y^{k-1}, \mathbf{m}^{k-1}]$ is the normalization constant. Under Assumption 3, the first joint PDF of the validated measurements on the right hand side of (20) is

$$\begin{aligned} &p[Y_k \mid \theta_{k,i_l}, m_k, \theta^{k-1,s}, Y^{k-1}, \mathbf{m}^{k-1}] \\ &= \begin{cases} V_k^{-m_k+1} P_G^{-1} f[y_{k,i_l}; \hat{y}_{k|k-1}^s, S_k^s], & i_l = 1,\cdots,m_k \\ V_k^{-m_k}, & i_l = 0. \end{cases} \end{aligned} \tag{21}$$

The second density on the right hand side of (20) is as follows:

$$P[\theta_{k,i_l} \mid m_k, \theta^{k-1,s}, Y^{k-1}, \mathbf{m}^{k-1}] = P[\theta_{k,i_l} \mid m_k] = \gamma_{i_l}(m_k)$$

$$= \begin{cases} \dfrac{1}{m_k} P_D P_G \{P_D P_G + (1-P_D P_G)\dfrac{\mu_F(m_k)}{\mu_F(m_k-1)}\}^{-1}, & i_l = 1\cdots m_k \\ (1-P_D P_G)\dfrac{\mu_F(m_k)}{\mu_F(m_k-1)}\{P_D P_G & i_l = 0 \\ \quad + (1-P_D P_G)\dfrac{\mu_F(m_k)}{\mu_F(m_k-1)}\}^{-1}, & \end{cases} \tag{22}$$

where $\mu_F(m_k)$ is the probability mass function of the number of false measurements. Using the diffuse prior model by Assumption 2, (22) is rewritten as follows:

$$\gamma_{i_l}(m_k) = \begin{cases} \dfrac{P_D P_G}{m_k}, & i_l = 1\cdots m_k \\ 1 - P_D P_G, & i_l = 0. \end{cases} \tag{23}$$

The third density in (20) is available from the previous step as follows:

$$\beta^{k-1,s} = P[\theta^{k-1,s} \mid Y^{k-1}, m_k, \mathbf{m}^{k-1}]. \tag{24}$$

The substitution of (21)-(24) into (20) provides the following form:

$$\beta^{k,l} = \begin{Bmatrix} V_k^{-m_k+1} P_G^{-1} f[y_{k,i_l} - \hat{y}_{k|k-1}^s; 0, S_k^s] \cdot & \\ \times \left(\dfrac{P_D P_G}{m_k} \right), & i_l = 1,\cdots,m_k \\ V_k^{-m_k}(1-P_D P_G), & i_l = 0 \end{Bmatrix} \beta^{k-1,s}.$$

Using $V_k = C_{n_y} | \gamma S_k^s |^{1/2} = C_{n_y} \gamma^{n_y/2} | S_k^s |^{1/2}$, the above equation becomes

$$\beta^{k,l} = \begin{Bmatrix} (C_{n_y} \gamma^{n_y/2} | S_k^s |^{1/2})^{-m_k+1} P_D P_G \\ \times \dfrac{P_G^{-1} \exp\{-\frac{1}{2}(y_{k,i_l} - \hat{y}_{k|k-1}^s)' S_k^{-s}(y_{k,i_l} - \hat{y}_{k|k-1}^s)\}}{| 2\pi S_k^s |^{1/2} m_k}, \end{Bmatrix} \beta^{k-1,s}$$

$$i_l = 1,\cdots,m_k$$

$$\beta^{k,l} = \left\{ (C_{n_y} \gamma^{n_y/2} | S_k^s |^{1/2})^{-m_k}(1-P_D P_G), \quad i_l = 0 \right\} \beta^{k-1,s},$$

where C_{n_y} is the volume of the n_y-dimensional unit hypersphere and γ is the threshold of the gate. By normalizing this result, according to the value of i_l, $\beta^{k,l}$ is rewritten as follows:

a) In case of $i_l = 1,\cdots,m_k$,

$$\beta^{k,l} = \frac{\exp\left[-\frac{1}{2}(y_{k,i_l} - \hat{y}_{k|k-1}^s)' S_k^{-s}(y_{k,i_l} - \hat{y}_{k|k-1}^s)\right] \beta^{k-1,s}}{\sum_{i_l=1}^{m_k} \left\langle \exp\left[-\frac{1}{2}(y_{k,i_l} - \hat{y}_{k|k-1}^s)' S_k^{-s}(y_{k,i_l} - \hat{y}_{k|k-1}^s)\right] + (1-P_D P_G) | 2\pi S_k^s |^{1/2} \frac{m_k}{P_D V_k} \right\rangle}.$$

b) In case of $i_l = 0$,

$$\beta^{k,l} = \frac{|2\pi S_k^s|^{1/2} \frac{(1-P_D P_G)m_k}{P_D V_k} \beta^{k-1,s}}{\sum_{i_l=1}^{m_k} \left\langle \exp\left[-\frac{1}{2}(y_{k,i_l} - \hat{y}_{k|k-1}^s)' S_k^{-s} (y_{k,i_l} - \hat{y}_{k|k-1}^s)\right] + (1-P_D P_G)|2\pi S_k^s|^{1/2} \frac{m_k}{P_D V_k} \right\rangle}.$$

Therefore, the *a posterior* probability conditioned on the validated measurements in the receding horizon $[k_N, k]$ is given by

$$\beta^{k,l} = \begin{bmatrix} \dfrac{e^s}{b^s + \sum_{j_l=1}^{m_k} e_{j_l}^s} & i_l = 1 \cdots m_k \\ \dfrac{b^s}{b^s + \sum_{j_l=1}^{m_k} e_{j_l}^s} & i_l = 0 \end{bmatrix} \times \beta^{k-1,s}.$$

4. COMPUTATIONAL COMPLEXITY

The computational burden of each of the two tracking algorithms is presented in this section. The scheme of the suboptimal receding horizon Bayesian filter derived in Theorem 1 and Theorem 2 is outlined in Fig. 1. Using the algorithm in Fig. 1, the computational complexity and storage requirements of the tracking filter can be suboptimally reduced compared to the standard optimal Bayesian filter. Tables 1 and 2 illustrate the computational burden of the suggested suboptimal receding horizon Bayesian filter and the optimal Bayesian filter, respectively. From the Tables it is evident that the computational complexities of the suggested suboptimal receding horizon Bayesian filter is less than the optimal Bayesian filter using flops method [4]. In Table 1, the computational complexity of the suboptimal receding horizon Bayesian filter is shown. Table 2 also shows the computational complexity of the optimal Bayesian filter. In Table 3, an example demonstrates the computational burden of each filter for the two- dimensional case with $N = 3$. Although the optimal Bayesian filter of [13] can be preferred to the suboptimal receding horizon Bayesian filter in the aspect of performance, the latter has less computational complexity than the former.

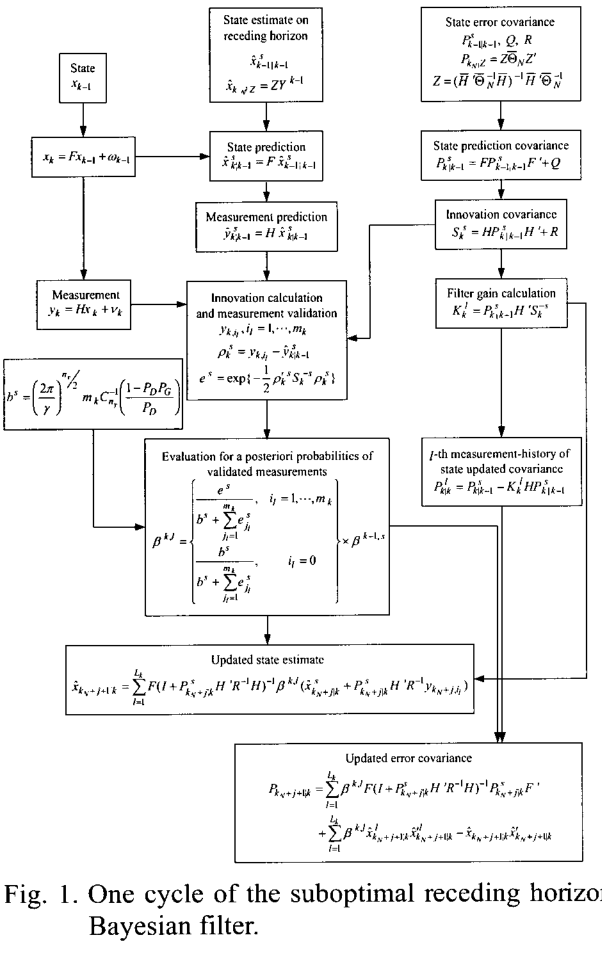

Fig. 1. One cycle of the suboptimal receding horizon Bayesian filter.

Table 1. Operation summary for the suboptimal receding horizon Bayesian filter.

Operation	Flops
$\hat{x}_{k\|k-1}^s = F\hat{x}_{k-1\|k-1}^s$	n^2
$\hat{y}_{k\|k-1}^s = H\hat{x}_{k\|k-1}^s$	mn
$P_{k-1\|k-1}^s F'$	n^3
$P_{k\|k-1}^s = FP_{k-1\|k-1}^s F' + Q$	n^3
$P_{k\|k-1}^s H'$	$n^2 m$
$S_k^s = H(P_{k\|k-1}^s H') + R$	$(m^2 n + mn)/2$
$(HP_{k\|k-1}^s H' + R)^{-1}$	$m^3 + m^2/2 + m/2$
$H'S_k^{-s}$	$m^2 n$
$K_k^l = P_{k\|k-1}^s H'S_k^{-s}$	$n^2 m$
$S_k^{-s}\rho_k^s$	m^2
$(\rho_k^s)^T S_k^{-s}\rho_k^s$	n
$HP_{k\|k-1}^s$	$n^2 m$
$K_k^l(HP_{k\|k-1}^s)$	$(mn^2 + nm)/2$
$e^s/\{b^s + \sum_{j_l=1}^{m_k} e_{j_l}^s\}$	$n/(1+m_k n)$
$b^s/\{b^s + \sum_{j_l=1}^{m_k} e_{j_l}^s$	$1/(1+m_k n)$
$R^{-1} y_{k_N+j,i_l}$	m^2
$P_{k_N+j\|k}^s H'(R^{-1} y_{k_N+j,i_l})$	n^2
$R^{-1}H$	$m^2 n$
$(P_{k_N+j\|k}^s H')(R^{-1}H)$	$(mn^2 + nm)/2$
$A = (I + P_{k_N+j\|k}^s H'R^{-1}H)^{-1}$	$n^3 + n^2/2 + n/2$
$A\beta^{k,l}(\hat{x}_{k_N+j\|k}^s + P_{k_N+j\|k}^s H'R^{-1} y_{k_N+j,i_l})$	n^2
$\sum_{l=1}^{L_k} FA\beta^{k,l}(\hat{x}_{k_N+j\|k}^s + P_{k_N+j\|k}^s H'R^{-1} y_{k_N+j,i_l})$	$\prod_{j=k-N}^{k}(1+m_j)n^2$
$A(P_{k_N+j\|k}^s F')$	n^3
$\sum_{l=1}^{L_k}\beta^{k,l}\hat{x}_{k_N+j+1\|k}^l(\hat{x}_{k_N+j+1\|k}^l)' + \sum_{l=1}^{L_k}\beta^{k,l}FA(P_{k_N+j\|k}^s F') - \hat{x}_{k_N+j+1\|k}(\hat{x}_{k_N+j+1\|k})'$	$\prod_{j=k-N}^{k}(1+m_j)n^3 + n(1+n)/2 + \prod_{j=k-N}^{k}(1+m_j)n(1+n)/2$

Table 2. Operation summary for the optimal Bayesian filter.

Operation	Flops
$\hat{x}^s_{k\|k-1} = F\hat{x}^s_{k\|k-1}$	n^2
$\hat{y}^s_{k\|k-1} = H\hat{x}^s_{k\|k-1}$	mn
$(FP^s_{k-1\|k-1})$	n^3
$F(P^s_{k-1\|k-1}F') + Q$	n^3
$HP^s_{k\|k-1}$	n^2m
$S^s_k = HP^s_{k\|k-1}H' + R$	$(m^2n + mn)/2$
$(HP^s_{k\|k-1}H' + R)^{-1}$	$m^3 + m^2/2 + m/2$
$H'S^{-s}_k$	m^2n
$K^l_k = P^s_{k\|k-1}H'S^{-s}_k$	n^2m
$S^{-1}_k\rho^s_k$	m^2
$(\rho^s_k)'(S^{-1}_k\rho^s_k)$	n
$e^s / \{b^s + \sum_{j_l=1}^{m_k} e^s_{j_l}\}$	$n/(1+m_k n)$
$b^s / \{b^s + \sum_{j_l=1}^{m_k} e^s_{j_l}\}$	$1/(1+m_k n)$
$K^l_k(HP^s_{k\|k-1})$	$(mn^2 + nm)/2$
$\hat{x}^l_{k\|k}(\hat{x}^l_{k\|k})'$	$(n+1)n/2$
$\sum_{l=1}^{L_k}\beta^{k,l}P^l_{k\|k} - \hat{x}_{k\|k}(\hat{x}_{k\|k})' + \sum_{l=1}^{L_k}\beta^{k,l}\hat{x}^l_{k\|k}(\hat{x}^l_{k\|k})'$	$\prod_{j=1}^{k}(1+m_j)n^2 + (n+1)n/2 + \prod_{j=1}^{k}(1+m_j)(n+1)n/2$

Table 3. Computation example for a 2-dimensional system.

Steps, k	# of Validated Measurements m_i	Total Flips	
		OBF	RHBF ($N=3$)
⋮	⋮	⋮	⋮
4	3	32,358	425,604
5	4	608,166	709,316
6	2	6,948,006	531,996
7	3	9,264,006	1,063,920
8	4	2,076,393,606	1,329,936
⋮	⋮	⋮	⋮

5. CONCLUSIONS

In a cluttered environment, the use of the optimal Bayesian filter, as a possible solution to the target-tracking problem, is often recommended. However, the computational burden and growing memory are known to be the main drawbacks in its use. The suboptimal algorithm proposed in this paper uses the measurements on the receding horizon and diminishes the computational complexity and storage requirement. Since prior information outside the horizon was not available, the horizon initial state estimate and its covariance were obtained using the measurements in the receding horizon.

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