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Some Stochastic Properties of Imperfect Repair Model with Random Repair Time

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Abstract. Maintenance models involving minimal imperfect repair frequently appear in the literature of reliability and operations research. Most of the literatures concerning the stochastic behavior of repairable systems assume that it takes negligible time to repair a failed system and so the length of repair time does not affect the maintenance strategy. It is more realistic to consider the length of repair times in developing maintenance model, however. In this paper, we consider an imperfect repair model with random repair time and investigate some stochastic properties of the number of perfect repairs and the number of minimal repairs. Also we derive the expressions for evaluating the expected numbers of perfect and minimal repairs in general and apply these formulas for certain parametric life distributions.

Key Words : *imperfect repair model, repair time, perfect repair, minimal repair.*

1. INTRODUCTION

We consider a system that is subject to failure. Upon failure of the system, there are two types of actions to be taken. The failed system can be either repaired or replaced by a new identical one. Among these repair actions, we can consider two types of repairs: a perfect repair is performed and the system is returned to the "good-as-new" state, otherwise a minimal imperfect repair is performed and the device is returned to the working state but it is only as good as the item of age equal to the age of the device at

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failure. Various maintenance models involving minimal imperfect repair have been discussed frequently in the literature of reliability and operations research.

Brown and Proschan(1983) examine a maintenance action, which is referred to as an imperfect repair under which a perfect repair is performed with probability p and a minimal repair is performed with probability $q = 1 - p$. The minimal repair restores the failed system to its condition just prior to failure. Assuming that the repair takes negligible time, they prove that the distribution of the waiting time , F_p, for the first perfect repair and its corresponding failure rate function r_p are

$$r_p(t) = pr_1(t) \quad \text{and} \quad \overline{F}_p(t) = \overline{F}^p(t),$$

where F and r_1 are the life distribution of a new unit and its failure rate. They also proved that some properties of F_p are presented for the nonparametric classes of IFR(DFR), IFRA(DFRA), NBU(NWU) and DMRL(IMRL) under imperfect repair model.

Their model has been generalized by Block, Borges and Savits(1985) to the case in which the probability of perfect repair is state-dependent, by Shaked and Shanthikumar(1986) to the multivariate case. A related model has been studied by Lim, Lu and Park(1998) in Bayesian point of view. They propose a new Bayesian imperfect repair model where the probability of perfect repair, p, is considered to be a random variable. Recently, Lim, Park and Sohn(1999) consider some stochastic properties for imperfect repair model under the assumption that the repair time is negligible and investigated the number of perfect repairs and minimal repairs under the imperfect repair model.

In most of the repair models proposed so far in the literatures, it is assumed that it takes negligible time to perform a repair. In all real situations such as maintaining facilities, power plant and switching systems, it is quite true that it takes a certain amount of repair times. In this paper, we discuss an imperfect repair model proposed by Brown and Proschan(1983) with random repair times. In Section 2, we derive some stochastic properties of the number of perfect repairs and the number of minimal repairs with respect to the value of p. We also present mathematical expressions to obtain the expected numbers of perfect repairs and minimal repairs during a finite time interval. In Section 3, we consider an exponential distribution and Weibull distribution as failure time distributions and an exponential distribution as a repair time distribution and compute the expected number of repairs during a finite time period. Section 4 is devoted to conduct simulation studies for numerical comparison of our results discussed.

2. SOME PROPERTIES FOR IMPERFECT REPAIR MODEL WITH RANDOM REPAIR TIME

Notations

X time to failure of unit with p.d.f. f and d.f. F

Y repair time of unit with p.d.f. g and d.f. G

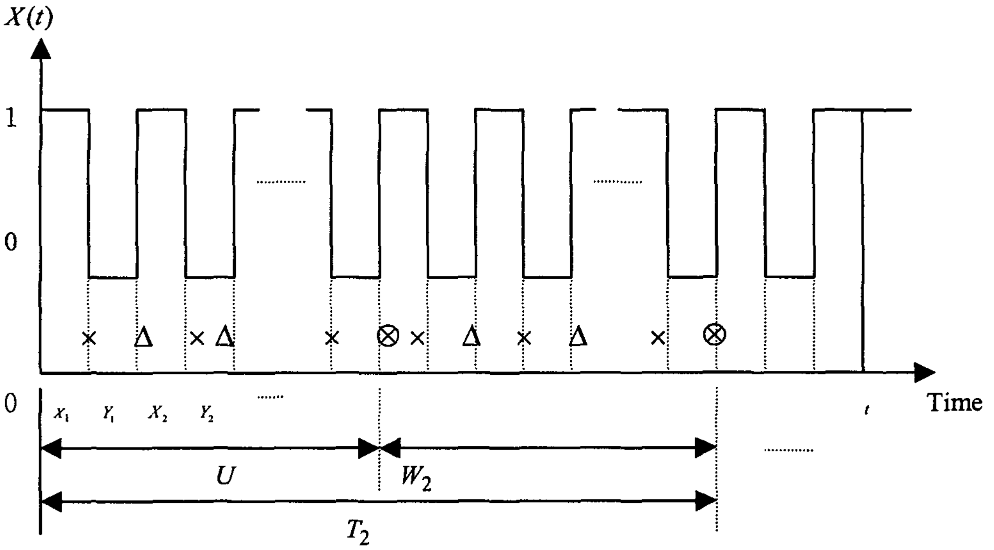
$Z(t)$	c.d.f. of $X+Y$, convolution of the distributions $F(t)$ and $G(t)$
$\overline{Z}(t)$	$1-Z(t)$
$r(t)$	failure rate fa life distribution $Z(t)$
$R(t)$	$-\ln \overline{Z}(t)$
$\overline{H}_p(t)$	reliability function of the length of time between two successive perfect repairs, $\overline{Z}^p(t)$
$H_p(t)$	$1-\overline{H}_p(t)=1-\overline{Z}^p(t)$
$H_p^{(k)}(t)$	k^{th} convolution of $H_p(t)$
p	probability of perfect repair
q	$1-p$, probability of minimal repair
$N^P(t,p)$	number of perfect repairs during [0, t] under imperfect repair model
$N^P(t,1)$	number of perfect repairs during [0, t], $t \geq 0$
$N^M(t,p)$	number of minimal repairs during [0, t] under imperfect repair model
$N^M(t,0)$	number of minimal repairs during [0, t], $t \geq 0$
U	waiting time to the first perfect repair under the imperfect repair model
T_i	waiting time to the i th perfect repair, $i=2,\cdots,N^P(t,p)$ under the imperfect repair model
W_i	length of time between perfect repairs, $T_i - T_{i-1}$, $i=1,2,\cdots,N^P(t,p)$

The following assumptions are postulated throughout this paper.

(1) At each failure, the failed unit is perfectly repaired with a probability p or minimally repaired with probability q =1-p.
(2) The probability of perfect repair, p, is fixed and known.
(3) The length of repair time is random.

To describe the imperfect maintenance model with random repair time proposed in this paper graphically, we present a diagram as in Figure 1. Here X(t) is the indicator function which denotes the operating status of the unit, where X(t)=1 implies that the unit is operating at time t and X(t)=0 otherwise. We refer to this model as a random repair model.

The following results are similar to those in Lim, Park, Sohn(1999), except that the repair time is considered to be random in this paper. Consequently, Lim, Park and Sohn's results can be obtained as special cases of the following results. Hereafter, such model will be referred to as a random repair model. Let $N^P(t,p)$ and $N^M(t,p)$ be the number of perfect and minimal repairs during an interval $[0,t), t \geq 0$ under the random repair model and let U denote the waiting time to the first perfect repair. Then, applying the similar technique as in Lim, Park and Sohn(1999), we obtain the following stochastic properties regarding $N^P(t,p)$ and $N^M(t,p)$.



× : failure-occurrence time ⊗ : perfect repair Δ : imperfect repair

Figure 1. Sample behavior of the imperfect repair model with random repair time

Theorem 2.1. (*i*) For every fixed $t \geq 0$ and every U, $\{N^P(t,p) : 0 \leq p \leq 1\}$ is a stochastically increasing family in p.
(*ii*) Given that $U > t$, that is the first perfect repair occurs later than *t*, $\{N^M(t,p) : 0 \leq p \leq 1\}$ is a stochastically decreasing family in p.

Proof. (*i*) Let $H_p(t)$ be the distribution of times between two successive perfect repairs under the random repair model. From the results of Brown and Proschan(1983), we can show that $\overline{H}_p(t) = \overline{Z}^p(t)$, where $Z(t)$ is the convolution of $F(t)$ and $G(t)$ and $\overline{Z}(t) = 1 - Z(t)$.

Let $k \geq 1$ be an integer. Then it is obvious that

$$P\{N^P(t,p) \geq k\} = H_p^{(k)}(t),$$

for $0 \leq p \leq 1$, where $H_p^{(k)}(\cdot)$ is the k^{th} convolution of H_p, which is the c.d.f. of the waiting time until the k th perfect repair occurs.

To show the monotonically increasing property of the family, we adopt the mathematical induction. Let $0 \leq p_1 \leq p_2 \leq 1$. Then

$$H_{p_1}^{(1)}(t) = 1 - \overline{Z}^{p_1}(t) \leq 1 - \overline{Z}^{p_2}(t) = H_{p_2}^{(1)}(t).$$

Next, by assuming that $H_{p_1}{}^{(n-1)}(t) \le H_{p_2}{}^{(n-1)}(t)$, we have

$$H_{p_1}{}^{(n)}(t) = \int_0^t H_{p_1}{}^{(n-1)}(t-x)dH_{p_1}(x) \le \int_0^t H_{p_2}{}^{(n-1)}(t-x)dH_{p_2}(x) = H_{p_2}{}^{(n)}(t).$$

This completes the proof.
(ii) Since the minimal repairs generate a nonhomogenous Poisson process, it follows from the result of Fontenot and Proschan(1984) (see Lemma 1.1 for details) that

$$P\left(N^M(t,p) = k \mid U > t\right) = \frac{\left(q(R(t))\right)^k e^{-qR(t)}}{k!}, \tag{2.1}$$

where $R(t) = -\ln \overline{Z}(t)$ is the hazard function of X+Y.
Let $0 \le p_1 \le p_2 \le 1$. Then, it is sufficient to prove that the inequalities

$$P\left(N^M(t,p_1) \ge k \mid U > t\right) \ge P\left(N^M(t,p_2) \ge k \mid U > t\right) \tag{2.2}$$

hold for all positive integer k. Since given $Z > t$, $N^M(t,p)$ follows a Poisson distribution with a mean of $qR(t)$ from (2.1) and $q_1R(t) > q_2R(t)$, where $q_i = 1 - p_i, i = 1,2$, (2.2) immediately follows. This completes the proof. □

By taking $p_1 = p, p_2 = 1$ for result (i) and $p_1 = 0, p_2 = p$ where $0 < p < 1$, we obtain Corollary 2.1.

Corollary 2.1. Let $0 < p < 1$.
(i) For $t \ge 0$, $N^P(t,1) \ge^{st.} N^P(t,p)$
and
(ii) Given that $U > t$, $N^M(t,p) \le^{st.} N^M(t,0)$,
where the notation, $\ge^{st.}$, represents “stochastically larger than ”.

Since $\{N^P(t,1), t \ge 0\}$ is a renewal process with interoccurance distribution $Z(t)$ and $\{N^M(t,0), t \ge 0\}$ is a nonhomogenous Poisson Process with intensity function $r(t)$, we have Theorem 2.2. Note that in this random repair model, we must con- sider not only the waiting tim to failure, but also the length of repair time, the intensity function, $r(t)$, is obtained with respect to the convolution of the waiting time to failure time and the length of repair time.

Theorem 2.2. Let $t > 0$ be given. Then (*i*) $E[N^P(t,1)] = M_Z(t) = \sum_{k=1}^{\infty} Z^{(k)}(t)$

(*ii*) $E[N^M(t,0)] = -\ln[\overline{Z}(t)]$

where $Z^{(k)}(\cdot)$ is the k th convolution of $Z(\cdot)$, which is the convolution of F and G and $M_Z(t)$ is the renewal function.

The following lemma is needed to derive the formulas for the expected numbers of perfect repairs and minimal repairs under the random repair model. The result is an extension of the one given in Lim, Park and Sohn(1999).

Lemma 2.1. For the repair process based on the random repair model, let $w_i = t_i - t_{i-1}$ be the realizations of $W_i = T_i - T_{i-1}$, where $i = 1,2,\cdots,N^P(t,p)$ and $T_0 = 0$. Then,

$$E[N^M(t,p) \mid N^P(t,p) = n, T_1 = t_1, \cdots T_n = t_n] = q\left\{\sum_{i=1}^{n} R(w_i) + R(t - t_n)\right\}.$$

It is noted that $\{N^P(t,p), t \ge 0\}$ is a renewal process with interoccurrence distribution H. Taking expectations with regard to $(T_1, T_2, \cdots, T_n)$ based on both sides of the result of Lemma 2.1 yields Theorem 2.3.

Theorem 2.3. Let $0 < p < 1$ and $t > 0$ be given. Then

(*i*) $E[N^P(t,p)] = M_{H_p}(t) = \sum_{k=1}^{\infty} H_p^{(k)}(t)$,

(*ii*) $E[N^M(t,p)] =$

$$(1-p)\left\{E\left[E\left(\sum_{i=1}^{N^P(t,p)} R(W_i) \mid N^P(t,p)\right)\right] + E\left[E\left[\left(R(t - T_{N^P(t,p)}) \mid N^P(t,p)\right)\right]\right]\right\}$$

To study the distribution of $\sum_{i=1}^{N^P(t,p)} R(W_i)$ and $R(t - T_{N^P(t,p)})$, we present the following theorem. Note that $W_i, i = 1,2,\cdots,N^P(t,p)$, represents the i th interoccurrence time of perfect repairs with the c.d.f. $H_p(\cdot)$ and $N^P(t,p)$ is the number of perfect repairs carried out during $[0,t)$. Our results are summarized in Theorem 2.4.

Theorem 2.4. Let $\{Z_n\}_{n=1}^{\infty}$ be a sequence of i.i.d. random variables from $H_p(\cdot)$ and let $N_{H_p}(t)$ be the number of renewals in $[0,t)$ with interoccurrence distribution $H_p(\cdot)$, where $H_p(t) = 1 - \overline{Z}^p(t)$ for $0 < p < 1$. Denote $V_i = -\ln \overline{Z}(Z_i)$, $i = 1,2,\cdots$ and let $S_n = \sum_{i=1}^{n} V_i$. Then

(*i*) $\{V_n\}^{\infty}_{n=1}$ is a sequence of i.i.d. random variables from $G(t) = 1 - e^{-pt}$ for $t \geq 0$.

(*ii*) If $Z(t)$ is NBU, then

$$P\left(S_{N_{H_p}(t)} \geq x\right) = \sum_{n=0}^{\infty} P\left(x \leq S_{N_{H_p}(t)} \leq t^*,\ N_{H_p}(t) = n\right)$$
$$= \sum_{n=0}^{\infty} \left[G_n(t^*) - G_n(x)\right] P\left[N_{H_p}(t) = n\right],$$

where $t^* = -\ln \overline{Z}(t)$ and $G_n(v) = \int_0^v \frac{p^n}{\Gamma(n)} y^{n-1} e^{-py} dy$.

(*iii*) $E\left[S_{N_{H_p}(t)}\right] = \sum_{n=0}^{\infty} \left[G_n(t^*)t^* - \int_0^{t^*} G_n(x)dx\right] P\left[N_{H_p}(t) = n\right]$

Proof. (*i*) It is obvious that V_i's are i.i.d. random variables. And the distribution of V_i is easily obtained by using the technique of the probability integral transformation.

(*ii*) It is clear that $\sum_{i=1}^{N_{H_p}(t)} Z_i \leq t$. Then

$$\sum_{i=1}^{N_{H_p}(t)} \overline{Z}(Z_i) \geq \overline{Z}\left(\sum_{i=1}^{N_{H_p}(t)} Z_i\right) \geq \overline{Z}(t)$$

The first inequality holds due to NBU assumption of $\overline{Z}(t)$ and the second inequality is true since $\overline{Z}(t)$ is a decreasing function in t. It follows immediately that $\ln\left(\prod_{i=1}^{N_{H_p}(t)} \overline{Z}(Z_i)\right) \geq \ln \overline{Z}(t)$ from the above inequality by taking the logarithm on both side.

Thus

$$S_{N_{H_p}(t)} = \sum_{i=1}^{N_{H_p}(t)} \left[\,-\ln \overline{Z}(Z_i)\right] \leq -\ln \overline{Z}(t) = t^*.$$

Hence, we have

$$P\left[S_{N_{H_p}(t)} \geq x\right] = P\left(x \leq S_{N_{H_p}(t)} \leq t^*\right)$$

$$= \sum_{n=0}^{\infty} P\left(x \le S_n \le t^* \mid N_{H_p}(t) = n\right) P\left(N_{H_p}(t) = n\right)$$

$$= \sum_{n=0}^{\infty} \left[G_n(t^*) - G_n(x)\right] P\left[N_{H_p}(t) = n\right].$$

(iii) As a consequence of (ii), it follows that

$$E\left(S_{N_H(t)}\right) = \int_0^{t^*} P\left(S_{N_H(t)} \ge x\right) dx$$

$$= \sum_{n=0}^{\infty} \left[G_n(t^*)t^* - \int_0^{t^*} G_n(x)dx \right] P\left(N_{H_p}(t) = n\right). \square$$

To derive the distribution of $R(t - T_{N^P(t,p)})$, we first note that $T_{N^P(t,p)}$ is stochastically equal to the sum of $N^P(t, p)$ i.i.d. random variables, each having a distribution $H_p(\cdot)$. Let $\delta_t = t - T_{N^P(t,p)}$. Then δ_t is known to be an age random variable (stortage random variable or current life random variable) and the distribution of δ_t is given by

$$P(\delta_t \le x) = P(t - T_{N^P(t,p)} \le x)$$

$$= P\left(t - x \le T_{N^P(t,p)}\right)$$

$$= \begin{cases} \int_{t-x}^{t} \overline{H}_p(t-u) dM_{H_p}(u) & \text{if } 0 < x < t \\ 1 & \text{otherwise} \end{cases}$$

where $\overline{H}_p(t) = \overline{Z}^p(t)$ and $M_{H_p}(u) = \sum_{k=1}^{\infty} H_p^{(k)}(u)$ is a renewal function.

Hence, we obtain

$$P\left(R(t - T_{N^P(t,p)}) \le x\right) = P\left(-\ln \overline{Z}(\delta_t) \le x\right)$$

$$= P\left(\delta_t \le \overline{Z}^{-1}(e^{-x})\right)$$

$$= \begin{cases} \int_{\zeta}^{t} \overline{H}(t-u) dM_H(u) & \text{if } x < t^* \\ 1 & \text{if } x \ge t^* \end{cases}$$

, where $\zeta = t - \overline{Z}^{-1}(e^{-x})$.

As mentioned in the beginning of the paper, this random repair model assumes that the length of repair time follows a distribution function G(t). Thus, if we take the special case that $\overline{G}(t) = 0$ for all $t > 0$, that is, the repair time is negligible, the results obtained in this section are reduced to those in Lim, Park and Sohn(1999). Also note that $\overline{Z}(t)$ in this section plays exactly same role as $\overline{F}(t)$ in Lim, Park and Sohn(1999).

3. THE EXAMPLES

In this section, we compute the expected numbers of perfect repairs and minimal repairs when the underlying life distributions of a unit are assumed to be exponential and Weibull and the repair time distribution is an exponential.

3.1 When both life time and repair time are exponentially distributed.

Suppose that the failure and repair times are exponentially distributed with means of $1/\lambda_1$ and $1/\lambda_2$, respectively. Since the mean repair time is quite naturally much smaller than the mean lifetime in practical field applications, we consider only the case of different means with $\lambda_1 \ll \lambda_2$. Then, it is easy to obtain that their convolution, $Z(t)$, has the following expression.

$$Z(t) = 1 - \left(\frac{\lambda_2}{\lambda_2 - \lambda_1}\right) e^{-\lambda_1 t} + \left(\frac{\lambda_1}{\lambda_2 - \lambda_1}\right) e^{-\lambda_2 t}$$

By applying the results of Theorem 2.2 and 2.3, we can obtain the expected numbers of perfect and minimal repairs during $[0, t)$. The results are summarized as follows.

(1) Case when $p = 0$ or $p = 1$.

$$E\left[N^P(t,1)\right] = \sum_{k=1}^{\infty} \left\{1 - \left(\frac{\lambda_2}{\lambda_2 - \lambda_1}\right) e^{-\lambda_1 t} + \left(\frac{\lambda_1}{\lambda_2 - \lambda_1}\right) e^{-\lambda_2 t}\right\}^{(k)} .$$

$$E\left[N^M(t,0)\right] = R(t) = -\log\left[\left(\frac{\lambda_2}{\lambda_2 - \lambda_1}\right) e^{-\lambda_1 t} - \left(\frac{\lambda_1}{\lambda_2 - \lambda_1}\right) e^{-\lambda_2 t}\right] .$$

(2) Case when $0 < p < 1$.

$$E\left[N^P(t,p)\right] = \sum_{k=1}^{\infty} \left\{1 - \left[\left(\frac{\lambda_2}{\lambda_2 - \lambda_1}\right) e^{-\lambda_1 t} - \left(\frac{\lambda_1}{\lambda_2 - \lambda_1}\right) e^{-\lambda_2 t}\right]^p\right\}^{(k)} .$$

$$E\left[N^M(t,p)\right]=\bar{p}\cdot E\left\{E\left[\sum_{i=1}^{N_P(t,p)}\left(-\log\left[\left(\frac{\lambda_2}{\lambda_2-\lambda_1}\right)e^{-\lambda_1 W_i}-\left(\frac{\lambda_1}{\lambda_2-\lambda_1}\right)e^{-\lambda_2 W_i}\right]\right)\mid N^P(t,p)\right]\right\}$$
$$+\bar{p}\cdot E\left\{E\left[\left(-\log\left[\left(\frac{\lambda_2}{\lambda_2-\lambda_1}\right)e^{-\lambda_1\left[t-T_{N^P(t,p)}\right]}-\left(\frac{\lambda_1}{\lambda_2-\lambda_1}\right)e^{-\lambda_2\left[t-T_{N^P(t,p)}\right]}\right]\right)\mid N^P(t,p)\right]\right\}.$$

When it takes negligible time to repair a failed unit, we investigate the behavior of the expected numbers of perfect and minimal repairs during $[0,t)$. To this end, we let the value of λ_2 increase to infinity. Then, we have the following results.

$$\lim_{\lambda_2\to\infty} E\left[N^P(t,p)\right]=\begin{cases}\sum_{k=1}^{\infty}\left\{1-e^{-\lambda_1 pt}\right\}^{(k)}=p\lambda_1 t & if \quad 0<p<1\\ \sum_{k=1}^{\infty}\left\{1-e^{-\lambda_1 t}\right\}^{(k)}=\lambda_1 t & if \quad p=1\end{cases}$$

and

$$\lim_{\lambda_2\to\infty} E[N^M(t,p)]$$
$$=\begin{cases}\bar{p}\cdot E\left\{E\left[\sum_{i=1}^{N_P(t,p)}\lambda_1 W_i\mid N^P(t,p)\right]\right\}+\bar{p}\cdot E\left\{E\left[\lambda_1\left[t-T_{N^P(t,p)}\right]N^P(t,p)\right]\right\}=\bar{p}\lambda t & if \quad 0<p<1\\ -\log\left[e^{-\lambda_1 t}\right]=\lambda_1 t & if \quad p=0\end{cases}$$

which are agree with the results obtained in the example of Lim, Park and Sohn(1999).

3.2 When the life time distribution is Weibull and the repair time distribution is exponential.

Suppose that the failure time distribution is Weibull with shape parameter of α and scale parameter of 1.0. The repair time distribution is exponential with mean of $1/\lambda$, which is assumed to be quite smaller relative to the mean failure time. Direct calculations show that the convolution of both distributions becomes

$$Z(t)=1-e^{-\lambda t}-\lambda\int_0^t e^{-[(t-y)^\alpha+\lambda y]}dy$$

By applying the results of Theorem 2.2 and 2.3 again, we can obtain the expected numbers of perfect and minimal repairs during $[0,t)$. The results are summarized as follows.

(1) Case when $p=0$ or $p=1$.

$$E\left[N^P(t,1)\right]=\sum_{k=1}^{\infty}\left\{1-e^{-\lambda t}-\lambda\int_0^{\infty}e^{-[(t-y)^{\alpha}+\lambda y]}dy\right\}^{(k)}$$

and

$$E\left[N^M(t,0)\right]=-log\left[e^{-\lambda t}+\lambda\int_0^{t}e^{-[(t-y)^{\alpha}+\lambda y]}dy\right]$$

(2) Case when $0<p<1$

$$E\left[N^P(t,p)\right]=\sum_{k=1}^{\infty}\left\{1-\left[e^{-\lambda t}+\lambda\int_0^{\infty}e^{-[(t-y)^{\alpha}+\lambda y]}dy\right]^{p}\right\}^{(k)}$$

and

$$E\left[N^M(t,p)\right]=\bar{p}\cdot\left\{E\left[E\left[\sum_{i=1}^{N^P(t,p)}\left[-\log\left(e^{-\lambda W_i}+\lambda\int_0^{W_i}e^{-[(W_i-y)^{\alpha}+\lambda y]}dy\right)\right]\mid N^P(t,p)\right]\right]\right\}$$
$$+\bar{p}E\left[E\left[-\log\left(e^{-\lambda(t-T_{N^P(t,p)})}+\lambda\int_0^{\delta_t}e^{-[(\delta_t-y)^{\alpha}+\lambda y]}dy\right)\mid N^P(t,p)\right]\right]$$

where $\delta_t=t-T_{N^P(t,p)}$.

4. SIMULATION STUDY

In this section, we assume that the failure times follow a Weibull distribution with shape parameter of α and scale parameter of 1.0 and that the repair times follow an exponential distribution with a mean of $1/\lambda$ for performing a simulation study. It is well known that Weibull distribution has strictly decreasing failure rate(DFR) when $\alpha<1$ while it has strictly increasing failure rate(IFR) when $\alpha>1$. In particular, when $\alpha=1$, it becomes an exponential distribution and then the failure rate is constant(CFR). We take the values of α as 0.5, 1.0 and 2.0 so that the life distributions become DFR, CFR and IFR, respectively. And the values of $1/\lambda$ are assumed to be equal to 0.000001, 0.0001 and 0.01. To evaluate the simulated values, operating time is specified by $t=10$ and each simulation is iterated 10^4 times. It is noted that the cases for p = 0 and p=1 are excluded from this simulation study since it takes too many repetitions to generate a proper random number for both cases. Tables 1-3 present the simulation results for $E[N^P(t,p)]$, $E[N^M(t,p)]$, $E[N^P(t,p)]+E[N^M(t,p)]$ for various choice of p,α and $1/\lambda$. Note that $E[N^P(t,p)]+E[N^M(t,p)]$ denotes the number of failures during $[0,t)$.

From Tables 1-3, it is shown that as p increases, the expected number of perfect repairs increases while the expected number of minimal repairs decreases. It is of great interest to note that as p increases, the expected number of failures decreases for α = 2.0,

Table 1. Simulated values for the expected numbers of perfect and minimal repairs and its corresponding expected costs for $\alpha = 0.5$ and $t = 10$.

$1/\lambda$	P	$E[N^P(t,p)]$	$E[N^M(t,p)]$	$E[N^P(t,p)]+E[N^M(t,p)]$
	0.1	0.349	3.099	3.448
	0.3	1.229	2.863	4.092
0.000001	0.5	2.402	2.392	4.794
	0.7	3.839	1.630	5.469
	0.9	5.619	0.622	6.241
	0.1	0.344	3.154	3.498
	0.3	1.215	2.834	4.049
0.0001	0.5	2.394	2.387	4.781
	0.7	3.825	1.669	5.494
	0.9	5.662	0.635	6.297
	0.1	0.342	3.083	3.425
	0.3	1.204	2.855	4.059
0.01	0.5	2.388	2.378	4.766
	0.7	3.824	1.626	5.450
	0.9	5.587	0.616	6.203

Table 2. Simulated values for the expected numbers of perfect and minimal repairs and its corresponding expected costs for $\alpha = 1.0$ and $t = 10$.

$1/\lambda$	P	$E[N^P(t,p)]$	$E[N^M(t,p)]$	$E[N^P(t,p)]+E[N^M(t,p)]$
	0.1	1.515	8.469	9.984
	0.3	3.135	6.833	9.968
0.000001	0.5	5.003	4.945	9.948
	0.7	7.016	3.001	10.017
	0.9	9.049	0.985	10.034
	0.1	1.501	8.517	10.018
	0.3	3.117	6.860	9.977
0.0001	0.5	5.054	4.987	10.041
	0.7	6.967	3.018	9.985
	0.9	9.011	1.014	10.025
	0.1	1.490	8.417	9.907
	0.3	3.090	6.764	9.854
0.01	0.5	5.011	4.936	9.947
	0.7	6.895	2.966	9.861
	0.9	8.905	0.987	9.892

while the expected number of failures increases for α=0.5. When the mean repair time becomes shorter, the expected number of failures increases in all cases, which is as expected. In case $\alpha = 1.0$, the failure time distribution is exponential, but the convolution of an exponential failure time distribution and an exponential repair time distribution has an increasing failure rate. Such property has very little effect on our simulation results in which the expected number of failures is constantly around 10.0 since the mean repair time is much smaller than the mean mean operating time. However, it would be different when the mean repair time is relatively large. Finally, we note that when the mean repair time is extremely small, i.e. $1/\lambda$=0.000001, which represent the negligible repair time, the values in our simulation results are very close to those in the simulation results of Lim, Park and Sohn(1999).

Table 3. Simulated values for the expected numbers of perfect and minimal repairs and its corresponding expected costs for α = 2.0 and t = 10.

$1/\lambda$	P	$E[N^P(t,p)]$	$E[N^M(t,p)]$	$E[N^P(t,p)]+E[N^M(t,p)]$
	0.1	4.037	19.500	23.537
	0.3	5.968	12.552	18.520
0.000001	0.5	7.614	7.480	15.094
	0.7	9.105	3.825	12.930
	0.9	10.338	1.151	11.489
	0.1	4.036	19.496	23.532
	0.3	5.963	12.421	18.384
0.0001	0.5	7.637	7.451	15.088
	0.7	9.066	3.865	12.931
	0.9	10.346	1.144	11.490
	0.1	3.930	18.986	22.916
	0.3	5.834	12.161	17.995
0.01	0.5	7.508	7.367	14.875
	0.7	8.964	3.813	12.777
	0.9	10.245	1.135	11.380

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