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Channel Reallocation Methodologies for Restorable Transmission Networks*

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ABSTRACT

This paper develops channel reallocation methodologies for survival transmission networks. Any failure on a high-speed telecommunication network needs to be restored rapidly and a channel reallocation methodology has an important role for the fast restoration. This paper considers the channel reallocation problems under a line restoration with distributed control, where the line restoration restores the failed channels by rerouting the channels along the two alternative routes. The objective is to determine the optimal number of rerouting channels on the alternative rerouting paths of a given transmission network, where the optimality criteria are the first, the last and the mean restoration times. For each criterion, the problem is formulated as a mixed integer programming and the optimal channel reallocation algorithm is suggested based upon the characterization of the optimal solution.

1. INTRODUCTION

Any failure on the high-speed telecommunication networks makes large cost, data loss, and loss of goodwill. Accordingly, survivability is an important factor in constructing the high-speed networks. One of popular restoration technique is a line restoration under distributed control which is composed of five main phases of detection, instigation, route selection, rerouting and return-to-normal phases (see [7, 10, 12, 13]). The criteria generally used to design network restoration methodologies are the minimization of the restoration time and the major part of the restoration time is switching time of the rerouting phase as described by Kobrinski and Azuma [7] and Wu *et al.* [14]. This paper considers channel reallocation methodologies at the rerouting phase of the restoration procedure to mini-

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mize the first, the last and the mean restoration time on STM/SDH (Synchronous Transfer Mode / Synchronous Digital Hierarchy) transmission networks. In the methodologies, a decision problem is how many failed channels are assigned (rerouted) to bifurcated paths for the fast restoration.

As the related researches on the bifurcated paths, Banerjee and Mukherjee [2] have formulated a linear programming problem for the virtual topology design in a wavelength routed optical network to minimize hop distance, where a bifurcated routing is allowed for each packet traffic. Skorin-Kapov and Labourdette [11] have formulated a mixed integer programming for designing a logical network to minimize congestion, where a flow from a source is allowed to be split and reached a sink via different routes and they have developed a heuristic algorithm based on tabu search. It is noticed that there are bifurcated routes between a pair of source and destination nodes on a ring topology. For example, Grover and Stamatelakis [4] have suggested a restoration algorithm based on the formation of pre-configured cycles (rings), where a link failure can be restored via bifurcated routes. As researches on channel reallocation methodologies, Fujii *et al.* [3] have reallocated the secondary rerouting path the exceeding failed channels of the first selected one when the first selected one has not sufficient spare channels to restore the whole failed channel. Joo and Lee [7] have assigned the more failed channels to the to the rerouting paths according to the non-increasing order of their spare channels. Although several performance evaluations are reported on the restoration time of the restoration algorithm by [5, 7, 8, 9, 14], those are based on numerical methods and no previous research is appeared on the channel reallocation problem for the restoration algorithm by using an analytical method. This paper considers the channel reallocation problems using bifurcated routes and develops the optimal solution algorithms based upon the analytic characterization of the optimal solution.

Similar problems to the channel reallocation ones also occur on a high-speed computer with several parallel processors for a single input and output devices. A protection for span-cut failures or setting the configuration on a bi-directional ring network can be processed via bifurcated routes. The channel assignment problem can also be found in the initial setting of the switching configuration on a transmission network. For the transmission service, a routing mechanism is required to assign a number of channels on the possible paths between each pair of source and destination nodes.

This paper is composed as follows. Section 2 describes the problem in detail and formulates the problems with mathematical programming. Section 3 finds the optimal solution characteristics and then suggests the optimal solution algorithms for minimizing the first, the last and the mean restoration times. Some concluding remarks are added at the final section.

2. PROBLEM MODELING

Consider a number of channels in service between a pair of source and destination nodes on a transmission network. If a link in service fails, then a network restoration algorithm will be activated to restore the failed channels by using spare channels on dynamically found rerouting paths, where each rerouting path has its restricted number of spare channels. This paper considers a line restoration situation under a distributed control on TDM (Time-Division Multiplexing) or WDM (Wavelength-Division Multiplexing) transmission networks. The line restoration method restores a failure via five main phases of detection, instigation, route selection, rerouting and return-to-normal phases. For the failed link, the failure situation is detected by two nodes connected the failed link and one of two nodes detecting the failure is assigned as *sender* and the other is assigned as *chooser*. Then, alternative routes are searched between the sender and chooser, where each route will have different length and bandwidth and it can be utilized to reroute some amount of failed channels by using its available spare channels. The route selection phase reserves the spare bandwidth on the selected alternative routes to restore the failed channels. A new switching from the failed route to the alternative one is performed at the rerouting phase and the new configuration remains during the state of the link failure. If the link is repaired and operated normally, then the return-to-normal phase reroutes the channels to the repaired link as the original routing. For the line restoration, the sender initiates a message flooding for finding rerouting routes and selects the rerouting paths and then reroutes the failed channels along to the selected rerouting paths. The chooser initiates transmission a message of reserving spare channels on the paths along the possible rerouting routes. The chooser also may play the role of a trigger for return-to-normal phase when the failure is repaired. The remaining nodes on the rerouting path are called *tandem* nodes and used to transmit each message to the adjacent node on the path.

This paper considers channel reallocation methodologies at the rerouting phase of the restoration procedure to minimize the first, the last and the mean restoration time, where it is assumed that bifurcated rerouting paths are selected at the route selection phase and the required time till the selection phase is negligible.

For the problem modeling, let us define some notations as follows:

m = total number of rerouting paths

s_j = speed factor of path j excluding the sender and chooser, $j = 1, 2, \cdots, m$

s_s = speed factor of the sender

s_d = speed factor of the chooser

d = total number of channels to be restored

n_j = total number of assigned channels on path j, where $\sum_{j=1}^{m} n_j = d$

H_j = total number of tandem nodes on path j, $j = 1, 2, \cdots, m$

U_j = maximum capacity of assignable channels on path j, $n_j \le U_j$, $j = 1, 2, \cdots, m$

t = basic processing time per channel

Suppose that there exist m rerouting paths and each failed channel requires the same basic processing time t for switching and transmission at a tandem node. Consider d channels between the sender and chooser whose rerouting processing times at the sender and chooser are s_s and s_d per channel, respectively. Assume that each tandem node processes rerouting the channels independently and there exist H_j tandem nodes on path j. Then, total required time on path j will become tH_j per channel and let us denote the tH_j as s_j depicted in Figure 1. So the problem can be regarded as one with series-parallel structure, where the parallel structure is composed of m nodes with speed factor s_j for node j, $j = 1, 2, \cdots, m$, and the node is called *intermidiate node* through the remaining of the paper.

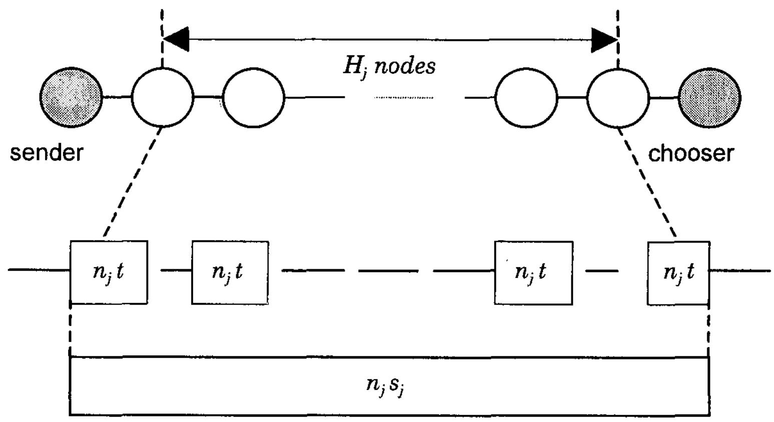

Figure 1. Tandem nodes on rerouting path j

Each path j has capacity restriction U_j on the assignable number of restoration channel n_j, where it is assumed that $\sum_{j=1}^{m} U_j \geq d$ for feasibility. Since a line restoration algorithm is considered, the message for switching the n_j channels at the sender and intermediate node are transmitted to the adjacent node only when all the n_j channels are processed at the nodes, where it is assumed that there is no failure on switching(rerouting) at each node. Therefore, all the rerouted n_j channels are transmitted after $n_j\, s_s$ and $n_j\, s_j$ unit time duration at the sender and intermediate node, respectively. The objective of the paper is to develop an exact algorithm for finding the value of $\{n_j\}$ which minimizes restoration time, where the first, the last and the mean restoration times are considered each other as the restoration time metrics.

For the problem formulation, some notations are additionally defined as follows:

C_j^i = completion time of n_j channels at stage i, where i = 1 at the sender, i = 2 at the intermidiate node and i = 3 at the chooser, $j = 1, 2, \cdots, m$

x_{jk} = 1 if n_j channels precede n_k channels at the chooser, 0 otherwise

y_j = 0 or 1, $j = 1, 2, \cdots, m$

M = very large positive number

It is assumed without loss of generality that n_{j+1} channels are processed after completion of the n_j channels processing at the sender, $j = 1, 2, \cdots, m-1$. Then it is dominant that all the channels are processed without idle time at the sender and the processed n_j channels are transmitted immediately to the intermediate node j. However, there may be exist intermediate idle times at the chooser. Accordingly, the completion time of the n_j channels is derived as equation (1).

$$C_j^3 \geq (\sum_{i=1}^{j} n_i) s_s + n_j (s_j + s_d), \; j = 1, 2, \cdots, m \tag{1}$$

Furthermore, since no two channels are processed simultaneously by the chooser, the decision variables C_j^i must satisfy the relationship (2)-(5).

$$C_j^3 - C_k^3 + M(1 - x_{jk}) \geq n_j s_d, \quad j, k = 1, 2, \cdots, m \tag{2}$$

$$C_k^3 - C_j^3 + Mx_{jk} \geq n_k s_d, \quad j, k = 1, 2, \cdots, m \tag{3}$$

$$x_{jk} = 0, 1, \quad j, k = 1, 2, \cdots, m \tag{4}$$

$$C_j^3 \geq 0, \quad j = 1, 2, \cdots, m \tag{5}$$

The decision variable n_j is an integer whose value is less than or equal to its path capacity. It is assumed that all the failed channels are restored as formulated in equations (6)-(8).

$$n_j \leq U_j, \quad j = 1, 2, \cdots, m \tag{6}$$

$$\sum_{j=1}^{m} n_j = d \tag{7}$$

$$n_j \geq 0 \text{ and integer}, \quad j = 1, 2, \cdots, m \tag{8}$$

The restoration time is measured as the first, the last and the mean ones. Firstly, the first restoration time is defined as the completion time of the firstly restored channel. Its importance is primarily to assess the impact on high priority demands and to characterize speed of the restoration algorithm when it acts in a mode imitative of an APS(Automatic Protection Switching) system for single-link failures. The problem can be formulated as following problem (P1).

$$\begin{aligned} &\textit{Min.}\ F_1 && \text{(P1)} \\ &\quad s.t.\ F_1 \geq C_j^3 - (n_j - 1)\, s_d + M(y_j - 1), \quad j = 1, 2, \cdots, m \\ &\quad \sum_{j=1}^{m} y_j = 1 \\ &\quad y_j = 0, 1, \quad j = 1, 2, \cdots, m \\ &\quad \text{Equations (1)-(8)} \end{aligned}$$

The first and second constraints in (P1) denote that the first restoration time F_1 is not smaller than the completion time of any channel at the chooser.

Secondly, the last restoration time is the time to complete rerouting all the failed channels. The minimal last restoration time is preferred when the worst level on continuity of service is a critical factor of the service. The first constraint of the problem (P2) implies that the last restoration time $F_{\max}$ is the time for restoration of all the failed channels.

$$\begin{aligned} &Min. \quad F_{\max} && \text{(P2)} \\ &s.t. \quad F_{\max} \geq C_j^3, \; j = 1,2,\cdots,m \\ &\qquad \text{Equations (1)-(8)} \end{aligned}$$

Finally, the mean restoration time is defined as the arithmetic mean of each restoration time of the restored channels and it is used as one popular measure for survivable transmission networks. The problem for the minimal mean restoration time can be formulated as the following problem (P3).

$$\begin{aligned} &Min. \quad \sum_{j=1}^{m} n_j (C_j^3 - n_j s_d) + \sum_{j=1}^{m} \frac{n_j(n_j - 1)}{2} s_d && \text{(P3)} \\ &s.t. \quad \text{Equations (1)-(8)} \end{aligned}$$

As shown in problems (P1), (P2) and (P3), the rerouting problems can be formulated as mixed integer programming and the required computational time for finding the optimal solution is generally very large. Therefore, the optimal solution of the problems (P1), (P2) and (P3) may not be useful for the fast restoration. However, it can be easily shown that the restoration times are derived as $F_1 = d(s_s + s_I) + s_d$, $F_{\max} = d(s_s + s_I) + s_d$ and $\overline{F} = (d)^2 (s_s + s_I + s_d) + \frac{d(d+1)}{2} s_d$ for the first, the last and the mean restoration ones when a single restoration path with speed factor s_I is used for the restoration, respectively. And Section 3 considers the channel reallocation problems with two alternative rerouting paths, $m = 2$.

3. ANALYSIS

The restoration time may depend on the sequence of processing at the sender and chooser and the processing order is determined according to a dispatching rule. The following property characterizes the dispatching rule at the chooser.

Property 1. For the first, the last and the mean restoration time criteria, the processing sequence at the chooser has no effect on the criteria.

Proof. For the waiting channels at the chooser, the first restoration time is minimized if the earliest transmitted channels are processed at the chooser regardless of their sequence. The last restoration time also minimized if all the channels are processed early as soon as possible without intermediate idle time. Similarly, the mean restoration time is a constant value independently on their sequence. This completes the proof.

Property 1 implies that the sequence for waiting channels at the chooser has no effect on the considered measures. However, this paper considers a sequence of the nondecreasing order of { C_j^2 } at the chooser for convenience. At the remaining of the section, we characterize the optimal solution properties at the sender and suggest channel reallocation methodologies for each criterion.

3.1 The First Restoration Time

The first restoration time indicates the minimum interruption that a priority service would get. For the fast restoration, one may arrange priority use of the fast path found in a restoration plan so as to have minimal outage for priority services.

The following property characterizes the optimal reallocation method for the first restoration time

Property 2. If the first one channel at the sender is assigned and restored along the fastest rerouting route(intermediate node), then the first restoration time is minimized.

Proof. There exist two types of the first restoration time according to the relative length of $C_j^2, j = 1, 2$. The one is $n_1(s_s+s_1)+s_d$ under $C_1^2 \le C_2^2$ which leads to the constraint $n_1 \le (\frac{s_s+s_2}{s_s+s_1+s_2})d$. Accordingly, the optimal solution of the first type is $n_1^*=1$ with the first restoration time $s_s+s_1+s_d$. The other is $ds_s+n_2s_2+s_d$ under $C_1^2 > C_2^2$ which can be re-expressed as the constraint $n_2 \le (\frac{s_1}{s_s+s_1+s_2})d$. Therefore, the optimal solution is $n_2^*=1$ with the restoration time $ds_s+s_2+s_d$ for the case $C_1^2 > C_2^2$. Therefore, the minimal first restoration time is $s_s+s_1+s_d$ when $s_1 \le s_2$ and $n_1^*=1$. This completes the proof.

Notice that the capacity restriction U_j on the number of allocable channels of path j has no effect since the optimal number of restoration channels is one. Furthermore, Property 2 can be easily shown that its optimality is remained for the problem (P1) with any number of rerouting paths m.

3.2 The Last Restoration Time

The last restoration time is the time to complete rerouting all the failed channels since all the failed channels are assumed to be restoration. The minimal last restoration time is preferred when the worst level on continuity of service is a critical factor of the service. Suppose that there exist bifurcated routes and the path capacity is very large. The following property characterizes the optimal reallocation method when $C_1^2 \le C_2^2$. It is noticed that the relationship $C_1^2 \le C_2^2$ equals to $n_1 \le (\frac{s_s + s_2}{s_s + s_1 + s_2})d$ or $n_2 \ge (\frac{s_1}{s_s + s_1 + s_2})d$.

Property 3. If $C_1^2 \le C_2^2$, then there exist two types of the last restoration time, $F^1_{\max}(n_1^*)$ and $F^2_{\max}(n_2^*)$, for the optimal solution, where $F^1_{\max}(n_1^*) = n_1^*(s_s + s_1) + ds_d$ for $n_1^* = \left\lceil (\frac{s_s + s_2}{s_s + s_1 + s_2 + s_d})d \right\rceil$ when $n_1^* \le \text{Min}\{(\frac{s_s + s_2}{s_s + s_1 + s_2})d, d-1\}$, $F^2_{\max}(n_2^*) = ds_s + n_2^*(s_2 + s_d)$ for $n_2^* = \left\lceil (\frac{s_1 + s_d}{s_s + s_1 + s_2 + s_d})d \right\rceil$ when $n_2^* \le d-1$, and $\lceil a \rceil$ denotes the smallest integer larger than or equal to a.

Proof. If $C_1^2 \le C_2^2$, the channel reallocation can be performed according to either $C_2^2 < C_1^3$ or $C_2^2 \ge C_1^3$, and each case of the $C_2^2 < C_1^3$ and $C_2^2 \ge C_1^3$ has the last restoration time $F^1_{\max}(n_1)$ and $F^2_{\max}(n_2)$, respectively.

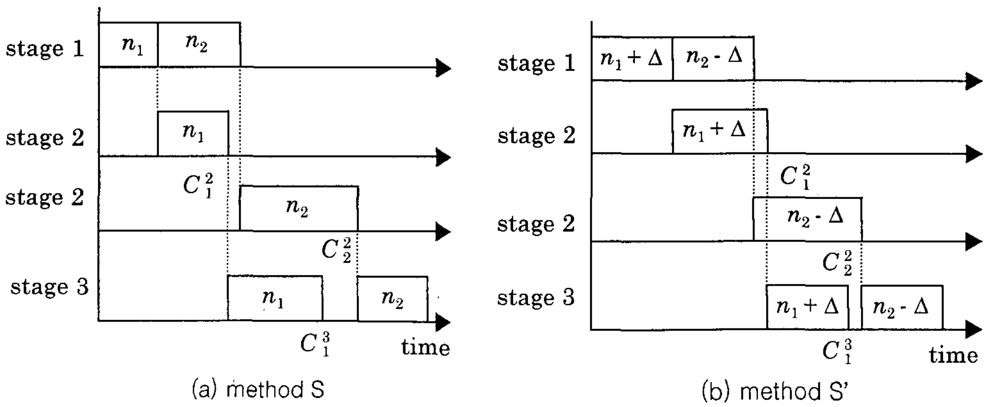

Figure 2. Channel assignment methods for the last restoration time criterion

For the case of $C_2^2 \geq C_1^3$, there exist intermediate idle time at stage 3 (chooser) as depicted in Figure 2, where the vertical axis represents three stages and the horizontal axis denotes the required processing time. The allocations S and S' in Figure 2 make the same type of the last restoration time, $F_{\max}^2(n_2) = ds_s + n_2(s_2 + s_d)$ since $n_1(s_s + s_1 + s_d) \leq (n_1 + n_2)s_s + n_2 s_2$. The relationship equals to the constraint $n_2 \geq (\frac{s_1 + s_d}{s_s + s_1 + s_2 + s_d})d$ and it is noticed that $(\frac{s_1 + s_d}{s_s + s_1 + s_2 + s_d})d > (\frac{s_1}{s_s + s_1 + s_2})d$. And their difference in the last restoration times is calculated as $F_{\max}^{s} - F_{\max}^{s'} = \Delta(s_2 + s_d) \geq 0$. Therefore, the smallest integer value of n_2 satisfying the relationship $n_2 \geq (\frac{s_1 + s_d}{s_s + s_1 + s_2 + s_d})d$ is the optimal solution for $F_{\max}^2(n_2^*)$.

Similarly, the other type of assignment with $F_{\max}^1(n_1)$ must be satisfied the relationship $(\frac{s_s + s_2}{s_s + s_1 + s_2})d \geq n_1 \geq (\frac{s_s + s_2}{s_s + s_1 + s_2 + s_d})d$. Therefore, the smallest value of n_1 on the possible ones leads to minimal value of $F_{\max}^1(n_1)$ such that $F_{\max}^1(n_1^*) = n_1^*(s_s + s_1) + ds_d$. However, since general superiority between $F_{\max}^1(n_1^*)$ and $F_{\max}^2(n_2^*)$ does not exist, the optimal solution is the one with the smaller value of the last restoration time among $F_{\max}^1(n_1^*)$ and $F_{\max}^2(n_2^*)$ for the case $C_1^2 \leq C_2^2$. This completes the proof.

It is necessary that the value of $F_{\max}^1(n_1^*)$ is letting to be infinity if there is no integer n_1 such that $(\frac{s_s + s_2}{s_s + s_1 + s_2 + s_d})d \leq n_1 \leq \text{Min}\{(\frac{s_s + s_2}{s_s + s_1 + s_2})d, d-1\}$. Let $F_{\max}^2(n_2^*) = \infty$ also if $n_2^* = d$ since the problem is assumed to use two alternative paths. For example, d = 10 and $(s_s, s_1, s_2, s_d) = (1, 10, 1, 10)$ lead to $F_{\max}^2(n_2^*) = \infty$ since $n_2^* = 10$ by Property 3.

Similarly to Property 3, the following property characterizes the optimal channel reallocation for the last restoration time under $C_1^2 \geq C_2^2$ which can be re-expressed as $n_1 s_1 \geq n_2(s_s + s_2)$.

Property 4. If $C_1^2 \geq C_2^2$, then there exist two types of the last restoration time,

$F^3_{\max}(n^*_1)$ and $F^4_{\max}(n^*_2)$, for the optimal solution, where $F^3_{\max}(n^*_1) = n^*_1(s_s + s_1 + s_d)$ for $n^*_1 = \left\lceil (\frac{s_s + s_2 + s_d}{s_s + s_1 + s_2 + s_d})d \right\rceil$ when $n^*_1 \le d-1$, $F^4_{\max}(n^*_2) = d(s_s + s_d) + n^*_2 s_d$ for $n^*_2 = \left\lceil (\frac{s_1}{s_s + s_1 + s_2 + s_d})d \right\rceil$ when $n^*_2 \le \text{Min}\{(\frac{s_1}{s_s + s_1 + s_2})d, d-1\}$ and $\lceil a \rceil$ denotes the smallest integer larger than or equal to a.

Proof. There exist two types of channel assignment forms in the sense of last restoration time when $C^2_1 \ge C^2_2$. The channel reallocation can be performed according to the either $C^2_1 > C^3_2$ with the last restoration time $F^3_{\max}(n^*_1)$ or $C^2_1 \le C^3_2$ with the time $F^4_{\max}(n^*_2)$, where the constraint $C^2_1 \le C^3_2$ equals to $ds_s + n_2(s_2 + s_d) \le n_1(s_s + s_1)$. The allocations satisfying the relationship $n_1 s_1 \ge n_2 (s_s + s_2)$ have the last restoration time $F^3_{\max}(n_1)$ and $F^4_{\max}(n_2)$ when the constraints $ds_s + n_2(s_2 + s_d) \ge n_1(s_s + s_1)$ and $ds_s + n_2(s_2 + s_d) \le n_1(s_s + s_1)$ are imposed, respectively. For the $F^3_{\max}(n_1)$, the constraints $n_1 s_1 \ge n_2(s_s + s_2)$ and $ds_s + n_2(s_2 + s_d) \ge n_1(s_s + s_1)$ lead to $n_1 \ge (\frac{s_s + s_2 + s_d}{s_s + s_1 + s_2 + s_d})d$. Therefore, the optimal value of n_1 is $n^*_1 = \left\lceil (\frac{s_s + s_2 + s_d}{s_s + s_1 + s_2 + s_d})d \right\rceil$ which minimizes the last restoration time $F^3_{\max}(n_1)$ so that $F^3_{\max}(n^*_1) = n^*_1(s_s + s_1 + s_d)$ when $n^*_1 \le d-1$.

For the case $F^4_{\max}(n^*_2)$, the optimality of n^*_2 within its domain $(\frac{s_1}{s_s + s_1 + s_2 + s_d})d \le n_2 \le \text{Min}\{(\frac{s_1}{s_s + s_1 + s_2})d, d-1\}$ can be proved similarly to that of $F^3_{\max}(n^*_1)$. This completes the proof.

Since Properties 3 and 4 characterize the optimal solution under the infinite path capacity, the properties cannot be directly used to find the optimal solution when the path capacity is finite. For the optimal solution algorithm, let us denote $l^1_1 = (\frac{s_s + s_2}{s_s + s_1 + s_2 + s_d})d$ and $l^2_2 = (\frac{s_1 + s_d}{s_s + s_1 + s_2 + s_d})d$, $l^3_1 = (\frac{s_s + s_2 + s_d}{s_s + s_1 + s_2 + s_d})d$ and $l^4_2 = (\frac{s_1}{s_s + s_1 + s_2 + s_d})d$.

Algorithm 1

Step 1. Calculate l_1^1, l_2^2, l_1^3 and l_2^4.

Step 2. If $l_1^1 > U_1$ or $d - U_2 > \text{Min}\{(\frac{s_s + s_2}{s_s + s_1 + s_2})d, d-1\}$, then let $F_{\max}^1(n_1^1) = \infty$ and go to Step 3. Otherwise, let $n_1^1 = \lceil l_1^1 \rceil$ and $n_2^1 = d - n_1^1$. If $n_2^1 \rangle U_2$, then let $n_2^1 \rangle U_2$ and $n_1^1 = d - n_2^1$. Calculate $F_{\max}^1(n_1^1)$.

Step 3. If $l_2^2 > \text{Min}\{U_2, d-1\}$, then let $F_{\max}^2(n_2^2) = \infty$ and go to Step 4. Otherwise, let $n_2^2 = \lceil l_2^2 \rceil$ and $n_1^2 = d - n_2^2$. If $n_1^2 > U_1$, then let $n_1^2 = U_1$ and $n_2^2 = d - n_1^2$. Calculate $F_{\max}^2(n_2^2)$.

Step 4. If $l_1^3 > \text{Min}\{U_1, d-1\}$, then let $F_{\max}^3(n_1^3) = \infty$ and go to Step 5. Otherwise let $n_1^3 = \lceil l_1^3 \rceil$ and $n_2^3 = d - n_1^3$. If $n_2^3 > U_2$, then let $n_2^3 = U_2$ and $n_1^3 = d - n_2^3$. Calculate $F_{\max}^3(n_1^3)$.

Step 5. If $l_2^4 > U_2$ or $d - U_1 > \text{Min}\{(\frac{s_1}{s_s + s_1 + s_2})d, d-1\}$, then let $F_{\max}^4(n_2^4) = \infty$ and go to Step 6. Otherwise, let $n_2^4 = \lceil l_2^4 \rceil$ and $n_1^4 = d - n_2^4$. If $n_1^4 > U_1$, then let $n_1^4 = U_1$ and $n_2^4 = d - n_1^4$. Calculate $F_{\max}^4(n_2^4)$.

Step 6. Find the minimum last restoration time $F_{\max}^*$ such that $F_{\max}^* = \text{Min}\{F_{\max}^1(n_1^1), F_{\max}^2(n_2^2), F_{\max}^3(n_1^3), F_{\max}^4(n_2^4)\}$ and find the optimal channel reallocation (n_1^*, n_2^*) such that $(n_1^*, n_2^*) = (n_1^k, n_2^k)$ for $k = \arg \min_{i \le j \le 4} \{F_{\max}^j(n^j)\}$.

All the sub-optimal solutions of Properties 3 and 4 are modified to satisfy the path capacity restrictions at Steps 2, 3, 4 and 5 of Algorithm 1. The optimal solution is obtained at Step 6 by selecting one with the minimum last restoration time.

Property 5. Algorithm 1 guarantees the optimal reallocation for problem (P2) with two alternative routes.

Proof. To show the optimality of Algorithm 1, we will prove that the algorithm always finds the best solution among all dominant types of reallocation methods.

It is noticed that there exist four types of functions for the last restoration time. All the types are considered at Steps 2 through 5 which are characterized their optimality at Properties 3 and 4 when $U_j = \infty$, $j = 1, 2$. To comply the path capacity restriction, Step 2 let $F^1_{\max}(n^1_1) = \infty$ if $l^1_1 > U_1$ or $d - U_2 > \text{Min}\{(\frac{s_s + s_2}{s_s + s_1 + s_2})d, d-1\}$ since the domain of n_1 is $(\frac{s_s + s_2}{s_s + s_1 + s_2 + s_d})d \le n_1 \le \text{Min}\{(\frac{s_s + s_2}{s_s + s_1 + s_2})d, d-1\}$. If $l^1_1 \le \text{Min}\{U_1, (\frac{s_s + s_2}{s_s + s_1 + s_2})d\}$, then the optimal value of n_1 is let $n^*_1 = \lceil l^1_1 \rceil$ according to Property 3. However, if the resulted solution (n^*_1, n^*_2) does not satisfy the path capacity restriction, it need to modify for the capacity restriction so that $n^1_2 = U_2$ and $n^1_1 = d - n^1_2$ under the relationship $n^*_2 > U_2$. The modified value of n^1_1 is the smallest value within its domain and thus it is optimal value for $F^1_{\max}(n^*_1)$ even though $n^1_1 > n^*_1$. The other steps of Steps 3 through 5 are constructed similarly to Step 2. Therefore, the minimum of four types of functions leads to the global optimal solution. This completes the proof.

As a numerical example, consider a network restoration problems for 10 failed channels with 2 alternative rerouting paths between the sender and chooser, where the paths have speed factors $(s_1, s_2) = (4, 6)$, $s_s = 7$ and $s_d = 5$ as depicted in Figure 3. And each path has spare channels $(U_1, U_2) = (3, 8)$ for restoration the failed channels.

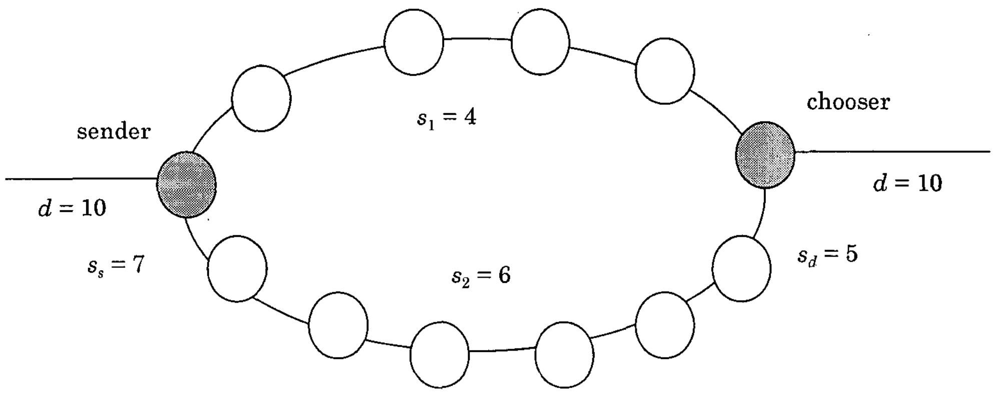

Figure 3. Link disjointed two alternative paths

At Step 1, the associated values are calculated as $l_1^1 = (\frac{s_s + s_2}{s_s + s_1 + s_2 + s_d})d = \frac{13(10)}{22} = 5\frac{20}{22}$ and $l_2^2 = \frac{9}{22}d = 4\frac{2}{22}$, $l_1^3 = 8\frac{2}{22}$ and $l_2^4 = 1\frac{18}{22}$. Since $l_1^1 = 5\frac{20}{22} > U_1 = 3$, $F_{\max}^1(n_1^1) = \infty$ at Step 2. For the value of n_2^2, let $n_2^2 = \left\lceil 4\frac{2}{22} \right\rceil = 5$ and $n_1^2 = d - n_2^2 = 5$ since $l_2^2 < U_2$. However, the relationship $n_1^2 > U_1 = 3$ requires modification the values as $n_1^2 = U_1 = 3$ and $n_2^2 = d - n_1^2 = 7$ at Step 3. Accordingly, $F_{\max}^2(n_2^2)$ is calculated as 10(7) + 7 (6 + 5) = 147. For the same reason as that of $F_{\max}^1(n_1^1)$, $F_{\max}^3(n_1^3) = \infty$ at Step 4. At Step 5, $F_{\max}^4(n_2^4) = \infty$ because $d - U_1 = 7 > \text{Min}\{(\frac{s_1}{s_s + s_1 + s_2})d, d-1\} = \text{Min}\{\frac{40}{17}, 9\} = \frac{40}{17}$. Therefore, the optimal solution is obtained at Step 6 as $(n_1^*, n_2^*) = (3, 7)$ with the last restoration time 147.

As discussed at Section 2, the optimal restoration time is $d(s_s + s_1 + s_d) = 7$ if only the fastest route is used to restore the channels. Therefore, if the sender waits the secondary route for less than $d(s_s + s_1 + s_d) = 7 \text{ - } F_{\max}^* = 160 - 147 = 13$ unit time, it will have better performance in the sense of the last restoration time than a restoration algorithm with a single route selection mechanism.

3.3 The Mean Restoration Time

The mean restoration time is defined as the arithmetic mean of each restoration time of the restored channels and it is used as one popular measure for survivable transmission networks.

For a solution algorithm, the following property characterizes the optimal channel reallocation methods when $C_1^2 \le C_2^2$.

Property 6. If $C_1^2 \le C_2^2$, then there exist two types of the mean restoration time, $\overline{F}^1(n_1^*)$ and $\overline{F}^2(n_2^*)$, for the optimal solution, where $\overline{F}^1(n_1^*) = n_1^*(s_s + s_1)d + \frac{d(d+1)}{2}s_d$ for $n_1^* = \left\lceil (\frac{s_s + s_2}{s_s + s_1 + s_2 + s_d})d \right\rceil$ when $n_1^* \le \text{Min}\{(\frac{s_s + s_2}{s_s + s_1 + s_2})d, d-1\}$,

$$\overline{F}^2(n_2^*) = (n_2^*)^2(s_2 + s_1 + s_2 + s_d) - (n_2^*)(s_2 + 2s_1 + s_d)d + (d)^2(s_2 + s_1 + \frac{s_d}{2}) + \frac{d}{2}s_d$$

for $n_2^* = \text{Max}\{\left\lceil (\frac{s_1+s_d}{s_s+s_1+s_2+s_d})d \right\rceil, \ \textit{round}\ [(\frac{s_s+2s_1+s_d}{2(s_s+s_1+s_2+s_d)})d\ ]\}$ when $n_2^* \le d-1$, $\lceil a \rceil$ and round [a] denote the smallest integer larger than or equal to a and the nearest integer to a, respectively.

Proof. There exist two types of channel assignment forms in the sense of the mean restoration time when $C_1^2 \le C_2^2$. The channel reallocation can be performed according to the either $C_2^2 < C_1^3$ with the mean restoration time $\overline{F}^1(n_1^*)$ or $C_2^2 \ge C_1^3$ with the time $\overline{F}^2(n_2^*)$.

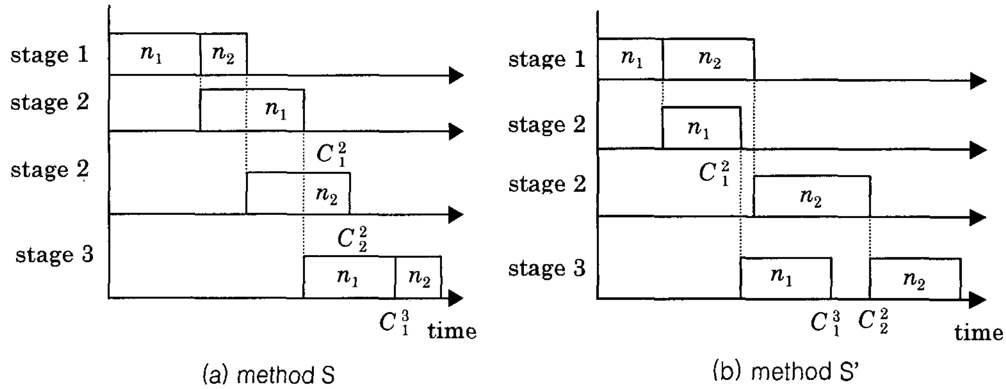

Figure 4. Channel assignment methods for the mean restoration time criterion

For the case of $C_2^2 < C_1^3$, there exist no intermediate idle time at stage 3 (chooser) as depicted in Figure 4 (a). The allocation S in Figure 4 (a) makes a type of the mean restoration time $\overline{F}^1(n_1^*)$ because $n_1(s_s+s_1+s_d) \ge (n_1+n_2)s_s + n_2 s_2$ which equals to the relationship $n_1 \ge (\frac{s_s+s_2}{s_s+s_1+s_2+s_d})d$. Since $\overline{F}^1(n_1)$ is derived as $\overline{F}^1(n_1) = n_1(s_s+s_1)d + \frac{d(d+1)}{2}s_d$, it is sufficient to minimize n_1 for the optimal mean restoration time. Therefore, the optimal solution is obtained as $n_1^* = \left\lceil (\frac{s_s+s_2}{s_s+s_1+s_2+s_d})d \right\rceil$ if n_1^* is less than or equal to $\text{Min}\{(\frac{s_s+s_2}{s_s+s_1+s_2})d, d-1\}$.

Similarly, the assignment S' in Figure 4 (b) has intermediate idle time at chooser and leads to another type of the mean restoration time $\overline{F}^2(n_2)$ whose value is derived as $\overline{F}^2(n_2)=(n_1)^2(s_2+s_1)+(ds_s+n_2s_2)n_2+[\frac{(n_1)^2+(n_2)^2+d}{2}]d=$ $(n_2)^2(s_2+s_1+s_2+s_d)-n_2(s_2+2s_1+s_d)d+(d)^2(s_s+s_1+\frac{s_d}{2})+\frac{d}{2}s_d$. It can be easily shown that $\overline{F}^2(n_2)$ is minimized when n_2 is the nearest integer to the value $[\frac{s_s+2s_1+s_d}{2(s_s+s_1+s_2+s_d)}]d$. However, the constraints $n_1(s_s+s_1+s_d)\le ds_s+n_2s_2$ and $n_1(s_s+s_1)\le ds_s+n_2s_2$ restrict the domain of n_2 as $n_2\ge(\frac{s_1+s_d}{s_s+s_1+s_2+s_d})d$. Therefore, the optimal value of n_2 is obtained as $n_2^*=\text{Max}\{\left\lceil(\frac{s_1+s_d}{s_s+s_1+s_2+s_d})d\right\rceil$ round $[\frac{s_s+2s_1+s_d}{2(s_s+s_1+s_2+s_d)}d]\}$ when $n_2^*\le d-1$. By the way, since there is no general superiority between $\overline{F}^1(n_1^*)$ and $\overline{F}^2(n_2^*)$, the optimal solution is the one with the smaller value of the mean restoration time among $\overline{F}^1(n_1^*)$ and $\overline{F}^2(n_2^*)$ for the case $C_1^2\le C_2^2$. This completes the proof.

Since one reallocation method with either $C_1^2\le C_2^2$ or $C_1^2\ge C_2^2$ generally does not better than the other one, the following property is also derived for the remaining case $C_1^2\ge C_2^2$.

Property 7. If $C_1^2\ge C_2^2$, then there exist two types of the mean restoration time, $\overline{F}^3(n_1^*)$ and $\overline{F}^4(n_2^*)$, for the optimal solution, where $\overline{F}^3(n_1^*)=(n_1^*)^2(s_s+s_1+s_2+s_d)-n_1^*(s_s+2s_2+s_d)d+(d)^2(s_2+s_2+\frac{s_d}{2})+\frac{d}{2}s_d$ for $n_1^*=\text{Max}\{\left\lceil(\frac{s_s+s_2+s_d}{s_s+s_1+s_2+s_d)})d\right\rceil$ $round[\frac{s_s+2s_2+s_d}{2(s_s+s_1+s_2+s_d)}d]\}$ when $n_1^*\le d-1$, $\overline{F}^4(n_2^*)=(ds_s+n_2^*s_2)d+\frac{d(d+1)}{2}s_d$ for $n_2^*=\left\lceil(\frac{s_1}{s_s+s_1+s_2+s_d})d\right\rceil$ when $n_2^*\le\text{Min}\{(\frac{s_1}{s_s+s_1+s_2})d, d-1\}$, $\lceil a\rceil$ and *round* $[a]$ denote the smallest integer larger than or equal to a and the nearest integer to a, respectively.

Proof. Two types of channel assignment forms are derived according to the either

$C_1^2 > C_2^3$ with the mean restoration time $\overline{F}^3(n_1^*)$ or $C_1^2 \le C_2^3$ with the time $\overline{F}^4(n_2^*)$ If the channels are allocated to satisfy the relationships $n_1(s_s + s_1) \ge ds_s + n_2 s_2$ and $n_1(s_s + s_1) \ge ds_s + n_2(s_2 + s_d)$, the mean restoration time becomes $\overline{F}^3(n_1)$,

$$\overline{F}^3(n_1) = (n_1)^2(s_s + s_1 + s_2 + s_d) - n_1(s_s + 2s_2 + s_d) + d + (d)^2(s_s + s_2 + \frac{s_d}{2}) + \frac{d}{2}s_d$$

since $n_2 = d - n_1$. It is noticed that the relationships lead to $n_1 \ge (\frac{s_s + s_2 + s_d}{s_s + s_1 + s_2 + s_d})d$ and $\overline{F}^3(n_1)$ is minimized when n_1 is the nearest integer to the value $\frac{s_s + 2s_2 + s_d}{2(s_s + s_1 + s_2 + s_d)}d$. Therefore, the optimal value of n_1 is obtained such that

$$n_1^* = \text{Max}\{\left\lceil (\frac{s_s + s_2 + s_d}{s_s + s_1 + s_2 + s_d})d \right\rceil, \textit{round}\ [(\frac{s_s + 2s_2 + s_d}{2(s_s + s_1 + s_2 + s_d)})d]\} \text{ when } n_1^* \le d - 1.$$

By the way, if the allocation satisfies the relationships $n_1(s_s + s_1) \ge ds_s + n_2 s_2$ and $n_1(s_s + s_1) \le ds_s + n_2(s_2 + s_d)$, the mean restoration time becomes the minimal value $\overline{F}^4(n_2^*) = (ds_s + n_2^* s_2)d + \frac{d(d+1)}{2}s_d$ when the optimal value of n_2 is n_2^*, $n_2^* = \left\lceil (\frac{s_1}{s_s + s_1 + s_2 + s_d})d \right\rceil$ under the constraint $n_2^* \le \text{Min}\{(\frac{s_1}{s_s + s_1 + s_2})d, d - 1\}$, where the optimality can be shown similarly to that of $\overline{F}^3(n_1^*)$. It is noticed that there does not exist general superiority of either $\overline{F}^3(n_1^*)$ or $\overline{F}^4(n_2^*)$. Accordingly, a good solution can be obtained by selecting the one with the smaller value of the mean restoration time among $\overline{F}^3(n_1^*)$ and $\overline{F}^4(n_2^*)$. This completes the proof.

Since Properties 6 and 7 characterize the optimal solution under the infinite path capacity, the properties cannot be directly used to find the optimal solution under the finite path capacity. For the optimal solution algorithm, let us denote $e_1^1 = (\frac{s_s + s_2}{s_s + s_1 + s_2 + s_d})d$, $e_2^2 = (\frac{s_1 + s_d}{s_s + s_1 + s_2 + s_d})d$, $e_o^2 = [\frac{s_s + 2s_1 + s_d}{2(s_s + s_1 + s_2 + s_d)}]d$, $e_1^3 = (\frac{s_s + s_2 + s_d}{s_s + s_1 + s_2 + s_d})d$, $e_o^3 = [\frac{s_s + 2s_2 + s_d}{2(s_s + s_1 + s_2 + s_d)}]d$, and $e_2^4 = (\frac{s_1}{s_s + s_1 + s_2 + s_d})d$.

Then, the optimal channel reallocation algorithm can be derived similar to Algorithm 1 as follow.

Algorithm 2

Step 1. Calculate e_1^1, e_2^2, e_o^2, e_1^3, e_o^3 and e_2^4.

Step 2. If $e_1^1 > U_1$ or $d - U_1 > \text{Min}\{(\frac{s_s + s_2}{s_s + s_1 + s_2})d, d-1\}$, then let $\overline{F}^1(n_1^1) = \infty$ and go to Step 3. Otherwise, let $n_1^1 = \lceil e_1^1 \rceil$ and $n_2^1 = d - n_1^1$. If $n_2^1 > U_2$, then let $n_2^1 = U_2$ and $n_1^1 = d - n_2^1$. Calculate $\overline{F}^1(n_1^1)$.

Step 3. If $e_2^2 > \text{Min}\{U_2, d-1\}$, then let $\overline{F}^2(n_2^2) = \infty$ and go to Step 4. Otherwise, let $n_2^2 = \text{Max}\{\lceil e_2^2 \rceil, \textit{round}\ (e_o^2)\}$ and $n_1^2 = d - n_2^2$. If $n_1^2 > U_1$, then let $n_1^2 = U_1$ and $n_2^2 = d - n_1^2$. Calculate $\overline{F}^2(n_2^2)$.

Step 4. If $e_1^3 > \text{Min}\ \{U_1, d-1\}$, then let $\overline{F}^3(n_1^3) = \infty$ and go to Step 5. Otherwise, let $n_1^3 = \text{Max}\{\lceil e_1^3 \rceil, \textit{round}\ (e_o^3)\}$ and $n_2^3 = d - n_1^3$. If $n_2^3 > U_2$, then let $n_2^3 = U_2$ and $n_1^3 = d$. Calculate $\overline{F}^3(n_1^3)$.

Step 5. If $e_2^4 > U_2$ or $d - U_2 > \text{Min}\{(\frac{s_1}{s_s + s_1 + s_2})d, d-1\}$, then let $\overline{F}^4(n_2^4) = \infty$ and go to Step 6. Otherwise, let $n_2^4 = \lceil e_2^4 \rceil$ and $n_1^4 = d - n_2^4$. If $n_1^4 > U_1$, then let $n_1^4 = U_1$ and $n_2^4 = d - n_1^4$. Calculate $\overline{F}^4(n_2^4)$.

Step 6. Find the minimum mean restoration time $\overline{F}^*$ such that $\overline{F}^* = \text{Min}\ \{\overline{F}^1(n_1^1), \overline{F}^2(n_2^2), \overline{F}^3(n_1^3), \overline{F}^4(n_1^4)\}$ and the optimal channel reallocation is $(n_1^*, n_2^*) = (n_1^k, n_2^k)$ for $k = \arg\min_{1 \le j \le 4} \{\overline{F}^j(n_1^j)\}$.

Property 8. Algorithm 2 guarantees the optimal reallocation for problem (P3) with bifurcated routes.

Proof. The optimality of Algorithm 2 can be shown similarly to that of Property 5. It is noticed that there exist four types of the mean restoration time. Their optimality is characterized at Properties 6 and 7 and the restrictions on the finite path capacity are incorporated at Steps 2 through 5. Therefore, the minimum of four types of the mean restoration times leads to the optimal solution of the algorithm. This completes the proof.

For an example, let us reconsider the problem as discussed at Section 3.2. Ten failed channels are required to be rerouted along two alternative paths between the sender and chooser with their speed factors $(s_s, s_1, s_2, s_d) = (7, 4, 6, 5)$ and capacity $(U_1, U_2) = (3, 8)$.

For the optimal reallocation, Step 1 calculates the values as $e_1^1 = (\frac{s_s + s_2}{s_s + s_1 + s_2 + s_d})d = 5\frac{20}{22}$, $e_2^2 = (\frac{s_1 + s_d}{s_s + s_1 + s_2 + s_d})d = 4\frac{2}{22}$, $e_o^2 = 4\frac{24}{22}$, $e_1^3 = 8\frac{4}{22}$, $e_o^3 = 5\frac{20}{44}$, and $e_2^4 = 1\frac{18}{22}$. A sub-optimal solution is obtained as $\overline{F}^1(n_1^1) = \infty$ at Step 2 since $e_1^1 = 5\frac{20}{44} > U_1 = 3$. At Step 3, $n_2^2 = \mathrm{Max}\{\lceil e_2^2 \rceil, round\,(e_o^2)\} = \mathrm{Max}\{5, 5\} = 5$ and $n_1^2 = d - n_2^2 = 10 - 5 = 5$. However, a sub-optimal solution of Step 3 is $\overline{F}^2(n_2^2) = (7)^2(22) - 7(20)(10) + (10)^2(13.5) + 5(5) = 1053$ for $n_1^2 = U_1 = 3$ and $n_2^2 = d - n_1^2 = 7$ because $n_1^2 = 3 \le (\frac{s_s + s_2}{s_s + s_1 + s_2})d = 7\frac{11}{17}$. And $\overline{F}^3(n_1^3) = \infty$ at Step 4 since $e_1^3 = 8\frac{4}{22} > U_1 = 3$
As the final candidate for the optimal solution, Step 5 finds $\overline{F}^4(n_2^4) = \infty$ because there exists no solution (n_1^4, n_2^4) such that $n_2^4 \le (\frac{s_1}{s_s + s_1 + s_2})d = 2\frac{6}{17}$ and $n_1^4 + n_2^4 = d$. Therefore, the optimal solution is obtained at Step 6 as $(n_1^*, n_2^*) = (3, 7)$ with $\overline{F}^* = 1053$, $1053 = \mathrm{Max}\{\overline{F}^1(n_1^1), \overline{F}^2(n_2^2), \overline{F}^3(n_1^3), \overline{F}^4(n_2^4)\} = \mathrm{Min}\{\infty, 1053, \infty, \infty\}$.

It is noticed that the mean restoration time is $(d)^2(s_s + s_1) + \frac{d(d+1)}{2}s_d$ if only the fastest route is used to restore the channels. Therefore, if the sender waits the secondary route for less than $(d)^2(s_s + s_1) + \frac{d(d+1)}{2}s_d - \overline{F}^* = 1375 - 1053 = 322$ unit time, the restoration algorithm with two route selection mechanism will have better performance in the sense of the mean restoration time than a restoration algorithm with a single route selection mechanism.

4. CONCLUSION

The recent telecommunication network is designed for fast service to the various types of huge data including voice, text and image. Therefore, any failure on the network leads to serious problems such as inconvenience, large cost on lost of profit and loss of goodwill. This paper considers channel reallocation problems of rerouting phase at a line restoration under the distributed control. The objective of the channel reallocation problems is to find the optimal reallocation and rerouting methodologies for the failed channels along the alternative routes between the sender and chooser, where each alternative route has its hop count and path capacity. The first, the last and the mean restoration times of the failed channels are considered as the metrics of the channel reallocation problems. For each criterion, a mathematical programming is formulated explicitly and the optimal solution algorithm is developed based upon the optimal solution characterization when the failed channels are restored via bifurcated routes. These channel reallocation methodologies can be utilized to design the fast restoration algorithm on STM/SDH transmission networks since the rerouting time of the methodologies will be short due to the parallel processing on two alternative routes.

As a further study, a problem with three or more than routes is a general version of the paper. But, the problem seems to be a tedious subject for the network restoration algorithm. For an example, finding the optimal solution for the last restoration time is a hard problem as discussed by Joo[6]. And another problem with stochastic factors such as possibility of failure in alternative paths and uncertainty in the processing speed need to be studied.

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