An Implementation Method of Linearized Equations of Motion for Multibody Systems with Closed Loops

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Abstract

This research proposes an implementation method of linearized equations of motion for multibody systems with closed loops. The null space of the constraint Jacobian is first pre-multiplied to the equations of motion to eliminate the Lagrange multiplier and the equations of motion are reduced down to a minimum set of ordinary differential equations. The resulting differential equations are functions of all relative coordinates, velocities, and accelerations. Since the variables are tightly coupled by the position, velocity, and acceleration level constraints, direct substitution of the relationships among these variables yields very complicated equations to be implemented. As a consequence, the reduced equations of motion are perturbed with respect to the variations of all variables, which are coupled by the constraints. The position, velocity and acceleration level constraints are also perturbed to obtain the relationships between the variations of all relative coordinates, velocities, and accelerations and variations of the independent ones. The perturbed constraint equations are then simultaneously solved for variations of all variables only in terms of the variations of the independent variables. Finally, the relationships between the variations of all variables and these of the independent ones are substituted into the variational equations of motion to obtain the linearized equations of motion only in terms of the independent variables variations.

Key Words : Linearization, Vibration Analysis, Natural Frequency; Null Space; Constrained Multibody System

1. Introduction

Linearization is an important tool in understanding the system behavior of a nonlinear system at a certain state. As an example, the eigenvalues of the linearized equations of motion are very useful information in developing control logics. Linearization of an unconstrained system is relatively easier than that of the constrained systems due to the algebraic constraint equations and corresponding Lagrange multipliers. This research proposes a linearization method for the constrained mechanical systems and compares the results with those obtained from other

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methods.

Sohoni[1] presented an approach for automatically generating a linearized dynamical model which is derived from the nonlinear equations of motion. The Lagrange multiplier term was kept constant in the linearized equations of motion. The velocity and acceleration level constraints have not been considered in the resulting linearized equations of motion. Neuman symbolically generated the dynamic robot model by Lagrange-Euler formulation and linearized the dynamic model about a nominal trajectory[2]. Balafoutis presented a computational method for recursive evaluation of linearized dynamic robot model about a nominal trajectory[3]. The formulation was applied to the constrained robot systems. This formulation was generalized by Gontier[4] for general unconstrained mechanical systems. Similar formulations have been developed by the variational approach in Refs. [5,6]. A recursive formulation using the relative coordinates was proposed by Bae in Ref. [7]. The equations of motion were derived in a compact matrix form by using the velocity transformation method. The actual computation was carried out by using the recursive formulas developed for each joints. Realtime simulation of a vehicle system has been carried out by the recursive method in Ref. [8]. The Jacobian matrix was updated once in a while during time marching of the numerical integration. The recursive method was extended to the flexible body dynamics of constrained mechanical systems in Ref. [9]. A virtual body concept was employed to relieve the implementation burden of the flexible body dynamics coding. A compliant track link model was developed for tracked vehicles in Ref. [10]. A minimum set of the equations of motion was obtained by the recursive method. Concept of the configuration design variable with the recursive formulation was introduced in Ref. [11].

The equations of motion for multibody systems are highly nonlinear with respect to the relative positions, velocities, and accelerations. The equations of motion are perturbed to obtain the linearized equations of motion. Since the equations of motion are highly nonlinear, their perturbation involves with many arithmetic operations for a multibody system consisting of many bodies and joints. In case of open loop systems which do not have any constraints, the equations of motion result in the ordinary differential equations whose partial derivatives with respect to the relative coordinates, velocities, and accelerations has been obtained by several different methods in Refs.[2,3,4]. In case of closed loop systems which have constraints, these method cannot be used directly any more due to the constraints and corresponding Lagrange multipliers.

One of the intuitive methods to handle the constraints is to directly express the equations of motion only in terms of the independent relative positions, velocities, and accelerations. In order to achieve this goal, the relative coordinates must be divided into the independent and dependent variables must be directly expressed in terms of independent ones. However, the independent and dependent variables are tightly and nonlinearly coupled by the position, velocity, and acceleration level constraints and the equations of motion are implicit function of the coordinates, velocities, and accelerations. As a result, it is very difficult to directly express the dependent variables in terms of independent ones and consequently to express the equations of motion only in terms of the independent variables.

Relative coordinates and equations of motion of a constrained multibody system are introduced in Chapter 2 and 3. In Chapter 4, the null space of the constraint Jacobian is introduced first to eliminate the Lagrange multiplier in the constrained equations of motion, and which are reduced down to a minimum set of ordinary differential equations. The reduced equations of motion are perturbed with respect to the variations of all variables, which are coupled by the constraints. The position, velocity and acceleration level constraints are also perturbed to obtain the relationships between the variations of all variables and variations of the independent ones. Finally, the relationships between the variations of all variables and these of the independent ones are substituted into the variational equations of motion to obtain the linearized equations of motion only in terms of the independent coordinate, velocity, and acceleration variations.

The proposed method is implemented in the commercial program RecurDyn. Vibration analyses of a four bar mechanism and a cantilever beam with a driving constraint are carried out to demonstrate the validity of the proposed method.

2. Relative Coordinate Kinematics

Figure 1 shows the coordinate system fixed on a body i. In the figure, the $x'_i - y'_i - z'_i$ frame is the body reference frame and the X-Y-Z frame is the inertial reference frame. Point is the origin of X-Y-Z, point O_i is the origin of $x'_i - y'_i - z'_i$, and $\mathbf{r}_i$ is the position vector of O_i from O. The $\mathbf{f}_i$, $\mathbf{g}_i$, and $\mathbf{h}_i$ are unit vectors along the x'_i, y'_i, and z'_i axes, respectively. Orientation matrix of the body is given as

$$\mathbf{A}_i = [\mathbf{f}_i \quad \mathbf{g}_i \quad \mathbf{h}_i] \tag{1}$$

Velocities and virtual displacements of point O_i in the X-Y-Z frame are defined as (see Refs.[4-5])

$$\mathbf{Y}_i = \begin{bmatrix} \dot{\mathbf{r}}_i \\ \boldsymbol{\omega}_i \end{bmatrix} \tag{2}$$

$$\delta\mathbf{Z}_i = \begin{bmatrix} \delta\mathbf{r}_i \\ \delta\boldsymbol{\pi}_i \end{bmatrix} \tag{3}$$

Their corresponding quantities in the $x'_i - y'_i - z'_i$ frame are defined as

$$\mathbf{Y}'_i = \begin{bmatrix} \dot{\mathbf{r}}'_i \\ \boldsymbol{\omega}'_i \end{bmatrix} \equiv \begin{bmatrix} \mathbf{A}_i^T \dot{\mathbf{r}}_i \\ \mathbf{A}_i^T \boldsymbol{\omega}_i \end{bmatrix} \tag{4}$$

$$\delta\mathbf{Z}'_i = \begin{bmatrix} \delta\mathbf{r}'_i \\ \delta\boldsymbol{\pi}'_i \end{bmatrix} \equiv \begin{bmatrix} \mathbf{A}_i^T \delta\mathbf{r}_i \\ \mathbf{A}_i^T \delta\boldsymbol{\pi}_i \end{bmatrix} \tag{5}$$

A pair of contiguous bodies is shown in Figure 2. Body is assumed to be an inboard body of body and the position of point O_i is

$$\mathbf{r}_i = \mathbf{r}_{(i-1)} + \mathbf{s}_{(i-1)i} + \mathbf{d}_{(i-1)i} - \mathbf{s}_{i(i-1)} \tag{6}$$

By using Eq. (5), the angular virtual displacement of body in its local reference frame is

$$\delta\boldsymbol{\pi}'_i = \mathbf{A}^T_{(i-1)i}\delta\boldsymbol{\pi}'_{(i-1)} + \mathbf{A}^T_{(i-1)i}\mathbf{H}'_{(i-1)i}\delta\mathbf{q}_{(i-1)i} \tag{7}$$

where $\mathbf{H}'_{(i-1)i}$ is determined by the axis of rotation and $\mathbf{A}'_{(i-1)i}$ is defined as

$$\mathbf{A}_{(i-1)i} = \mathbf{A}^T_{(i-1)}\mathbf{A}_i \tag{8}$$

Taking variation of Eq. (6) yields

$$\begin{aligned} \delta\mathbf{r}'_i &= \mathbf{A}^T{}_{(i-1)i}\ \delta\mathbf{r}'_{(i-1)} \\ &- \mathbf{A}^T{}_{(i-1)i}\ (\ \tilde{\mathbf{s}}'_{(i-1)i} + \tilde{\mathbf{d}}'_{(i-1)i} - \mathbf{A}_{(i-1)i}\ \tilde{\mathbf{s}}'_{i(i-1)}\ \mathbf{A}^T{}_{(i-1)i}\ )\ \delta\boldsymbol{\pi}'_{(i-1)} \\ &+ \mathbf{A}^T{}_{(i-1)i}\ (\ (\mathbf{d}'_{(i-1)i})_{\mathbf{q}_{(i-1)i}} + \mathbf{A}_{(i-1)i}\ \tilde{\mathbf{s}}'_{i(i-1)}\ \mathbf{A}^T{}_{(i-1)i}\ \mathbf{H}'_{(i-1)i}\ )\ \delta\mathbf{q}_{(i-1)i} \end{aligned} \tag{9}$$

where symbols with tildes denote skew symmetric matrices comprised of their vector elements that implement

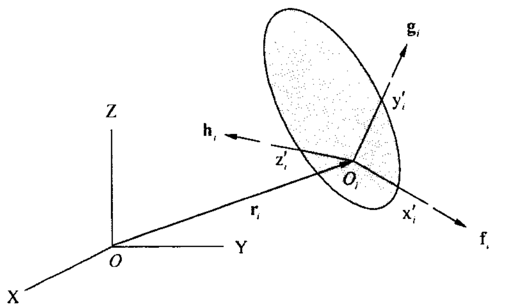

Figure 1. Coordinate systems and a rigid body

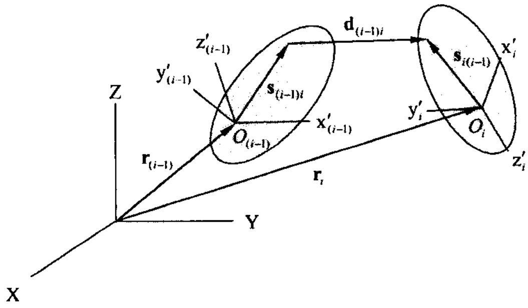

Figure 2. Kinematic relationships between two adjacent rigid bodies

the vector product operation and $\mathbf{q}'_{(i-1)i}$ denotes the relative coordinate vector.

Combining Eqs. (7) and (9) yields the recursive virtual displacement equation for a pair of contiguous bodies

$$\delta\mathbf{Z}_i' = \mathbf{B}_{(i-1)i1}\ \delta\mathbf{Z}'_{(i-1)} + \mathbf{B}_{(i-1)i2}\ \delta\mathbf{q}_{(i-1)i} \tag{10}$$

where

$$\mathbf{B}_{(i-1)i1} = \begin{bmatrix} \mathbf{A}^T_{(i-1)i} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^T_{(i-1)i} \end{bmatrix} \begin{bmatrix} \mathbf{I} & -(\widetilde{\mathbf{s}}'_{(i-1)i} + \widetilde{\mathbf{d}}'_{(i-1)i} - \mathbf{A}_{(i-1)i}\widetilde{\mathbf{s}}'_{i(i-1)}\mathbf{A}^T{}_{(i-1)i}) \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \tag{11}$$

$$\mathbf{B}_{(i-1)i2} = \begin{bmatrix} \mathbf{A}^T_{(i-1)i} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^T_{(i-1)i} \end{bmatrix} \begin{bmatrix} (\mathbf{d}'_{(i-1)i})_{\mathbf{q}_{(i-1)i}} + \mathbf{A}_{(i-1)i}\widetilde{\mathbf{s}}'_{i(i-1)}\mathbf{A}^T{}_{(i-1)i}\ \mathbf{H}'_{(i-1)i} \\ \mathbf{H}'_{(i-1)i} \end{bmatrix} \tag{12}$$

It is important to note that matrices $\mathbf{B}'_{(i-1)i1}$ and $\mathbf{B}'_{(i-1)i2}$ are functions of only relative coordinates of the joint between bodies (i-1) and i. As a consequence, further differentiation of the matrices $\mathbf{B}'_{(i-1)i1}$ and $\mathbf{B}'_{(i-1)i2}$ in Eqs. (11) and (12) with respect to other than $\mathbf{q}'_{(i-1)i}$ yields zero. The virtual displacement relationship between the absolute and relative coordinates for the whole system can be obtained by repetitive application of Eq. (10) as

$$\delta\mathbf{Z}' = \mathbf{B}\delta\mathbf{q} \tag{13}$$

where $\mathbf{B}$ is the velocity transformation matrix with relationship between Cartesian and relative coordinates. The relationship between Cartesian velocity $\mathbf{Y}'$ and relative velocity $\dot{\mathbf{q}}$ can be derived in the same manner.

$$\mathbf{Y}' = \mathbf{B}\dot{\mathbf{q}} \tag{14}$$

3. Equations of Motion

The variational form of the Newton-Euler equations of motion for a constrained multibody system is

$$\delta\mathbf{Z}'^T(\mathbf{M}\dot{\mathbf{Y}}' + \mathbf{\Phi}^T_{Z'}\lambda - \mathbf{Q}) = \mathbf{0} \tag{15}$$

where $\mathbf{M}$ and $\mathbf{Q}$ are the mass matrix and general force vector in Cartesian space, respectively. $\delta\mathbf{Z}'$ must be kinematically admissible for all joints except cut joints [12]. In the equation, $\mathbf{\Phi}$ and λ, respectively, denote the constraint equations and the corresponding Lagrange multiplier in R^m in which m is the number of the constraint equations. Substituting the virtual displacement relationship and acceleration relationships $\dot{\mathbf{Y}} = \mathbf{B}\ddot{\mathbf{q}} + \dot{\mathbf{B}}\dot{\mathbf{q}}$ into Eq. (15) yields (see Ref. [5])

$$\mathbf{F} = \mathbf{M}^*\ddot{\mathbf{q}} + \mathbf{\Phi}^T_{\mathbf{q}}\lambda - \mathbf{Q}^* = \mathbf{0} \quad \mathbf{F} \in R^n \tag{16}$$

where n is the number of generalized coordinates and the mass matrix $\mathbf{M}^*$ and force vector $\mathbf{Q}^*$ are defined as

$$\mathbf{M}^* = \mathbf{B}^T\mathbf{M}\mathbf{B} \tag{17}$$

$$\mathbf{Q}^* = \mathbf{B}^T(\mathbf{Q} - \mathbf{M}\dot{\mathbf{B}}\dot{\mathbf{q}}) \tag{18}$$

A recursive method has been proposed to compute Eqs. (17) and (18) in Ref. [7].

4. Linearization of the Equations of Motion

The relative coordinates $\mathbf{q}$ in R^n can be partitioned into dependent coordinates $\mathbf{q}_D$ in R^m and independent coordinates $\mathbf{q}_I$ in R^{n-m} such that the sub-Jacobian $\mathbf{\Phi}_{\mathbf{q}D}$ is well conditioned. The index of n and m are the numbers of the generalized coordinates and dependent coordinates, respectively. Variational form of the cut constraint equations can be written as

$$\delta\mathbf{\Phi} = \mathbf{\Phi}_{\mathbf{q}_D}\delta\mathbf{q}_D + \mathbf{\Phi}_{\mathbf{q}_I}\delta\mathbf{q}_I = \mathbf{0} \tag{19}$$

The $\delta\mathbf{q}_D$ can be obtained from Eq. (19) as

$$\delta\mathbf{q}_D = -\mathbf{\Phi}^{-1}_{\mathbf{q}_D}\mathbf{\Phi}_{\mathbf{q}_I}\delta\mathbf{q}_I \tag{20}$$

By using the relationship in Eq. (20), $\delta\mathbf{q}_D = -\mathbf{\Phi}^{-1}_{\mathbf{q}_D}\mathbf{\Phi}_{\mathbf{q}_I}\delta\mathbf{q}_I$ is represented as

$$\delta\mathbf{q} = \mathbf{N}\delta\mathbf{q}_I \tag{21}$$

where

$$\mathbf{N} = \begin{bmatrix} -\mathbf{\Phi}^{-1}_{\mathbf{q}_D}\mathbf{\Phi}_{\mathbf{q}_I} \\ \mathbf{I} \end{bmatrix}_{n\times(n-m)} \tag{22}$$

Direct calculation of $\mathbf{N}^T\mathbf{\Phi}^T_{\mathbf{q}}$ shows that $\mathbf{N}$ is the null

$\mathbf{\Phi}_q$ space of in $R^{m\times n}$ as

$$\mathbf{N}^T\mathbf{\Phi}_q^T = \left[-\mathbf{\Phi}_{q_I}^T(\mathbf{\Phi}_{q_D}^{-1})^T \quad \mathbf{I}\right]\begin{bmatrix}\mathbf{\Phi}_{q_D}^T \\ \mathbf{\Phi}_{q_I}^T\end{bmatrix} = \mathbf{0} \tag{23}$$

As a result, pre-multiplication of Eq. (16) by $\mathbf{N}^T$ gives

$$\mathbf{F}^* = \mathbf{N}^T\mathbf{M}^*\ddot{\mathbf{q}} - \mathbf{N}^T\mathbf{Q}^* = \mathbf{0} \tag{24}$$

where Lagrange multiplier λ in R^m can be eliminated since $\mathbf{N}$ is the null space of $\mathbf{\Phi}_q$. However, the equations of motion $\mathbf{F}^*$ in $\mathbf{R}^{n-m}$ are dependent on not only the dependent variables $\mathbf{q}_D$, $\dot{\mathbf{q}}_D$ and $\ddot{\mathbf{q}}_D$ but also independent variables $\mathbf{q}_I$, $\dot{\mathbf{q}}_I$ and $\ddot{\mathbf{q}}_I$. Taking variation of Eq. (24) yields

$$\delta\mathbf{F}^* = \mathbf{F}_q^*\delta\mathbf{q} + \mathbf{F}_{\dot{q}}^*\delta\dot{\mathbf{q}} + \mathbf{F}_{\ddot{q}}^*\delta\ddot{\mathbf{q}} = \mathbf{0} \tag{25}$$

Equation (25) can be rewritten in a matrix form as

$$\left[\mathbf{F}_q^* \quad \mathbf{F}_{\dot{q}}^* \quad \mathbf{F}_{\ddot{q}}^*\right]_{(n-m)\times 3n}\begin{Bmatrix}\delta\mathbf{q} \\ \delta\dot{\mathbf{q}} \\ \delta\ddot{\mathbf{q}}\end{Bmatrix}_{3n\times 1} = \mathbf{0} \tag{26}$$

Variations of position, velocity and acceleration level constraints are

$$\begin{aligned}&\mathbf{\Phi}_q\delta\mathbf{q} = \mathbf{0} \\ &\dot{\mathbf{\Phi}}_q\delta\mathbf{q} + \mathbf{\Phi}_q\delta\dot{\mathbf{q}} = \mathbf{0} \\ &\ddot{\mathbf{\Phi}}_q\delta\mathbf{q} + 2\dot{\mathbf{\Phi}}_q\delta\dot{\mathbf{q}} + \mathbf{\Phi}_q\delta\ddot{\mathbf{q}} = \mathbf{0}\end{aligned} \tag{27}$$

Appending the trivial identity relationships for the variations of independent coordinates, velocities and accelerations to Eq. (19) yields

$$\begin{bmatrix}\mathbf{\Phi}_q & \mathbf{0} & \mathbf{0} \\ \dot{\mathbf{\Phi}}_q & \mathbf{\Phi}_q & \mathbf{0} \\ \ddot{\mathbf{\Phi}}_q & 2\dot{\mathbf{\Phi}}_q & \mathbf{\Phi}_q \\ & \mathbf{C} & \end{bmatrix}_{3n\times 3n}\begin{Bmatrix}\delta\mathbf{q} \\ \delta\dot{\mathbf{q}} \\ \delta\ddot{\mathbf{q}}\end{Bmatrix}_{3n\times 1} = \begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ & \mathbf{I} & \end{bmatrix}_{3n\times 3(n-m)}\begin{Bmatrix}\delta\mathbf{q}_I \\ \delta\dot{\mathbf{q}}_I \\ \delta\ddot{\mathbf{q}}_I\end{Bmatrix}_{3(n-m)\times 1} \tag{28}$$

where $\mathbf{C}$ matrix in $\mathbf{R}^{3(n-m)\times 3n}$ is the coefficient matrix which has zero in the column of a dependent coordinate or one in the column of independent coordinate and $\mathbf{I}$ matrix is the identity matrix in $\mathbf{R}^{3(n-m)\times 3(n-m)}$.

Equation (28) is solved for the $\{\delta\mathbf{q} \quad \delta\dot{\mathbf{q}} \quad \delta\ddot{\mathbf{q}}\}^T$ and substituted into the linearized equations of motion in Eq. (26) to yield the following linearized equations of motion only in terms of the variations of independent coordinates, velocities and accelerations:

$$\left[\mathbf{F}_q^* \quad \mathbf{F}_{\dot{q}}^* \quad \mathbf{F}_{\ddot{q}}^*\right]\begin{bmatrix}\mathbf{\Phi}_q & \mathbf{0} & \mathbf{0} \\ \dot{\mathbf{\Phi}}_q & \mathbf{\Phi}_q & \mathbf{0} \\ \ddot{\mathbf{\Phi}}_q & 2\dot{\mathbf{\Phi}}_q & \mathbf{\Phi}_q \\ & \mathbf{C} & \end{bmatrix}^{-1}\begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ & \mathbf{I} & \end{bmatrix}\begin{Bmatrix}\delta\mathbf{q}_I \\ \delta\dot{\mathbf{q}}_I \\ \delta\ddot{\mathbf{q}}_I\end{Bmatrix} = \mathbf{0} \tag{29}$$

Direct comparison of Eq. (29) and the following linearized equations of motion yields the $\hat{\mathbf{M}}$, $\hat{\mathbf{C}}$ and $\hat{\mathbf{K}}$ matrices in $\mathbf{R}^{3(n-m)\times 3(n-m)}$:

$$\delta\mathbf{F}^*\big|_{q^*} = \hat{\mathbf{M}}\delta\ddot{\mathbf{q}}_I + \hat{\mathbf{C}}\delta\dot{\mathbf{q}}_I + \hat{\mathbf{K}}\delta\mathbf{q}_I = \mathbf{0} \tag{30}$$

5. Eigenvalue Problem

In Chapter 4, the equation of motion was linearized about the independent variables $\mathbf{x}_I^* = \{\mathbf{q}_I^* \quad \dot{\mathbf{q}}_I^* \quad \ddot{\mathbf{q}}_I^*\}^T$ at any time as following

$$\delta\mathbf{F}^*\big|_{x_I^*} = \hat{\mathbf{M}}\delta\ddot{\mathbf{q}}_I + \hat{\mathbf{C}}\delta\dot{\mathbf{q}}_I + \hat{\mathbf{K}}\delta\mathbf{q}_I = \mathbf{0} \tag{31}$$

where $\hat{\mathbf{M}}$, $\hat{\mathbf{C}}$ and $\hat{\mathbf{K}}$ matrices are mass, damping and stiffness matrices of a multibody system, respectively.

Assume a solution of the homogeneous set of equation in the form

$$\begin{aligned}\delta\mathbf{q}_I &= [\psi]e^{wt} \\ \delta\dot{\mathbf{q}}_I &= \omega[\psi]e^{wt} \\ \delta\ddot{\mathbf{q}}_I &= \omega^2[\psi]e^{wt}\end{aligned} \tag{32}$$

Substituting for Eq. (32) into Eq. (31),

$$(\omega^2\hat{\mathbf{M}} + \omega\hat{\mathbf{C}} + \hat{\mathbf{K}})[\psi] = \mathbf{0} \tag{33}$$

Equation (33) has a nontrivial solution only if the determinant of the coefficients is zero

$$\left|\omega^2\hat{\mathbf{M}} + \omega\hat{\mathbf{C}} + \hat{\mathbf{K}}\right| = \mathbf{0} \tag{34}$$

Equation (34) is the characteristic equation that is an algebraic equation of in order 2n. The roots can be real, purely imaginary, or complex. If the roots are real they must be negative, which corresponds to an overdamped system for which a periodic decaying motion is obtained. If the roots are complex they must appear in pairs of complex conjugates with negative real part. The corresponding modal columns [Ψ] must also be complex conjugates. A pair of complex conjugate modes multiplied by the corresponding time-dependent exponential functions can be combined to obtain a damped oscillatory motion. This is the case in which the system is underdamped. For undamped systems, one obtains purely imaginary roots.

6. Numerical Examples

6.1 A fourbar mechanism with a spring

Figure 3 shows a four bar mechanism with a spring. The system consists of four revolute joints and one spring and their material properties are defined in Table 1. As a result, three generalized coordinates, θ_1, θ_2 and θ_3 are defined for the first three revolute joints and the remaining one revolute joint is defined as a cut joint. The constraint equations are introduced from the cut joint.

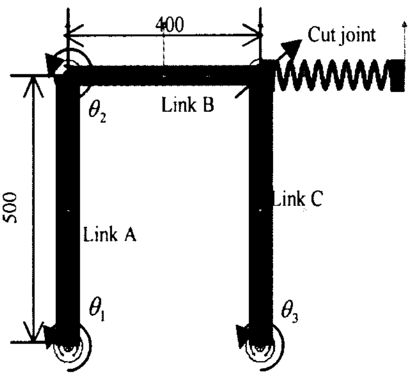

Figure 3. A fourbar mechanism with a spring

Table 1 Material property of bodies and a spring

		Mass (kg)	Inertia Moment (kg*mm^2)
Body	Link A	7.707	161760.83
	Link B	3.946	53005.79
	Link C	7.707	161760.83
Spring	Stiffness (N/mm)		Damping (N*sec/mm)
	10.0		0

Dynamic analysis of the mechanism is performed to obtain the time domain response. FFT of the time response is performed to extract dominant frequency domain response. Figures 4 and 5 show the time and frequency responses, respectively.

The proposed linearization method is applied for the system. The dominant frequency and corresponding normalized mode shape are shown in Table 2. The frequency obtained from the proposed method and that obtained from FFT analysis of the time domain responses are shown to be very close, which validates the proposed method.

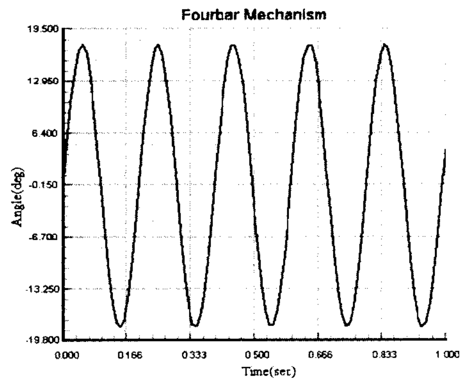

Figure 4. Angle of link C in time domain

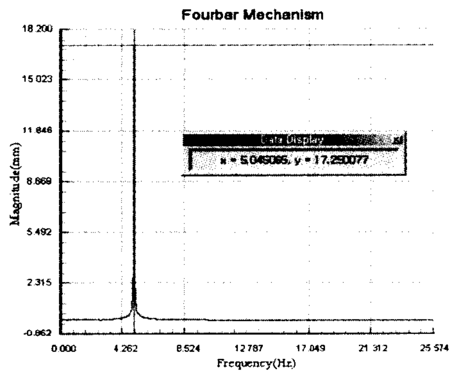

Figure 5. Response in frequency domain

Table 2 Undamped natural frequency and mode shape from the proposed method

Undamped Natural Frequency (Hz)	Mode		
	θ_1	θ_2	θ_3
5.04E+00	5.773503E-01	-5.773503E-01	5.773503E-01

6.2 Cantilever beam driven by a motion

The system characteristics of a rotating cantilever beam differ from those of beam in a static state, because the stiffness of the beam is changed by a centrifugal force due to the rotational motion. (see Ref. [13]). A cantilever beam rotating with the angular velocity **w** is shown in Figure 6.

Length of the beam is 6.8 m, density of the material is 14705.88 kg/m^3, Young's modulus of the material is 7.0×10^8 N/m^2. Area of the cross section is 0.002 m^2, the moment of inertia 4.0×10^{-7} m^4. The beam is divided into 21 lumped mass and 20 beam elements in Ref.[14]. Figure 7 shows the lowest three natural frequencies of the rotating beam. To change the angular velocity of rotor, a driving constraint is applied as follows.

$$^{D}\Phi(\mathbf{q},t)=\theta-50t^2=0$$

As the angular speed increases, the bending natural frequencies are shown to be increased.

Figure 6. A rotating cantilever beam

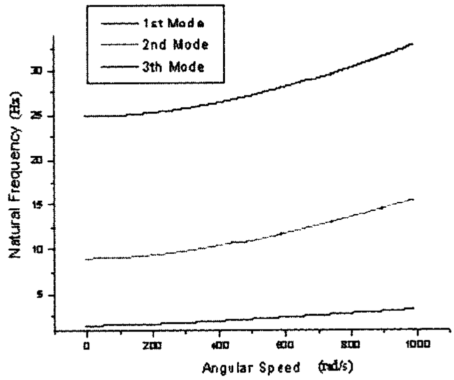

Figure 7. The relationship between angular velocity and natural frequencies

7. Conclusions

In this paper, a linearization method for constrained multibody systems is proposed for the non-linear equations of motion employing the relative coordinates. Null space of the constraint Jacobian is pre-multiplied to the equations of motion to eliminate the Lagrange multipliers and to reduce the number of equations. The set of differential equations are perturbed in terms of all relative positions, velocities and accelerations. The position, velocity and acceleration level constraints are perturbed to express the variations of all relative positions, velocities and accelerations in terms of the variations of independent positions, velocities and accelerations, which are substituted into the perturbed equations of motion. The equations of motion perturbed with respect to the $\mathbf{q}$, $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ finally become the corresponding equations perturbed with respect to the $\mathbf{q}_1$, $\dot{\mathbf{q}}_1$ and $\ddot{\mathbf{q}}_1$. Eigenvalues and eigenvectors are then computed from the equations of motion perturbed with respect to the $\mathbf{q}_1$, $\dot{\mathbf{q}}_1$ and $\ddot{\mathbf{q}}_1$. The proposed method is implemented in a commercial program RecurDyn. Numerical results obtained from the proposed method are in good agreement with the results reported in the literature and obtained by other methods.

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