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Empirical Comparisons of Disparity Measures for Partial Association Models in Three Dimensional Contingency Tables

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Abstract

This work is concerned with comparison of the recently developed disparity measures for the partial association model in three dimensional categorical data. Data are generated by using simulation on each term in the log-linear model equation based on the partial association model, which is a proposed method in this paper. This alternative Monte Carlo methods are explored to study the behavior of disparity measures such as the power divergence statistic $I(\lambda)$, the Pearson chi-square statistic X^2, the likelihood ratio statistic G^2, the blended weight chi-square statistic $BWCS(\lambda)$, the blended weight Hellinger distance statistic $BWHD(\lambda)$, and the negative exponential disparity statistic $NED(\lambda)$ for moderate sample sizes. We find that the power divergence statistic $I(2/3)$ and the blended weight Hellinger distance family $BWHD(1/9)$ are the best tests with respect to size and power.

Keywords : Disparity measure, Goodness-of-fit, Hellinger distance, Log-linear model, Power divergence statistic.

1. Introduction

Three-dimensional $I\times J\times K$ contingency tables are considered with observed cell frequencies $\{x_{ijk}\}$ corresponding to the probabilities $\{\pi_{ijk}\}$ of the multinomial distribution. The saturated log-linear model (say, [123]) of the expected frequencies $\{m_{ijk}\}$ is

$$\ln m_{ijk} = u + u_{1(i)} + u_{2(j)} + u_{3(k)} + u_{12(ij)} + u_{13(ik)} + u_{23(jk)} + u_{123(ijk)} \quad (1)$$

with the usual restrictions on parameters u's (see Bishop et al., 1975).

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The concept of three-factor (or second-order) interaction was first introduced by Bartlett (1935) in $2\times2\times2$ contingency table. Goodman (1964) proposed several methods for analyzing the three-factor interaction in a three dimensional $I\times J\times K$ table (also see Bhapker and Koch (1968), Grizzle, Starmer and Koch (1969)). Larntz (1978) and Haber (1984) compared several tests of no three-factor interaction for only $3\times3\times3$ and $2\times2\times2$ contingency tables respectively.

The null hypothesis of no three-factor interaction is

$$\begin{aligned} H_0 &: \ u_{123(ijk)}=0 \\ &\text{or} \\ H_0 &: \ \ln m_{ijk}=u+u_{1(i)}+u_{2(j)}+u_{3(k)}+u_{12(ij)}+u_{13(ik)}+u_{23(jk)} \end{aligned} \tag{2}$$

for $i=1,\ldots,I-1$, $j=1,\ldots,J-1$ and $k=1,\ldots,K-1$, which is called as the partial association model (say, [12][13][23]). The constraint equations specification of H_0 is known to be (see, e.g., Roy and Kastenbaum (1956))

$$\begin{aligned} H_0 &: \ \ln\pi_{ijk}-\ln\pi_{Ijk}-\ln\pi_{iJk}-\ln\pi_{ijK}+\ln\pi_{iJK}+\ln\pi_{IJk}+\ln\pi_{IjK}-\ln\pi_{IJK}=0 \\ &\text{or} \\ H_0 &: \ \frac{\pi_{ijk}\pi_{IJk}}{\pi_{Ijk}\pi_{iJk}}=\frac{\pi_{ijK}\pi_{IJK}}{\pi_{IjK}\pi_{iJK}} \end{aligned} \tag{3}$$

for $i=1,\ldots,I-1$, $j=1,\ldots,J-1$ and $k=1,\ldots,K-1$.

Several chi-square statistics are available in the literature for testing the above hypothesis H_0 in $I\times J\times K$ contingency table. The Pearson's chi-square and log likelihood ratio statistics have long been used to perform tests of hypotheses about the parameters of a multinomial distribution.

Cressie and Read (1984), and Read and Cressie (1988) developed a class of goodness-of-fit test statistics called the family of the power divergence statistics denoted by $\{I(\lambda);\lambda\in \boldsymbol{R}\}$. They presented an analytical discussion of the asymptotic differences between different $I(\lambda)$ tests with a considerable amount of numerical finite sample results. Cressie and Read (1984) showed that the statistic $I(2/3)$ is an excellent and compromising alternative to any other goodness-of-fit test statistics based on a multimonial distribution.

An even more general class of goodness-of-fit test statistics of which the family of the power divergence statistics is a subclass was studied. These test statistics, which is called the disparity test statistics, have been derived following the minimum disparity estimation approach of Lindsay (1993, 1994). And Lindsay (1994) and Basu and Sarkar (1994) introduced two other subfamilies of disparity tests, which are named the blended weight chi-squared family $BWCS(\lambda)$ and the blended weight Hellinger distance family $BWHD(\lambda)$. Basu and Sarkar (1994) derived the asymptotic chi-square distribution of the disparity tests, and showed that the blended weight Hellinger distance family, like the power divergence statistics, is excellent compromise to any other goodness-of-fit statistics when $BWHD(1/9)$ as well as $BWCS(1/3)$ under a multimonial distribution. Another subfamily of disparity tests is investigated by Jeong and Sarkar (2000). This statistic is the negative exponential disparity test $NED(\lambda)$, whose family includes the Pearson's chi-square as a member. When $\lambda=4/3$, the negative exponential disparity test might be preferred to the power divergence statistics $I(2/3)$ based on a multimonial model.

These disparity test statistics including the Pearson chi-square statistics X^2, the log likelihood ratio statistics G^2, the power divergence statistics $I(2/3)$, the blended weight chi-squared statistics $BWCS(1/3)$, the blended weight Hellinger distance statistics $BWHD(1/9)$, and the negative exponential disparity statistics $NED(4/3)$ are introduced in Section 2. Many authors have compared these goodness-of-fit test statistics. See, for example, Cochran (1952), Hoeffding (1965), West and Kempthorne (1972), Chapman (1976), Lantz (1978), Koehler and Lantz (1980), and Moore and Spruill (1975).

Lots of literatures mentioned above have been studied for multinomial models in order to investigate behaviors of these goodness-of-fit test statistics. In this paper, $3\times3\times3$ categorical data are generated by using simulation on each term in the log-linear model equation based on partial association model. Numerical results for disparity statistics are compared and discussed in Section 3. Finally the conclusions are given in Section 4.

2. Disparity Measures

Suppose the sample space is a countable set, without any losses of generality $\boldsymbol{X}=\{0,1,\ldots,K\}$, with K possibly infinite, and that $m_\beta(x)$ is a family of probability densities on $\boldsymbol{X}$, indexed by $\beta\in\Omega$. To avoid technicalities, it will be assumed that $m_\beta(x)>0$ for all $x\in\boldsymbol{X}$. Moreover, suppose that n independent and identically distributed observations $X_1,\ldots,X_n$ are made from $m_\beta(x)$. Let $d(x)$ be the proportion of the n observations which had value x. Define the Pearson residual function $\delta(x)$ to be

$$\delta(x)=[d(x)-m_\beta(x)]/m_\beta(x) \ .$$

Note that the model-weighted sum of the squared residuals, $\sum m_\beta(x_i)\delta(x_i)^2$, is Pearson's chi-squared distance. And it is important to note that these residuals are not standardized to have identical variances, so that these residuals have range $[-1,\infty]$.

Suppose that $G(\cdot)$ is a real-valued thrice-differentiable function on $[-1,\infty)$, with $G(0)=0$. Lindsay (1994) defines the disparity measure determined by G to be

$$\rho(\mathbf{d},\mathbf{m}_\beta)=\sum m_\beta(x_i)\,G(\delta(x_i)) \ . \tag{4}$$

If G is assumed to be strictly convex, then Csiszar (1963) shows that the disparity measure is nonnegative, and is zero only when $\mathbf{d} = \mathbf{m}_\beta$ by Jensen's inequality. An important class of such measures is the Cressie-Read [Cressie and Read (1984), Read and Cressie (1988)] family of power divergence measures, defined by

$$I(\lambda) = \sum d(x_i) \frac{\{[d(x_i)/m_\beta(x_i)]^\lambda - 1\}}{\lambda(\lambda+1)}$$
$$= \sum m_\beta(x_i) \frac{\{(1+\delta(x_i))^{\lambda+1} - 1\}}{\lambda(\lambda+1)} .$$

For $\lambda = -2, -1, -0.5, 0,$ and 1, one obtains the well-known measures: Neyman chi-squared measure divided by 2, Kullback-Leibler divergence measure, the twice-squared Hellinger measure, the likelihood disparity, Pearson chi-squared measure divided by 2, respectively. Cressie and Read (1994) suggest that a member of this class, $I(2/3)$, could be used as a good alternative to any other measures.

For λ any fixed number in $[0,1]$ and $\bar{\lambda} = 1-\lambda$, Lindsay (1994) introduces two modified distance measures. One is the blended weight chi-squared disparity which is defined as

$$BWCS(\lambda) = \sum \frac{[d(x_i) - m_\beta(x_i)]^2}{2[\lambda d(x_i) + \bar{\lambda} m_\beta(x_i)]} .$$

Here Pearson chi-squared measure corresponds to $\lambda = 0$ and Neyman's corresponds to $\lambda = 1$. Le Cam (1986, page 47) considered the case $\lambda = 0.5$, showing that it is squared distance satisfying the triangle inequality. The other weighting scheme that generalizes Hellinger distance is the blended weight Hellinger distance measure such as

$$BWHD(\lambda) = \sum \frac{[d(x_i) - m_\beta(x_i)]^2}{2[\lambda\sqrt{d(x_i)} + \bar{\lambda}\sqrt{m_\beta(x_i)}]^2} .$$

This family includes Neyman chi-squared measure, Pearson chi-squared measure, and Hellinger distance for $\lambda = 1$, 0, and 0.5, respectively. Jeong and Sarkar (2000) propose the generalized negative exponential disparity family as the following:

$$NED(\lambda) = \sum \frac{\exp\left[-\lambda\left(\frac{d(x_i)}{m_\beta(x_i)} - 1\right)\right] - 1 + \lambda\left(\frac{d(x_i)}{m_\beta(x_i)} - 1\right)}{\lambda^2} m_\beta(x_i) .$$

This family includes Pearson chi-squared measure and the negative exponential disparity introduced by Lindsay (1994) for $\lambda = 0$ and 1, respectively. Basu and Sarkar (1994) show that the blended weighted Hellinger distance statistic and the blended weighted chi-square statistic are excellent compromise to any other goodness-of-fit statistics when $BWHD(1/9)$ and $BWCS(1/3)$. Jeong and Sarkar (2000) derive that the negative exponential disparity statistic, $NED(\lambda)$, might be preferred to the power divergence statistics $I(2/3)$ when $\lambda=4/3$.

For a sequence of n observations on a multinomial distribution with probability vector $\boldsymbol{\pi}=(\pi_1,\ldots,\pi_k)$ and $\sum_{i=1}^{k}\pi_i=1$, let $\rho_G(\mathbf{d},\boldsymbol{\pi})$ be a disparity measure defined in (4). Then consider $D_{\rho G}=2n\rho_G(\mathbf{d},\boldsymbol{\pi})$ as a test statistic for the simple null hypothesis $H_0: \boldsymbol{\pi}=\boldsymbol{\pi}_0$, $\pi_{i0}>0$ for all i. Basu and Sarkar (1994) show that the disparity test statistic $D_{\rho G}$ has an asymptotic chi-squared distribution with $k-1$ degrees of freedom, $\chi^2_{(k-1)}$, under the null hypothesis.

3. Numerical investigation

Many authors including Cochran (1952), Hoeffding (1965), West and Kempthorne (1972), Chapman (1976), Lantz (1978), Koehler and Lantz (1980), and Moore and Spruill (1975) worked to compare behaviors of a lot of goodness-of-fit test statistics for multinomial models. In this paper $3\times3\times3$ contingency tables are considered. These categorical data are generated by using the log-linear model. Values of eighteen u terms $\{u_{1(i)}, u_{2(j)}, u_{3(k)}, u_{12(ij)}, u_{13(ik)}, u_{23(jk)};\ i,j,k=1,2\}$ in the partial association model in (1) are simulated from the uniform distribution having appropriate intervals. The rest u terms are obtained satisfying usual restriction on u terms such as $u_{1(3)}=-(u_{1(1)}+u_{1(2)})$, $u_{2(3)}=-(u_{2(1)}+u_{2(2)})$, ..., $u_{23(13)}=-(u_{23(11)}+u_{23(12)})$, ... and for each $i,j,k\ (=1,2,3)$, cell counts $x_{ijk}(\geq 1)$ satisfying H_0 are calculated by the following equation

$$x_{ijk}=N\frac{\exp(u_{1(i)}+u_{2(j)}+u_{3(k)}+u_{12(ij)}+u_{13(ik)}+u_{23(jk)})}{\sum_{i,j,k}\exp(u_{1(i)}+u_{2(j)}+u_{3(k)}+u_{12(ij)}+u_{13(ik)}+u_{23(jk)})},$$

where N is a sample size. Fienberg (1979) suggested that the sample size might be as small as 4 or 5 times the total number of cells of the table, so that cell counts x_{ijk} are obtained with sample sizes 50, 70, 100 and 200. Then estimates of expected counts $\{\hat{m}_{ijk}\}$ are obtained using iterative proportional fitting method (see Bishop, Fienberg, and Holland, 1975 for more detail). Note that $d(x_{ijk})=x_{ijk}/N$ and $m_\beta(x_{ijk})=\hat{m}_{ijk}/N$ in (4).

Independent random samples were obtained 1,000 times for the partial association log-linear model. We explore behaviors of disparity test statistics discussed in Section 2. For each simulation, disparity statistics $I(2/3)$, X^2, G^2, $BWCS(1/3)$, $BWHD(1/9)$ and $NED(4/3)$

computed for testing H_0. The empirical levels of significance attained, viz. $\hat{\alpha}$, were computed as the proportion of times the values of test statistics exceed the asymptotic critical value $\chi^2_{v,\alpha}$ for the nominal values $\alpha=0.05$, 0.01 with degrees of freedom $v=(I-1)(J-1)(K-1)=8$. The values of $\hat{\alpha}$ are given in [Table I and II] for the nominal values $\alpha=0.05$, and 0.01, respectively, satisfying the partial association model.

【Table I】 Empirical levels for $I(2/3)$, X^2, G^2, $BWCS(1/3)$, $BWHD(1/9)$ and $NED(4/3)$ statistics at $\alpha=0.05$ for testing [12][13][23].

Statistics	Sample size (*N*)			
	50	70	100	200
$I(2/3)$	22	52	56	48
X^2	49	101	117	67
G^2	3	9	20	38
$BWCS(1/3)$	2	2	13	32
$BWHD(1/9)$	17	47	49	46
$NED(4/3)$	0	1	11	27

【Table II】 Empirical levels for $I(2/3)$, X^2, G^2, $BWCS(1/3)$, $BWHD(1/9)$ and $NED(4/3)$ statistics at $\alpha=0.01$ for testing [12][13][23].

Statistics	Sample size (*N*)			
	50	70	100	200
$I(2/3)$	6	8	10	10
X^2	22	44	30	13
G^2	0	1	3	11
$BWCS(1/3)$	0	1	1	8
$BWHD(1/9)$	2	8	9	10
$NED(4/3)$	0	1	1	8

In this work all theses statistics perform more or less the same for large samples. For moderate samples, the statistics $I(2/3)$ and $BWHD(1/9)$ attain quite close levels to the nominal values $\alpha=0.05$ and 0.01, whereas X^2, G^2, $BWCS(1/3)$ and $NED(4/3)$ statistics do not perform well. Among them the statistic X^2 rejects much more often than expected. The statistics G^2, $BWCS(1/3)$ and $NED(4/3)$ seem to work very poorly compared to any other statistics considered in our study.

We also study the power of the disparity test statistics. Since the partial association log-linear model ([12][13][23]) nests the conditional independent log-linear model (say, [12][13] or [12][23] or [13][23]), which nests the collapsible log-linear model (say, [12][3] or [13][2] or [1][23]), it notes that one regards the null hypotheses with a conditional independent log-linear model ([12][13]) and a collapsible log-linear model ([12][3]), respectively, as follows :

$$[12][13] : \ln m_{ijk} = u + u_{1(i)} + u_{2(j)} + u_{3(k)} + u_{12(ij)} + u_{13(ik)}$$
$$[12][3] : \ln m_{ijk} = u + u_{1(i)} + u_{2(j)} + u_{3(k)} + u_{12(ij)}$$

The empirical powers of the disparity statistics which simulate for samples of size 50, 70, 100 and 200 to test the [12][13] model and the [12][3] model against the [12][13][23] model are listed in [Table Ⅲ] and [Table Ⅳ], respectively. These tables indicate that for the nominal values $\alpha=0.05, 0.01$ with degrees of freedom $I(J-1)(K-1)=12$ and $(IJ-1)(K-1)=16$, respectively, the empirical powers which are the values of test statistics exceed the asymptotic critical value $\chi^2_{v,\alpha}$ for the nominal values.

From [Table Ⅲ and Ⅳ], the powers of all theses statistics perform more or less the same for large samples. For moderate samples, the statistics $I(2/3)$ and $BWHD(1/9)$ attain similar powers. We might say that whereas the statistic X^2 is over-powered, the statistic G^2, $BWCS(1/3)$ and $NED(4/3)$ is under-powered than the statistics $I(2/3)$ and $BWHD(1/9)$. These investigations are consistent with the conclusions induced from [Table Ⅰ and Ⅱ]. One also found that the powers in [Table Ⅳ] are more likely than those of [Table Ⅲ], since the null model ([12][3]) for [Table Ⅳ] is nested by the null model ([12][13]) for [Table Ⅲ]. Since the completely independent model ([1][2][3]) is nested by [12][3], the empirical powers of the disparity statistics to test [1][2][3] against [12][13][23] have much larger than those of [Table Ⅳ]. Therefore the empirical powers of the disparity statistics to test [1][2][3] are not described in this paper.

4. Conclusion

We investigate in this work that the Cressie-Read's power divergence statistic $I(2/3)$ and the blended weight Hellinger distance family $BWHD(1/9)$ attain almost close levels that are very close to the nominal values and have similar powers. It is found that the statistic X^2 gets larger values than those empirical levels of $I(2/3)$ and $BWHD(1/9)$ and its powers are larger than those of $I(2/3)$ and $BWHD(1/9)$ whereas the statistics G^2, $BWCS(1/3)$ and $NED(4/3)$ obtain less levels and its powers are smaller than those of $I(2/3)$ and $BWHD(1/9)$. Based on these facts the Cressie-Read's power divergence statistic $I(2/3)$ and the blended weight Hellinger distance family $BWHD(1/9)$ are the best tests with respect to size and power for testing the partial association model.

【Table Ⅲ】 Empirical powers for $I(2/3)$, X^2, G^2, $BWCS(1/3)$, $BWHD(1/9)$ and $NED(4/3)$ statistics at $\alpha=0.05$, and 0.01 for testing [12][13] against [12][13][23].

Statistics	Sample size (N) 50	70	100	200
$I(2/3)$	233	403	559	792
	(103)	(254)	(377)	(659)
X^2	363	530	652	809
	(171)	(363)	(456)	(668)
G^2	143	273	478	780
	(87)	(170)	(317)	(655)
$BWCS(1/3)$	121	203	428	756
	(67)	(130)	(269)	(618)
$BWHD(1/9)$	227	393	552	792
	(100)	(246)	(375)	(659)
$NED(4/3)$	109	168	390	746
	(60)	(114)	(250)	(606)

*The lower figures in parentheses refer to $\alpha=0.01$

【Table Ⅳ】 Empirical powers for $I(2/3)$, X^2, G^2, $BWCS(1/3)$, $BWHD(1/9)$ and $NED(4/3)$ statistics at $\alpha=0.05$, and 0.01 for testing [12][3] against [12][13][23].

Statistics	Sample size (N) 50	70	100	200
$I(2/3)$	626	811	924	987
	(443)	(708)	(835)	(974)
X^2	716	856	939	988
	(527)	(763)	(864)	(975)
G^2	575	761	902	986
	(412)	(647)	(807)	(972)
$BWCS(1/3)$	475	690	867	984
	(328)	(548)	(752)	(968)
$BWHD(1/9)$	625	808	921	987
	(440)	(701)	(834)	(974)
$NED(4/3)$	439	648	834	980
	(312)	(497)	(719)	(967)

*The lower figures in parentheses refer to $\alpha=0.01$

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