A Study on Integral Equalities Related to a Laplace Transformable Function and its Applications

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Abstract: This paper establishes some integral equalities formulated by zeros located in the convergence region of a Laplace transformable function. Using the definition of the Laplace transform, it shows that Laplace transformable functions have to satisfy the integral equalities in the time-domain, which can be applied to the understanding of the fundamental limitations on the control system represented by the transfer function. In the unity-feedback control scheme, another integral equality is derived on the output response of the system with open-loop poles located in the convergence region of the output function. From these integral equalities, two sufficient conditions related to undershoot and overshoot phenomena in the step response, respectively, are investigated.

Keywords: Laplace transform, fundamental limitation, unity-feedback control, undershoot, overshoot.

1. INTRODUCTION

Linear transforms are well-known as providing techniques for solving problems in linear systems. In particular, the Laplace transform method is an operational method that can be used advantageously for solving linear differential equations. The Laplace transformation dates back to the work of the French mathematician, Pierre Simon Marquis de Laplace (1749-1827), who used it in his work on probability theory in the 1780s. It has played an important part in the theory of many branches of science and engineering [1]. In the field of control engineering, the Laplace transform theory is necessary for understanding the linear control theory since the systems used in the control engineering are usually represented by the transfer function which has been Laplace transform [1-3]. Moreover, the Laplace transform, together with the Fourier transform, has been applied to the formulation of the fundamental limitations on the transfer function, like the number of sign changes [4] or step response extrema [5-7].

Actually, there are always fundamental limitations involved with any feedback control system. Much research has been conducted to clarify these limitations imposed by the inherent characteristics of the physical system. Most is formulated in the frequency domain for linear SISO(Single-Input, Single-Output) systems [8-9] and is extended to MIMO(Multi-Input, Multi-Output) systems [10] as well as nonlinear systems [11] Other limitations have also been developed in the time-domain based on Laplace transform [4,6,9,12]. As a result, it has been realized that nonminimum phase systems [8-10,12] or systems with $j\omega$-axis zeros [15] compared with minimum phase systems, have more various fundamental limitations associated with the achievable closed-loop transfer function, closed-loop gain margin, loop transfer recovery, sensitivity or complementary sensitivity function, etc. In many cases, these limitations on the achievable performances are utilized to adjust trade-off relations between design specifications [7-10,12].

In this paper, we investigate some integral equalities about a Laplace transformable function, which can also be applied to understanding of the fundamental limitations on the control system represented by the transfer function. In the unity-feedback control scheme another integral equality is also derived on the output response of the system with open-loop poles located in the right area of the dominant pole of the output function. In particular, two examples show the explicit integral equalities which are satisfied by the impulse and the unit-step responses of the system. Based on these equalities, the performance limitations of the system, undershoot and overshoot phenomena, are established in the step response.

The layout of this paper is organized as follows: In

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Section 2, we present the mathematical preliminaries, notation, definitions, and previous results necessary for the understanding of the main results in this paper. In Section 3, the integral equalities in the time-domain for a class of Laplace transformable functions, which includes the physical systems with rational transfer function, are formulated. It is shown that the proposed results can be applied to the feedback control system in Section 4. A simple example is included in Sections 3 and 4. Two sufficient conditions related to undershoot and overshoot phenomena of the system, respectively, are shown in Section 5. The concluding remarks are given in Section 6.

2. PRELIMINARIES AND NOTATIONS

The intent of this section is to state the background necessary for the statement of the main results, which covers the definition of the Laplace integral and some properties of the Laplace transform utilized in this paper.

Let $f(t)$ be a function with the following properties:

(1) $f(t)=0$, for $-\infty < t < 0$.

(2) There exists a real number c such that $f(t)e^{-ct}$ is absolutely integrable over $-\infty < t < \infty$.

When $f(t)$ has the above properties, $f(t)$ is said to be Laplace transformable. If $f(t)$ is Laplace transformable, then the integral

$$F(s) \triangleq \mathcal{L}[f(t)] = \int_0^\infty e^{-st} f(t)dt \tag{1}$$

exists for a given value of s. The variable s is referred to as the Laplace operator, which is a complex variable. The integral defined in (1) is a well-known (one-sided) Laplace transform [1,4]. The functions $f(t)$ and $F(s)$ are a Laplace transform pair. Convergence properties of the integral (1) are examined in the following lemmas [1,4]:

Lemma 1: If the integral (1) converges for $s=\sigma_0+j\omega_0$, then it converges for all $s=\sigma+j\omega$ with $\sigma>\sigma_0$.

Lemma 2: The convergence region of the integral (1) is a half plane.

Lemma 1 and Lemma 2 allow the establishment of the convergence region of the Laplace transform.

The Laplace transformation has many useful properties, which are important to the control engineering [1],[2],[3]. In this paper, it used the complex shifting property of the Laplace transform as follows:

Lemma 3: The Laplace transform of $f(t)$ multiplied by $e^{-\alpha t}$, where α is a constant, is equal to $F(s)$ with s replaced by $s+\alpha$, *i.e*

$$\mathcal{L}\left[e^{-\alpha t} f(t)\right] = F(s+\alpha). \tag{2}$$

In addition, let us define undershoot and overshoot phenomena in the step response of the system provided that the DC gain of the system is positive[7]. Without loss of generality, let the step input be a positive value.

Definition 1: The step response is said to have undershoot if there is an open interval (a,b) such that

$$y_2(t)<0, \quad \forall t \in (a,b), \tag{3}$$

where y_2 is the step response of the system.

Definition 2: The step response $y_2(t)$ is said to have overshoot if there is an open interval (c,d) such that

$$y_2(t)<0, \quad \forall t \in (c,d), \tag{4}$$

where $y_2(t)$ is the step response of the system and K is the magnitude of the step input.

Note that Definition 1 includes Type A undershoot, *i.e.* the initial undershoot, as well as Type B undershoot [13], and Definition 2 is the overshoot for the system with DC gain smaller than or equal to the unity.

It is assumed in this paper that $f(t)$ is always Laplace transformable, and $F(s)$ is the proper, minimal function and has a real part of all poles less than $\gamma<0$. Note that the linear system is always Laplace transformable and can be represented by the rational transfer function.

3. INTEGRAL EQUALITIES

This section investigates some integral equalities on the time-domain representation of the function $f(t)$, which is imposed by the complex values located in the convergence region of $F(s)$.

Lemma 4: For $\mathrm{Re}[\chi]>\gamma$, the Laplace transform pair, $f(t)$ and $F(s)$, meets integral equalities as follows:

$$\int_0^\infty e^{-\alpha t}\cos(\omega t) f(t)dt = \mathrm{Re}[F(\chi)], \tag{5}$$

$$\int_0^\infty e^{-\alpha t}\sin(\omega t) f(t)dt = -\mathrm{Im}[F(\chi)], \tag{6}$$

where $s=\sigma+j\omega$.

Proof: From Lemma 3, the Laplace transform of $e^{-\chi t} f(t)$ can be written as

$$F(s+\chi)=\int_0^\infty e^{-st}e^{-\chi t}f(t)dt\lim_{x\to\infty} \\ =\int_0^\infty e^{-(s+\alpha)t}e^{-j\omega t}f(t)dt. \quad (7)$$

Since $F(s+\chi)$ is asymptotically stable for $\text{Re}[\chi]>\gamma$, it is clear that $F(s+\chi)$ has the closed RHP(right half plane) as its region of convergence. Hence, the value of $s=0$ is obviously in the convergence region of (7). Consequently, after evaluation (7) at $s=0$, the result follows by using the fact that

$$e^{-j\omega t}=\cos(\omega t)-j\sin(\omega t), \quad (8)$$

which completes the proof. □

Equations (5) and (6) in Lemma 4 have to always be satisfied by the function $f(t)$ for all complex value χ in the convergence region of $F(s)$, which has nothing to do with minimum or nonminimum phase functions. It is noted that the convergence region is the right area of the dominant pole. Lemma 4 implies some important results according to the particular values of χ. For example, if the value of χ is taken as 0 in Lemma 4, it can obtain the well-known result that the integral of $f(t)$ is equal to the DC gain of $F(s)$, *i.e.*

$$\int_0^\infty f(t)dt=\kappa, \quad (9)$$

where κ is the DC gain of $F(s)$.

Note that (6) in Lemma 4 is meaningless when the value of χ is taken as a real value since the right and left parts are identically all 0. If the function $f(t)$ is Laplace transformable, it has to satisfy the integral equalities such as (5), (6) and (9). Lemma 4 can be converted into the more simplified forms in the particular functions with zeros located in the convergence region of the Laplace transform.

Theorem 1: Let $F(s)$ have real zeros at $s=z_i$ for $i=1,2,\cdots,r_1$ and complex conjugate zeros at $s=a_k\pm jb_k$ for $k=1,2,\cdots,r_2$, which have all real parts larger than γ. Then $f(t)$ has to satisfy

$$\int_0^\infty[E_r(t)+\Gamma_c(t)+\Gamma_s(t)]f(t)dt=0, \quad (10)$$

where $E_r(t)$ is a linear combination of $e^{-z_i t}$, $\Gamma_c(t)$ is a linear combination of $e^{-a_k t}\cos(b_k t)$, and $\Gamma_s(t)$ is a linear combination of $e^{-a_k t}\sin(b_k t)$.

Proof: Since $F(s)$ has the right area of γ as its region of convergence, the real zeros z_i and the complex conjugate zeros $a_k\pm jb_k$ are located in the convergence region. Let us take $\chi=z_i$, and (5) in Lemma 4 can be rewritten as

$$\int_0^\infty E_r(t)f(t)dt=0 \quad (11)$$

since $F(z_i)=0$. Similarly, let us take $\chi=a_k\pm jb_k$. Then (5) and (6) in Lemma 4 can be reformulated as

$$\int_0^\infty[\Gamma_c(t)+\Gamma_s(t)]f(t)dt=0. \quad (12)$$

Hence, the result comes from (11) and (12), which completes the proof. □

The next example shows that the integral equalities presented in Theorem 1 are be satisfied by the impulse and the unit step responses of the system represented by the rational transfer function.

Example 1: Let us consider a 4th-order system $G(s)$ as follows:

$$G(s)=\frac{-72s^3+360s^2-648s+360}{s^4+18s^3+119s^2+342s+360}, \quad (13)$$

which has the poles at $s=-3,-4,-5,-6$ and the zeros at $s=1,2\pm j$. The impulse response $y_1(t)$ and the unit step response $y_2(t)$ can be computed as

$$y_1(t)=1248e^{-3t}-6660e^{-4t}+10800e^{-5t}-5460e^{-6t}, \quad (14)$$

$$y_2(t)=1-416e^{-3t}+1665e^{-4t}-2160e^{-5t}+910e^{-6t}, \quad (15)$$

respectively. From (9), we can see that

$$\int_0^\infty y_1(t)dt=1, \quad (16)$$

and from Theorem 1, we can also see that

$$\int_0^\infty e^{-t}y_1(t)dt=0, \quad (17)$$

$$\int_0^\infty e^{-2t}\cos(t)y_1(t)dt=0, \quad (18)$$

$$\int_0^\infty e^{-2t}\sin(t)y_1(t)dt=0, \quad (19)$$

$$\int_0^\infty e^{-t}y_2(t)dt=0, \quad (20)$$

$$\int_0^\infty e^{-2t}\cos(t)y_2(t)dt=0, \quad (21)$$

$$\int_0^\infty e^{-2t}\sin(t)y_2(t)dt=0, \quad (22)$$

The linear combinations of (17)-(19) and (20)-(22) have to also be satisfied by the impulse response $y_1(t)$ and the unit step response $y_2(t)$, respectively.

These integral equalities can be directly verified by using the impulse response (14) and unit step response (15).

Theorem 1 gives some information about the performance limitations on the response of the system with those zeros. For example, if the system with a real zero at $s = z_1$ is larger than γ, its impulse response $y_1(t)$ has to satisfy

$$\int_0^\infty e^{-z_1 t} y_1(t)dt = 0, \tag{23}$$

which states that $y_1(t)$ must have sign changes at some time instant since $e^{-z_1 t} \geq 0$ for all time $t \geq 0$. It means that the step response of the system has the extrema such as the undershoot and the overshoot. As a matter of fact, it is well-known that real zeros located between the dominant pole and the imaginary axis necessarily contribute to the overshoot, and RHP real zeros have to exhibit the initial undershoot in the step response [4],[6],[9],[14]. If $s = z_1$ is the RHP zero, the step response $y_2(t)$ also satisfies

$$\int_0^\infty e^{-z_1 t} y_2(t)dt = 0, \tag{24}$$

which confirms that RHP real zeros are the cause of the undershoot phenomena in the step response.

For the second example, let us consider the system with complex conjugate zeros on the imaginary axis at $s = \pm jb$. In this case, (10) in Theorem 1 can be simply rewritten as

$$\int_0^\infty \cos(bt) y_1(t)dt = 0, \tag{25}$$

where $y_1(t)$ is the impulse response of that system. Let τ be the settling time such that $y_1(t) = 0$ for all time $t \geq \tau$. Assume that $b\tau \ll \pi/2$. Then, using the Taylor series expansion for $\cos(bt)$, integral equality (25) yields

$$\int_0^\infty y_1(t)dt \simeq 0, \tag{26}$$

which contradicts (9) for the system with nonzero DC gain, *i.e.* the integral of the impulse response has to be equal to the nonzero DC gain. It means that the complex conjugate zeros on the imaginary axis necessarily imply a lower bound on the achievable settling time of the system. This result coincides with the work of G. C. Goodwin *et al.* [15], which has shown that fundamental limitations on the achievable settling time exist if systems have zeros on or near the $j\omega$-axis.

4. APPLICATIONS TO FEEDBACK CONTROL SYSTEM

Let us consider the unity-feedback control system as shown in Fig. 1. It is the most commonly used system configuration with the controller placed in series with the controlled plant [2]. In Fig. 1, the symbols have the following meaning:

$P(s)$ plant transfer function,
$K(s)$ controller transfer function,
$r(t)$ reference input,
$e(t)$ error signal,
$u(t)$ controller output or plant input,
$y(t)$ plant output.

Let us define the complementary sensitivity function by

$$T(s) \triangleq \frac{K(s)P(s)}{1 + K(s)P(s)}, \tag{27}$$

which is also the closed-loop transfer function between the reference input $r(t)$ and the plant output $y(t)$. Hence, $Y(s)$, which is the Laplace transform of $y(t)$, can be written as

$$Y(s) = T(s)R(s), \tag{28}$$

where $R(s)$ is the Laplace transform of $r(t)$. The closed-loop transfer function has the unionized zeros of the plant and controller provided that there is no pole-zero cancellation, *i.e.* the open-loop zeros are not changed in spite of the feedback control scheme as shown in Fig. 1. The results presented in the previous section can also be directly applied to the closed-loop system. Moreover, if p is the open-loop pole of the unity-feedback system, the closed-loop transfer function $T(s)$ always satisfies

$$T(p) = 1, \tag{29}$$

and equivalently,

$$Y(p) = R(p). \tag{30}$$

Similarly to Theorem 1, the open-loop poles located in the convergence region of $Y(s)$ yield the integral equalities as follows:

Corollary 1: If $Y(s)$ has the open-loop poles lo-

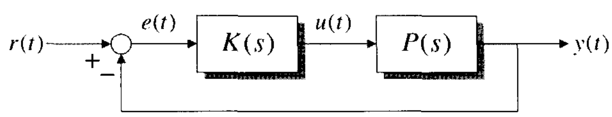

Fig. 1. The unity-feedback control system.

cated in the right area of the dominant pole, then $y(t)$ has to satisfy

$$\int_0^\infty e^{-\alpha t}\cos(\beta t)y(t)dt = \mathrm{Re}[R(p)], \quad (31)$$

$$\int_0^\infty e^{-\alpha t}\sin(\beta t)y(t)dt = \mathrm{Im}[R(\bar{p})], \quad (32)$$

where $p=\alpha+j\beta$ and $\bar{p}=\alpha-j\beta$ are those complex conjugate poles.

Proof: If we take $\chi = p$ in Lemma 4, the result immediately follows from (5) and (6) since $Y(p)=R(p)$, which completes the proof. □

In the case that the system has an open-loop real pole at $s=\alpha$ located in the convergence region, its response $y(t)$ has to satisfy

$$\int_0^\infty e^{-\alpha t}y(t)dt = R(\alpha) \quad (33)$$

from Corollary 1. It is noted that if the reference input is the impulse, then

$$R(p)=1, \quad (34)$$

and if the reference input is the step, then

$$R(p)=\frac{K}{p}, \quad (35)$$

where K is the magnitude of the step input. When $y(t)$ is the step response, the open-loop poles located in the convergence region of $Y(s)$ are equivalent to unstable open-loop poles. The next example shows that integral equalities presented in Corollary 1 are satisfied by the impulse and the unit step responses of the closed-loop system.

Example 2: Let us consider a plant $P(s)$ and a controller $K(s)$ as follows:

$$P(s)=\frac{1}{s^3-2s^2+2s}, \quad (36)$$

$$K(s)=\frac{9020s^2+3410s+5040}{s^3+29s^2+351s+2309}, \quad (37)$$

and the closed-loop transfer function can be computed as

$$T(s)=\frac{9020s^2+3410s+5040}{(s+2)(s+3)(s+4)(s+5)(s+6)(s+7)}. \quad (38)$$

It has the impulse response $y_1(t)$, and the unit step response $y_2(t)$ as follows:

$$y_1(t)=\sum_{i=1}^{6}A_i e^{-(i+1)t}, \quad (39)$$

$$y_2(t)=1+\sum_{k=7}^{12}A_k e^{-(k-5)t}, \quad (40)$$

where the coefficients A_1 - A_{12} can be properly computed by the closed-loop transfer function (38). The unity-feedback control system has the open-loop marginally stable and unstable poles at $s=0, 1\pm j$ on the complex plane. From (9) and Corollary 1, we can see that

$$\int_0^\infty y_1(t)dt=1, \quad (41)$$

$$\int_0^\infty e^{-t}\cos(t)y_1(t)dt=1, \quad (42)$$

$$\int_0^\infty e^{-t}\sin(t)y_1(t)dt=0, \quad (43)$$

$$\int_0^\infty e^{-t}\cos(t)y_2(t)dt=\frac{1}{2}, \quad (44)$$

$$\int_0^\infty e^{-t}\sin(t)y_2(t)dt=\frac{1}{2} \quad (45)$$

since the open-loop poles at $s=0, 1\pm j$ are located in the convergence region of $y_1(t)$, and the open-loop poles at $s=1\pm j$ are located in the convergence region of $y_2(t)$. Equations (41)-(45) can be verified by using impulse response (39) and unit step response (40). Note that the output responses of the closed-loop system also satisfy Theorem 1.

5. UNDERSHOOT AND OVERSHOOT PHENOMENA IN THE STEP RESPONSE

Based on the results of the previous sections, the performance limitations on the step response of the system are established in this section. Let us consider a situation in which the impulse response is equal to 0 after a finite time period, which is previously used in [15]. Although this assumption of an exact settling time would be unrealistic, corresponding results presented in this paper can be extended so that similar limitations hold under the less restrictive set of assumptions.

Definition 3: Let us define the exact settling time of a system as follows:

$$t_s=\inf\{\tau : y_1(t)=0,\ {}^\forall t\geq\tau\}, \quad (46)$$

where $y_1(t)$ is the impulse response of the system. Note that the exact settling time is also identically defined as

$$t_s=\inf\{\tau : y_2(t)=V_s,\ {}^\forall t\geq\tau\}, \quad (47)$$

where $y_2(t)$ and V_s are the step response and its steady-state value, respectively.

It is firstly shown which system has the undershoot phenomena in the sense of Definition 1 as follows:

Theorem 2: For a stable system with RHP complex conjugate zeros at $s = a \pm jb$ on the complex plane, the step response has the undershoot if

$$bt_s \le \frac{\pi}{2} + \tan^{-1}\frac{a}{b}, \tag{48}$$

where t_s is the exact settling time.

Proof: Let $E_r(t) = 0$, $\Gamma_c(t) = 0$, and $\Gamma_s(t) = e^{-at}\sin(bt)$ in Theorem 1, and the step response $y_2(t)$ has to satisfy

$$\begin{aligned} 0 &= \int_0^\infty e^{-at}\sin(bt) y_2(t)dt \\ &= \int_0^{t_s} e^{-at}\sin(bt) y_2(t)dt \\ &\quad + V_s \int_{t_s}^\infty e^{-at}\sin(bt)dt \end{aligned} \tag{49}$$

since the system has RHP complex conjugate zeros at $s = a \pm jb$, where V_s is the steady-state value of the step response. From (49), we can obtain the relation

$$\begin{aligned} &\int_0^{t_s} e^{-at}\sin(bt) y_2(t)dt \\ &= -\frac{V_s e^{-at_s}}{a^2+b^2}[a\sin(bt_s) + b\cos(bt_s)] \\ &= -\frac{V_s e^{-at_s}}{a^2+b^2}\cos\left(bt_s - \tan^{-1}\frac{a}{b}\right). \end{aligned} \tag{50}$$

Hence, if $bt_s \le \pi/2 + \tan^{-1}(a/b)$, then $y_2(t)$ will take both positive and negative signs since the right part of (50) is always negative, and $e^{-at}\sin(bt) > 0$ for all $t \in [0, t_s]$, which completes the proof. □

It is noted that the value of $\tan^{-1}(a/b)$ is given by

$$0 \le \tan^{-1}\frac{a}{b} \le \frac{\pi}{2} \tag{51}$$

since $a > 0$ and $b > 0$. If $a \ge b$, (48) in Theorem 2 leads to the result

$$bt_s \le \frac{3}{4}\pi. \tag{52}$$

Theorem 2 also implies that the system with RHP real zeros, *i.e.* the case of $b = 0$, always has the undershoot in the step response without any relation with the settling time. Although it is well-known that SISO LTI continuous-time systems with an odd number of RHP real zeros have the initial undershoot on the step-type reference input [6,9,13-14] the corresponding result has not been presented for the system with RHP complex conjugate zeros related to the undershoot phenomena in the step response [7].

For the unity-feedback system, another result related to the overshoot in the sense of Definition 2 can be induced by (33) [7,9].

Lemma 5: The step response of a stable unity-feedback system with an open-loop unstable real pole must have the overshoot in the sense of Definition 2.

Proof: Let $Y_{\max}$ and $s = \alpha$ be the maximum value of the step response and an open-loop unstable real pole of the system, respectively. Then (33) can be rewritten as

$$\begin{aligned} \frac{K}{\alpha} &= \int_0^\infty e^{-at} y(t)dt \\ &< Y_{\max}\int_0^\infty e^{-at}dt \\ &= \frac{Y_{\max}}{\alpha} \end{aligned} \tag{53}$$

since $R(\alpha) = K/\alpha$, where K is the magnitude of the step input. Hence, $Y_{\max} > K$, *i.e.* a stable unity-feedback system with an open-loop unstable real pole must have the overshoot in the sense of Definition 2, which completes the proof. □

Lemma 5 states that any unity-feedback controller, {\it *e.g.*} the conventional PID controller, does not make the step response without the overshoot in the sense of Definition 2 if the plant $P(s)$ has an unstable real pole. In the case that the DC gain of the system is larger than 1, the overshoot is obviously larger than the magnitude of the step input, but it might be smaller than the steady-state value of the step response.

6. CONCLUSIONS

In this paper, we have presented some time-domain integral equalities, which have to be satisfied by the Laplace transformable function. In the unity-feedback control scheme, it has derived another integral equality on the output response of the closed-loop system with open-loop poles located in the convergence region of the output function. These results have been verified by using simple examples. Using the integral equalities, it has been shown that a system, which satisfies Theorem 2, must have the undershoot phenomena in the step response. It has also been shown that a plant with an unstable real pole must have the overshoot in the step response provided that the unity-feedback scheme is used.

Integral equalities presented in this paper can be applied to the understanding of the fundamental limitations of the control system since it is represented by the transfer function which has been Laplace transform. It requires further research to find the other fundamental limitations by utilizing the proposed integral equalities.

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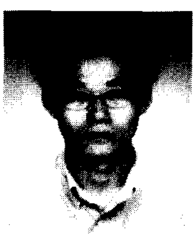

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