Robust High Gain Adaptive Output Feedback Control for Nonlinear Systems with Uncertain Nonlinearities in Control Input Term

Ryuji Michino, Ikuro Mizumoto, Zenta Iwai, and Makoto Kumon

Abstract: It is well known that one can easily design a high-gain adaptive output feedback control for a class of nonlinear systems which satisfy a certain condition called output feedback exponential passivity (OFEP). The designed high-gain adaptive controller has simple structure and high robustness with regard to bounded disturbances and unknown order of the controlled system. However, from the viewpoint of practical application, it is important to consider a robust control scheme for controlled systems for which some of the assumptions of output feedback stabilization are not valid. In this paper, we design a robust high-gain adaptive output feedback control for the OFEP nonlinear systems with uncertain nonlinearities and/or disturbances. The effectiveness of the proposed method is shown by numerical simulations.

Keywords: Adaptive control, high gain output feedback control, uncertain nonlinearity, nonlinear systems.

1. INTRODUCTION

The linear plant is said to be ASPR (almost strictly positive real) if there exists a static output feedback such that the resulting closed loop system is SPR (strictly positive real) [1]. It is well known that, for the ASPR plants, one can design a stable control system via adaptive output feedback with the very simple controller structure [2,3]. Unlike other adaptive methods, under the ASPR condition, we are able to design the adaptive controller without a priori information of the controlled plants (e.g. order of the plant and the size of the uncertainties). As for the nonlinear systems the condition of the high gain output feedback stabilization is recognized as output feedback exponential passivity (OFEP) [4,7]. The sufficient conditions for systems to be OFEP are that the system be globally exponential minimum phase, have a relative degree of one and that the nonlinearities of the system satisfy the Lipschitz conditions. Furthermore, the nonlinearities in the control input term are bounded. Under these conditions, it has been shown that one can stabilize uncertain nonlinear systems via high gain feedback based adaptive output feedback control with simple structure as well as by the method for ASPR linear systems [5-7]. It has also been shown that the adaptive method has a highly robustness for bounded disturbance and noise [5,7]. Unfortunately, the above-mentioned OFEP (or ASPR) conditions are very restrictive for practical systems and there might exist unbounded (state dependent) disturbances with which some of the OFEP conditions are not valid.

With this point in mind, some robust adaptive schemes based on the high gain output feedback strategy and alleviation methods for the restrictions on the linear controlled system have been proposed [3,8]. Recently, robust adaptive output feedback control schemes for OFEP nonlinear systems with output dependent uncertainties and/or disturbances have been proposed [7,9]. Considering the nonlinear uncertain function as a kind of output dependent disturbance, the methods are able to deal with robust stabilization problems via high gain adaptive output feedback for nonlinear systems, for which some Lipschitz conditions on nonlinear functions are not satisfied with respect to output signal. In these methods, however, it is assumed that the uncertainties in the control input term are bounded.

In this paper, we will show that we can remove the restriction that is imposed on the uncertainties in the control input term. That is, we propose a robust high gain adaptive output feedback strategy that can deal with a broader class of uncertain nonlinearities. Unlike previous high gain output feedback strategies, it is shown that we can design an adaptive output feedback controller for nonlinear systems with unbounded uncertainties in the control input term.

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2. PRELIMINARIES

Consider the following affine nonlinear systems:

$$\begin{aligned} \dot{x}(t) &= f(x) + g(x)u(t), \\ y(t) &= h(x), \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is a state vector, $u(t) \in R$ is an input, $y(t) \in R$ is output, and $f(x)$, $g(x): R^n \to R^n$ and $h(x): R^n \to R^n$ are sufficiently smooth (e.g. of class C^∞) functions such that $f(0) = 0, h(0) = 0$, i.e. it is assumed that $\dot{x}(t) = f(x)$ has an equilibrium at the origin.

It is well known [10] that if the system (1) has a strong relative degree of one, then there exists a smooth nonsingular change of coordinates, $z = \Phi(x)$, such that the system (1) can be transformed to the normal form:

$$\begin{aligned} \dot{y}(t) &= a(y,\eta) + b(y,\eta)u(t), \\ \dot{\eta}(t) &= q(y,\eta). \end{aligned} \tag{2}$$

Throughout this paper, we consider the case where the controlled system (1) has a relative degree of one so that it is described by the form of (2).

3. ROBUST ADAPTIVE OUTPUT FEEDBACK CONTROL

3.1. Problem statement

Consider the following SISO nonlinear system with a relative degree of one:

$$\begin{aligned} \dot{y}(t) &= a(y,\eta) + b(y,\eta)u(t) + f_1(t,y,\eta), \\ \dot{\eta}(t) &= q(y,\eta) + f_2(t,y,\eta), \end{aligned} \tag{3}$$

where $a(y,\eta)$, $b(y,\eta)$, $q(y,\eta)$ and $f_1(t,y,\eta)$, $f_2(t,y,\eta)$ are uncertain nonlinearities and/or disturbances.

We impose the following assumptions on the controlled system (3):

Assumptions: (A-1) The nominal part of the system (3) is exponentially minimum-phase. That is, the zero dynamics of the nominal system:

$$\dot{\eta}(t) = q(0,\eta) \tag{4}$$

is exponentially stable.

(A-2) The function $q(y,\eta)$ is globally Lipschitz, i.e., there exists a positive constant L_1 such that

$$\|q(y_1,\eta_1) - q(y_2,\eta_2)\| \le L_1\left(|y_1 - y_2| + \|\eta_1 - \eta_2\|\right). \tag{5}$$

(A-3) The function $a(y,\eta)$ is globally Lipschitz, i.e., there exists a positive constant L_2 such that

$$|a(y_1,\eta_1) - a(y_2,\eta_2)| \le L_2\left(|y_1 - y_2| + \|\eta_1 - \eta_2\|\right). \tag{6}$$

(A-4) The uncertain nonlinearities $f_1(t,y,\eta)$ can be evaluated by

$$|f_1(t,y,\eta)| \le \sum_{i=1}^{M_1} d_i \psi_i(y) + d_0 \tag{7}$$

with known positive functions $\psi_i(y)$, and unknown positive constants d_i and d_0.

(A-5) The uncertain nonlinearities $f_2(t,y,\eta)$ can be evaluated by

$$|f_2(t,y,\eta)| \le \sum_{i=1}^{M_2} g_i \phi_i(y) + g_0 \tag{8}$$

with unknown positive constants g_0 and g_i and known positive functions $\phi_i(y)$ which have the following property for any variables y_1 and y_2 :

$$\phi_i(y_1 + y_2) \le \phi_{1i}(y_1, y_2)|y_1| + \phi_{2i}(y_2) \tag{9}$$

with known positive function $\phi_{1i}(y_1, y_2)$ and unknown positive function $\phi_{2i}(y_2)$ which is smooth for all $y_2 \in R$.

(A-6) The function $b(y,\eta)$ can be evaluated as

$$b(y,\eta) \ge b_0 > 0 \tag{10}$$

where b_0 is an unknown positive constant.

The control objective is to achieve the goal:

$$\lim_{t\to\infty} |y(t) - y^*(t)| \le \delta \tag{11}$$

for a given positive constant δ and smooth function $y^*(t)$ such as

$$|y^*(t)| \le \beta_0, \quad |\dot{y}^*(t)| \le \beta_1 \tag{12}$$

Remark 1: It is noted that the restriction concerning the relative degree might be a severe restriction on practical systems. For this problem, the introduction of a parallel feedforward compensator (PFC) or shunt filter which renders the resulting augmented system OFEP has been proposed [6,7]. That is, it is possible to make an exponentially minimum-phase system with relative degree of one by introducing a PFC. Furthermore, the restrictions on uncertainties will be satisfied for a nonlinear system with *output-feedback form* under mild conditions for nonlinear functions.

Remark 2: The main modification on the proposed method to previous methods [5,7,9] is that we consider the further uncertain nonlinearity in the control input term and remove the restriction on the uncertainty which is bounded or known.

3.2. Robust adaptive controller design

Under assumptions (A-1) to (A-6), we design an adaptive controller as follows:

$$u(t) = -\left[k(t)v(t) + \sum_{i=1}^{M_1} u_{fi}(t) \right] \tag{13}$$

where $v(t) = y(t) - y^*(t)$ and $k(t)$ is an adaptive feedback gain which is adjusted by the following adjusting law:

$$k(t) = k_I(t) + k_P(t), \tag{14}$$

$$\dot{k}_I(t) = \gamma_I v(t)^2 - \sigma_I k_I(t), \quad k_I(0) \geq 0, \tag{15}$$

$$k_P(t) = \sum_{i=1}^{M_2} \gamma_{pi} \phi_{1i}(v, y^*)^4 v(t)^2, \tag{16}$$

where γ_I and σ_I are positive constants, and $u_{fi}(t)$ is the robust adaptive control term for $f_1(y,\eta)$ which is given by

$$u_{fi}(t) = \begin{cases} \left\{ \hat{d}_i(t)\psi_i(y) \right\}^2 v(t)/\varepsilon_{fi} & \text{if } \left| \hat{d}_i(t)\psi_i(y)v(t) \right| \leq \varepsilon_{fi} \\ \hat{d}_i(t)\psi_i(y)\text{sign}(v(t)) & \text{if } \left| \hat{d}_i(t)\psi_i(y)v(t) \right| > \varepsilon_{fi} \end{cases} \tag{17}$$

$$\dot{\hat{d}}_i(t) = \gamma_{di}\psi_i(y)|v(t)| - \sigma_{di}\hat{d}_i(t), \quad \hat{d}_i(0) \geq 0, \tag{18}$$

$$\varepsilon_{fi}, \gamma_{di}, \sigma_{di} > 0.$$

The following theorem shows the main results in this paper.

Theorem 1: Under assumptions (A-1) to (A-6), there exist γ_I, γ_{pi}, γ_{di}, ε_{fi} and the ideal feedback gain k^* such that all the signals in the closed-loop system with the controller (13) to (18) are bounded and the goal (11) is satisfied.

Proof: After the change of coordinates from (y,η) to (v,η), the system (3) with the controller (13) is transformed

$$\begin{aligned} \dot{v}(t) &= a(v + y^*, \eta) \\ &\quad - b(v + y^*, \eta)\left[k(t)v(t) + \sum_{i=1}^{M_1} u_{fi}(t, v + y^*) \right] \\ &\quad + f_1(t, v + y^*, \eta) - \dot{y}^*(t) \end{aligned} \tag{19}$$

$$\dot{\eta} = q(v + y^*, \eta) + f_2(t, v + y^*, \eta). \tag{20}$$

From assumption (A-1) and the Converse theorem of Lyapunov on exponential stability [11,12] , there exists a positive definite function $W(\eta)$ and positive constants α_1 to α_4 such that

$$\begin{aligned} &\frac{\partial W(\eta)}{\partial \eta} q(0,\eta) \leq -\alpha_1 \|\eta(t)\|^2, \quad \left\| \frac{\partial W(\eta)}{\partial \eta} \right\| \leq \alpha_2 \|\eta(t)\| \\ &\alpha_4 \|\eta(t)\|^2 \leq \|W(\eta)\| \leq \alpha_3 \|\eta(t)\|^2. \end{aligned} \tag{21}$$

Now, we consider the following positive definite function:

$$\begin{aligned} V(v,\eta,k,d) &= \mu W(\eta) + \frac{1}{2}v(t)^2 + \frac{b_0}{2\gamma_I}\left[k_I(t) - k^* \right]^2 \\ &\quad + \sum_{i=1}^{M_1} \frac{b_0}{2\gamma_{di}} \left[\hat{d}_i(t) - d_i / b_0 \right]^2 \end{aligned} \tag{22}$$

where μ is any positive constant and k^* is an ideal feedback gain to be determined later.

The time derivative of (22) along the trajectories of (15), (18) and (19), (20) yields

$$\begin{aligned} \frac{dV}{dt} &= \mu \frac{\partial W(\eta)}{\partial \eta}\left[q(v + y^*, \eta) + f_2(t, v + y^*, \eta) \right] \\ &\quad + v(t)\left[a(v + y^*, \eta) + f_1(t, v + y^*, \eta) - \dot{y}^*(t) \right. \\ &\quad \left. - b(v + y^*, \eta)\left\{ k(t)v(t) + \sum_{i=1}^{M_1} u_{fi}(t, v + y^*) \right\} \right] \\ &\quad + \frac{b_0}{\gamma_I}\left[k_I(t) - k^* \right]\left[\gamma_I v(t)^2 - \sigma_I k_I(t) \right] \\ &\quad + \sum_{i=1}^{M_1} \frac{b_0}{\gamma_{di}}\left[\hat{d}_i(t) - \frac{d_i}{b_0} \right] \\ &\qquad \times \left[\gamma_{di}\psi_i(v + y^*)|v(t)| - \sigma_{di}\hat{d}_i(t) \right]. \end{aligned} \tag{23}$$

It follows from assumptions (A-4) and (A-5) that (23) can be represented by

$$\begin{aligned} \frac{dV}{dt} &\leq \mu \frac{\partial W(\eta)}{\partial \eta} q(0,\eta) - b_0 k^* v(t)^2 \\ &\quad + \mu \left\| \frac{\partial W(\eta)}{\partial \eta} \right\| \left\| q(v + y^*, \eta) - q(0,\eta) \right\| \\ &\quad + \mu \left\| \frac{\partial W(\eta)}{\partial \eta} \right\| \left[\sum_{i=1}^{M_2} g_i \phi_i(v + y^*) + g_0 \right] \\ &\quad + |v(t)| \left| a(v + y^*, \eta) \right| - b(v + y^*, \eta) k(t) v(t)^2 \\ &\quad - v(t) b(v + y^*, \eta) \sum_{i=1}^{M_1} u_{fi}(t, v + y^*) \\ &\quad + |v(t)| \left[\sum_{i=1}^{M_1} d_i \psi_i(v + y^*) + d_0 \right] + \left| \dot{y}^*(t) \right| |v(t)| \\ &\quad + b_0 \frac{\sigma_I}{\gamma_I} k^* k_I(t) + b_0 k_I(t) v(t)^2 - b_0 \frac{\sigma_I}{\gamma_I} k_I(t)^2 \end{aligned} \tag{24}$$

$$-b_0\sum_{i=1}^{M_1}\frac{\sigma_{di}}{\gamma_{di}}\hat{d}_i(t)^2-\sum_{i=1}^{M_1}d_i\psi_i(v+y^*)|v(t)|$$
$$+b_0\sum_{i=1}^{M_1}\hat{d}_i(t)\psi_i(v+y^*)|v(t)|+\sum_{i=1}^{M_1}\frac{\sigma_{di}}{\gamma_{di}}d_i d_i(t).$$

Considering assumptions (A-2),(A-3) and taking (12) and (21) into account, we can evaluate the time derivative of V as follows:

$$\frac{dV}{dt}\le-\mu\alpha_1\|\eta\|^2+\mu\alpha_2\|\eta\|L_1\left(|v(t)|+\beta_0\right)-b_0k^*v(t)^2$$
$$+\mu\alpha_2\left[\sum_{i=1}^{M_2}g_i(\phi_{1i}(v,y^*)|v(t)|+\phi_{2i}(y^*))+g_0\right]\|\eta\|$$
$$+L_2\left(|v(t)|+|y^*|+\|\eta\|\right)|v(t)|+\beta_1|v(t)|+d_0|v(t)|$$
$$-b(v+y^*,\eta)\left[k_I(t)+k_p(t)\right]v(t)^2+b_0k_I(t)v(t)^2$$
$$-v(t)b(v+y^*,\eta)\sum_{i=1}^{M_1}u_{fi}(t,v+y^*)$$
$$+b_0\sum_{i=1}^{M_1}\hat{d}_i(t)\psi_i(v+y^*)|v(t)|$$
$$-\frac{b_0}{\gamma_I}\sigma_I\left[k_I(t)-k^*\right]^2-\frac{b_0}{\gamma_I}\sigma_I\left[k_I(t)-k^*\right]k^*$$
$$-b_0\sum_{i=1}^{M_1}\frac{\sigma_{di}}{\gamma_{di}}\left[\hat{d}_i(t)-d_i/b_0\right]^2$$
$$-\sum_{i=1}^{M_1}\frac{\sigma_{di}}{\gamma_{di}}\left[\hat{d}_i-d_i/b_0\right]d_i. \tag{25}$$

Here, we have from (15) and (16) that

$$k_I(t)=e^{-\sigma_I t}k_I(0)+\int_0^t e^{-\sigma_I(t-\tau)}\gamma_I v(\tau)^2d\tau\ge0 \tag{26}$$

and

$$k_p(t)\ge0. \tag{27}$$

It follows from (26),(27) and assumption (A-6) that

$$-b(v+y^*,\eta)\left[k_I(t)+k_p(t)\right]v(t)^2+b_0k_I(t)v(t)^2$$
$$\le-b_0\left[k_I(t)+k_p(t)\right]v(t)^2+b_0k_I(t)v(t)^2 \tag{28}$$
$$=-b_0k_p(t)v(t)^2.$$

From (28), we obtain

$$\frac{dV}{dt}\le-\mu\alpha_1\|\eta\|^2+(\mu\alpha_2L_1+L_2)\|\eta\||v(t)|-b_0k_p(t)v(t)^2$$
$$-(b_0k^*-L_2)v(t)^2+(L_2\beta_0+d_0+\beta_1)|v(t)|$$
$$+\mu\alpha_2(L_1\beta_0+\sum_{i=1}^{M_2}g_i\phi_{2i}(y^*)+g_0)\|\eta\|$$
$$+\mu\alpha_2\sum_{i=1}^{M_2}g_i\phi_{1i}(v,y^*)|v(t)|\|\eta\|$$
$$-b(v+y^*,\eta)\sum_{i=1}^{M_1}u_{fi}(t,v+y^*)v(t)$$
$$+b_0\sum_{i=1}^{M_1}\hat{d}_i(t)\psi_i(v+y^*)|v(t)| \tag{29}$$
$$-\frac{b_0}{\gamma_I}\sigma_I\left[k_I(t)-k^*\right]^2-\frac{b_0}{\gamma_I}\sigma_I\left[k_I(t)-k^*\right]k^*$$
$$-b_0\sum_{i=1}^{M_1}\frac{\sigma_{di}}{\gamma_{di}}\left[\hat{d}_i(t)-d_i/b_0\right]^2-\sum_{i=1}^{M_1}\frac{\sigma_{di}}{\gamma_{di}}\left[\hat{d}_i(t)-d_i/b_0\right]d_i.$$

Moreover, taking the following evaluation with any constants ρ_1 to ρ_{6i} into consideration

$$(\mu\alpha_2L_1+L_2)|v(t)|\|\eta\|$$
$$=\rho_1\|\eta\|^2-\rho_1(\|\eta\|-\frac{(\mu\alpha_2L_1+L_2)}{2\rho_1}|v(t)|)^2$$
$$+\frac{(\mu\alpha_2L_1+L_2)^2}{4\rho_1}|v(t)|^2 \tag{30}$$
$$\le\rho_1\|\eta\|^2+\frac{(\mu\alpha_2L_1+L_2)}{4\rho_1}|v(t)|^2,$$

$$\mu\alpha_2(L_1\beta_0+\sum_{i=1}^{M_2}g_i\phi_{2i}(y^*)+g_0)\|\eta\|$$
$$\le\rho_2\|\eta\|^2+\frac{\left[\mu\alpha_2(L_1\beta_0+\sum_{i=1}^{M_2}g_i\phi_{2i}(y^*)+g_0)\right]^2}{4\rho_2} \tag{31}$$

$$\mu\alpha_2\sum_{i=1}^{M_2}g_i\phi_{1i}(v,y^*)|v(t)|\|\eta\|$$
$$\le\sum_{i=1}^{M_2}\rho_{3i}\|\eta\|^2+\sum_{i=1}^{M_2}\frac{(\mu\alpha_2g_i)^2}{4\rho_{3i}}\phi_{1i}(v,y^*)^2v(t)^2 \tag{32}$$

$$(L_2\beta_0+d_0+\beta_1)|v(t)|$$
$$\le\rho_4|v(t)|^2+\frac{(L_2\beta_0+d_0+\beta_1)^2}{4\rho_4} \tag{33}$$

$$-\frac{b_0}{\gamma_I}\sigma_I\left[k_I(t)-k^*\right]^2-\frac{b_0}{\gamma_I}\sigma_I\left[k_I(t)-k^*\right]k^*$$
$$\le-(1-\frac{\gamma_I\rho_5}{b_0\sigma_I})\sigma_I\frac{b_0}{\gamma_I}\left[k_I(t)-k^*\right]^2+\frac{b_0^2\sigma_I^2}{4\rho_4\gamma_I^2}k^{*2} \tag{34}$$

$$-\sum_{i=1}^{M_1}\frac{b_0}{\gamma_{di}}\sigma_{di}\left[\hat{d}_i(t)-d_i/b_0\right]^2-\sum_{i=1}^{M_1}\frac{\sigma_{di}}{\gamma_{di}}[\hat{d}_i-d_i/b_0]d_i$$
$$\le-\sum_{i=1}^{M_1}(1-\frac{\gamma_{di}\rho_{6i}}{b_0\sigma_{di}})\sigma_{di}\frac{b_0}{\gamma_{di}}\left[\hat{d}_i(t)-d_i/b_0\right]^2$$
$$+\sum_{i=1}^{M_1}\frac{d_i^2\sigma_{di}^2}{4\rho_{6i}\gamma_{di}^2}. \tag{35}$$

The time derivative of V can be evaluated from (30) to (35) that

$$\begin{aligned}\frac{dV}{dt} &\le -(\mu\alpha_1 - \rho_1 - \rho_2 - \sum_{i=1}^{M_2}\rho_{3i})\|\eta\|^2 \\ &\quad -(b_0 k^* - L_2 - \rho_4 - \frac{(\mu\alpha_2 L_1 + L_2)^2}{4\rho_1}|v(t)|^2 \\ &\quad + \frac{\left[\mu\alpha_2(L_1\beta_0 + \sum_{i=1}^{M_2} g_i\phi_{2iM} + g_0\right]^2}{4\rho_2} \\ &\quad + \sum_{i=1}^{M_2}\frac{(\mu\alpha_2 g_i)^2}{4\rho_{3i}}\phi_{1i}(v,y^*)^2 v(t)^2 - b_0 k_p(t) v(t)^2 \\ &\quad + \frac{(L_2\beta_0 + d_0 + \beta_1)^2}{4\rho_4} + \frac{b_0\sigma_l k^{*2}}{4\gamma_l\rho_5'} + \sum_{i=1}^{M_1}\frac{\sigma_{di} d_i^2}{4b_0\gamma_{di}\rho_{6i}'} \\ &\quad + b_0\sum_{i=1}^{M_1}\hat{d}_i(t)\psi_i(v+y^*)|v(t)| \\ &\quad - b(v+y^*,\eta)\sum_{i=1}^{M_1} u_{fi}(t, v+y^*)v(t) \qquad (36) \\ &\quad -(1-\rho_5')b_0\frac{\sigma_l}{\gamma_l}\left[k_l(t) - k^*\right]^2 \\ &\quad - b_0\sum_{i=1}^{M_1}(1-\rho_{6i}')\frac{\sigma_{di}}{\gamma_{di}}\left[\hat{d}_i(t) - d_i/b_0\right]^2\end{aligned}$$

where

$\rho_5' = \frac{\gamma_l\rho_5}{b_0\sigma_l}$, $\rho_{6i}' = \frac{\gamma_{di}\rho_{6i}}{b_0\sigma_{di}}$ and ϕ_{2iM} is a constant such that $\phi_{2i}(y^*) \le \phi_{2iM}$. Such a constant exists from assumptions that $\phi_{2i}(y_2)$ is smooth for all $y_2 \in R$ and that y^* is bounded. Here it follows from (16) that

$$\begin{aligned}&\sum_{i=1}^{M_2}\frac{(\mu\alpha_2 g_i)^2}{4\rho_{3i}}\phi_{1i}(v,y^*)^2 v(t)^2 - b_0 k_p(t) v(t)^2 \\ &= \sum_{i=1}^{M_2}\frac{(\mu\alpha_2 g_i)^2}{4\rho_{3i}}\phi_{1i}(v,y^*)^2 v(t)^2 - b_0\sum_{i=1}^{M_2}\gamma_{pi}\phi_{1i}(v,y^*)^4 v(t)^4 \qquad (37) \\ &\le \sum_{i=1}^{M_2}\frac{1}{4b_0\gamma_{pi}}\left[\frac{(\mu\alpha_2 g_i)^2}{4\rho_{3i}}\right]^2.\end{aligned}$$

Furthermore, in the case where $|\hat{d}_i(t)\psi_i(v+y^*)v(t)| > \varepsilon_{fi}$, for any i we have from (17) that

$$\begin{aligned}&b_0\sum_{i=1}^{M_1}\hat{d}_i(t)\psi_i(v+y^*)|v(t)| \\ &- b(v+y^*,\eta)\sum_{i=1}^{M_1} u_{fi}(t, v+y^*)v(t) \\ &= b_0\sum_{i=1}^{M_1}\hat{d}_i(t)\psi_i(v+y^*)|v(t)| \qquad (38) \\ &- b(v+y^*,\eta)\sum_{i=1}^{M_1}\hat{d}_i(t)\psi_i(v+y^*)|v(t)|\end{aligned}$$

and since we obtain from (18) that $\hat{d}_i(t) \ge 0$, it follows from $\psi_i(v+y^*) \ge 0$ and assumption (A-6) that

$$\begin{aligned}&b_0\sum_{i=1}^{M_1}\hat{d}_i(t)\psi_i(v+y^*)|v(t)| \\ &- b(v+y^*,\eta)\sum_{i=1}^{M_1}\hat{d}_i(t)\psi_i(v+y^*)|v(t)| \\ &\le b_0\sum_{i=1}^{M_1}\hat{d}_i(t)\psi_i(v+y^*)|v(t)| \qquad (39) \\ &- b_0\sum_{i=1}^{M_1}\hat{d}_i(t)\psi_i(v+y^*)|v(t)| \\ &= 0.\end{aligned}$$

On the other hand, if $|\hat{d}_i(t)\psi_i(v+y^*)v(t)| \le \varepsilon_{fi}$, we have

$$\begin{aligned}&b_0\sum_{i=1}^{M_1}\hat{d}_i(t)\psi_i(v+y^*)|v(t)| \\ &- b(v+y^*,\eta)\sum_{i=1}^{M_1}\frac{\left[\hat{d}_i(t)\psi_i(v+y^*)\right]^2}{\varepsilon_{fi}}v(t)^2 \qquad (40) \\ &\le b_0\sum_{i=1}^{M_1}\left[\hat{d}_i(t)\psi_i(v+y^*)v(t)\right] \\ &\le b_0\sum_{i=1}^{M_1}\varepsilon_{fi}.\end{aligned}$$

Therefore, we have from (37),(39) and (40) that

$$\begin{aligned}\frac{dV}{dt} &\le -(\mu\alpha_1 - \rho_1 - \rho_2 - \sum_{i=1}^{M_2}\rho_{3i})\|\eta\|^2 \\ &\quad -(b_0 k^* - L_2 - \rho_4 - \frac{(\mu\alpha_2 L_1 + L_2)^2}{4\rho_1}|v(t)|^2 \\ &\quad -(1-\rho_5')\sigma_l\frac{b_0}{\gamma_l}\left[k_l(t) - k^*\right]^2 \\ &\quad -\sum_{i=1}^{M_1}(1-\rho_{6i}')\sigma_{di}\frac{b_0}{\gamma_{di}}\left[\hat{d}_i(t) - d_i/b_0\right]^2 \qquad (41) \\ &\quad + \frac{\left[\mu\alpha_2(L_1\beta_0 + \sum_{i=1}^{M_2} g_i\phi_{2iM} + g_0\right]^2}{4\rho_2} \\ &\quad \sum_{i=1}^{M_2}\frac{1}{4b_0\gamma_{pi}}\left[\frac{(\mu\alpha_2 g_i)}{4\rho_{3i}}\right]^2 + \frac{(L_2\beta_0 + d_0 + \beta_1)^2}{4\rho_4} \\ &\quad + \frac{b_0\sigma_l k^{*2}}{4\gamma_l\rho_5'} + \sum_{i=1}^{M_1}\frac{\sigma_{di} d_i^2}{4b_0\gamma_{di}\rho_{6i}'} + b_0\sum_{i=1}^{M_1}\varepsilon_{fi}.\end{aligned}$$

Finally, setting the values

$$\rho_1 = \rho_2 = \frac{\mu\alpha_1}{8},\quad \rho_{3i} = \frac{\mu\alpha_1}{4M_2},\quad \rho_5' = \rho_{6i}' = \frac{1}{2},$$

we obtain

$$\begin{aligned}\frac{dV}{dt} \le & -\frac{\mu\alpha_1}{2}\|\eta\|^2 \\ & -(b_0k^* - L_2 - \rho_4 - \frac{2(\mu\alpha_2L_1+L_2)^2}{\mu\alpha_1})|v(t)|^2 \\ & -\sigma_I\frac{b_0}{2\gamma_I}\left[k_I(t)-k^*\right]^2 - \sum_{i=1}^{M_1}\sigma_{di}\frac{b_0}{2\gamma_{di}}\left[\hat{d}_i(t)-d_i/b_0\right]^2 \\ & +2\mu\frac{\left[\alpha_2(L_1\beta_0+\sum_{i=1}^{M_2}g_i\phi_{2iM}+g_0\right]^2}{\alpha_1} \\ & +\sum_{i=1}^{M_2}\frac{1}{4b_0\gamma_{pi}}\left[\mu\frac{M_2(\alpha_2g_i)^2}{\alpha_1}\right]^2 + \frac{(L_2\beta_0+d_0+\beta_1)^2}{4\rho_3} \\ & +\frac{b_0\sigma_Ik^{*2}}{2\gamma_I} + \sum_{i=1}^{M_1}\frac{\sigma_{di}d_i^2}{2b_0\gamma_{di}} + b_0\sum_{i=1}^{M_1}\varepsilon_{fi}\end{aligned} \tag{42}$$

Since it follows from (21) that

$$\|\eta\|^2 \ge \frac{1}{\alpha_3}W(\eta) \tag{43}$$

we have

$$\begin{aligned}\frac{dV}{dt} \le & -\frac{\alpha_1}{2\alpha_3}\left[\mu W(\eta)+\frac{1}{2}|v(t)|^2\right] \\ & -\frac{b_0\sigma_I}{2\gamma_I}\left[k_I(t)-k^*\right]^2 - \sum_{i=1}^{M_1}\frac{b_0\sigma_{di}}{2\gamma_{di}}\left[\hat{d}_i(t)-d_i/b_0\right]^2\end{aligned} \tag{44}$$

by choosing the ideal feedback gain k^* as

$$k^* \ge \frac{1}{b_0}\left[\frac{\alpha_1}{4\alpha_3}+L_2+\frac{2(\mu\alpha_2L_1+L_2)^2}{\mu\alpha_1}+\rho_4\right] \tag{45}$$

where

$$\begin{aligned}\beta = & 2\mu\frac{\left[\alpha_2(L_1\beta_0+\sum_{i=1}^{M_2}g_i\phi_{2iM}+g_0)\right]^2}{\alpha_1} \\ & +\sum_{i=1}^{M_2}\frac{1}{4b_0\gamma_{pi}}\left[\mu\frac{M_2(\alpha_2g_i)^2}{\alpha_1}\right]^2 + \frac{(L_2\beta_0+d_0+\beta_1)^2}{4\rho_4} \\ & +\frac{b_0\sigma_Ik^{*2}}{2\gamma_I} + \sum_{i=1}^{M_1}\frac{\sigma_{di}d_i^2}{2b_0\gamma_{di}} + b_0\sum_{i=1}^{M_1}\varepsilon_{fi}\,.\end{aligned} \tag{46}$$

Consequently the time derivative of the positive definite function $V(t)$ given in (22) can be evaluated by

$$\frac{dV}{dt} \le -\alpha_v V + \beta\,, \tag{47}$$

$$\alpha_v = min\left[\frac{\alpha_1}{2\alpha_3}, \sigma_I, \sigma_{di}\right]. \tag{48}$$

It is apparent from (47), (48) that all signals in the closed-loop system with the controller (13) to (18) are bounded and we also obtain

$$\lim_{t\to\infty} V(t) \le \beta/\alpha_v\,. \tag{49}$$

From the fact that $|v(t)|^2 \le 2V(t)$, it follows that

$$\lim_{t\to\infty}|v(t)|^2 \le 2\beta/\alpha_v\,. \tag{50}$$

Thus, the goal (11) is achieved for $\delta^2 \ge 2\beta/\alpha_v$. It can also be confirmed that the appropriate choice of μ and ρ_4 and design parameters γ_I, γ_{pi}, γ_{di}, and ε_{fi} ensures the goal (11) for any δ

Remark 3: For example, one can set design parameters γ_I, γ_{pi}, γ_{di}, and ε_{fi} as follows in order to attain the goal (11) for a given δ.
Let's set μ and ρ_4 such that

$$\mu \le \frac{\alpha_1\alpha_v\delta^2}{24\{\alpha_2(L_1\beta_0+\sum_{i=1}^{M_2}g_i\phi_{2iM}+g_0)\}^2} \tag{51}$$

$$\rho_4 \ge \frac{3(L_2\beta_0+d_0+\beta_1)^2}{\alpha_v\delta^2} \tag{52}$$

and consider an ideal feedback gain k^* satisfying the inequality (45). Then, it is sufficient to choose design parameters such as

$$\begin{aligned}&\gamma_{pi} \ge \mu\frac{M_2^2\alpha_2^2g_i^4}{8b_0\alpha_1(L_1\beta_0+\sum_{i=1}^{M_2}g_i\phi_{2iM}+g_0)^2} \\ &\gamma_I \ge \frac{6b_0\sigma_Ik^{*2}}{\alpha_v\delta^2},\ \gamma_{di} \ge \frac{6M_1\sigma_{di}d_i^2}{b_0\alpha_v\delta^2},\ \varepsilon_{fi} \le \frac{\alpha_v\delta^2}{12M_1b_0}\,.\end{aligned} \tag{53}$$

Remark 4: Note that the design parameters γ_I, γ_{pi}, γ_{di}, and ε_{fi}, which are set in order to attain the goal (11) for a given small δ, depend on uncertain constants. However, as shown in (53), if we set sufficiently large γ_I, γ_{pi}, γ_{di} and sufficiently small ε_{fi}, then the control objective will be attained even if we do not know *a priori* information about uncertain constants. Hence, the inequalities (53) provide a design principle for design parameters in the controller.]

Remark 5: As shown in the proof of theorem 1, the control system can be stabilized by using an output feedback gain k^* which satisfies the inequality (47). This means that the resulting control system can be stabilized by an output feedback with sufficiently large feedback gain. This is the reason why we call the method `high gain output feedback control'. In the proposed method, since the controlled system is unknown, we can not determine the lower bound of the feedback gain and then we can not choose the ideal static feedback gain. Therefore, we adaptively determine the feedback gain by the adaptive adjusting law (15).

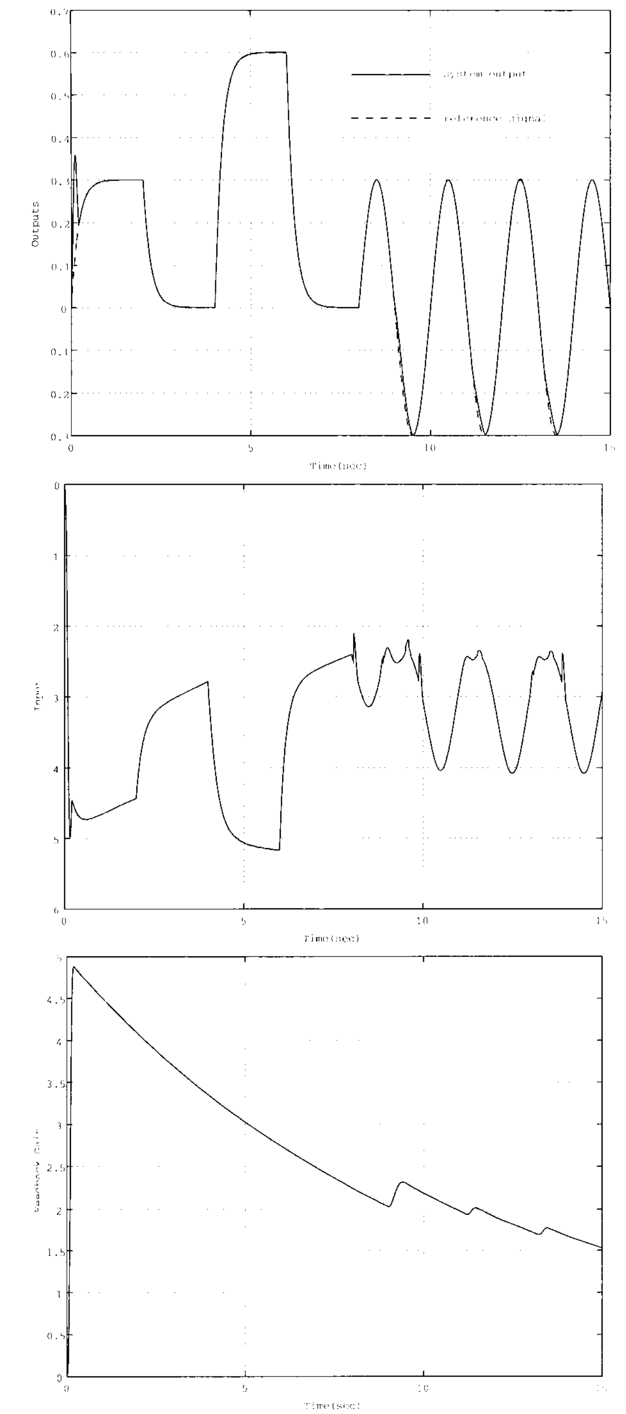

Fig. 1. Control result by the proposed control system.

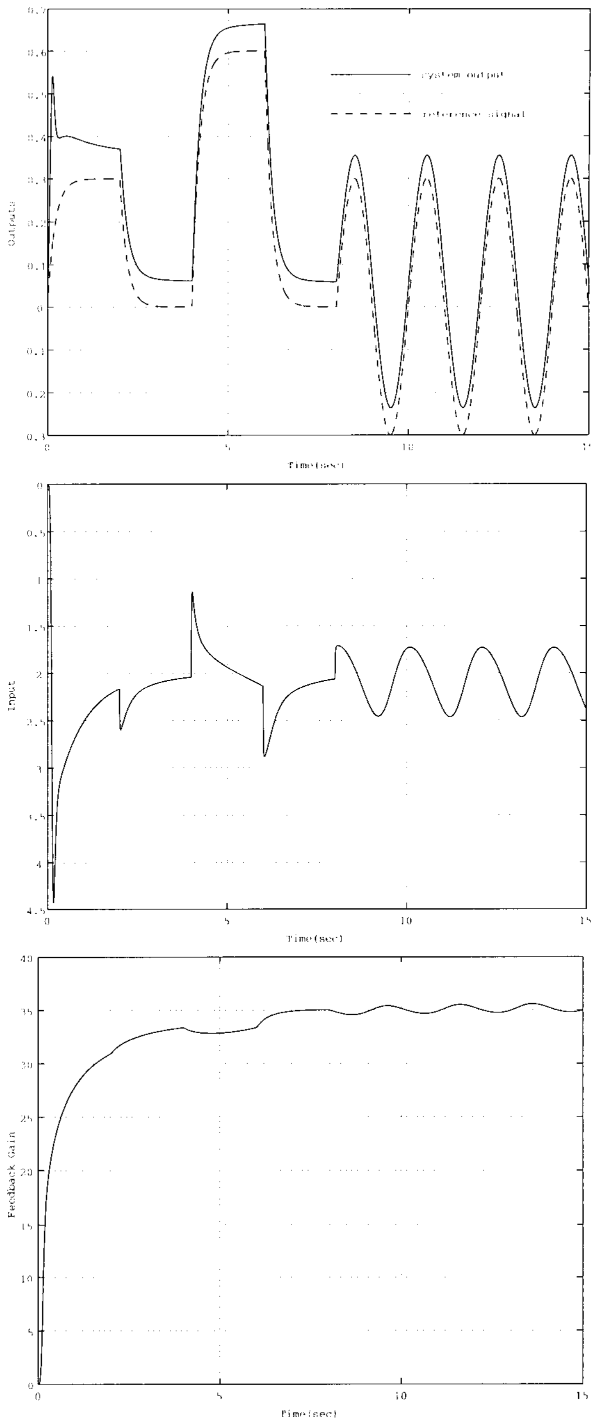

Fig. 2. Control result by the proposed control system without robust adaptive control term for f_1, f_2.

4. NUMERICAL SIMULATIONS

Here the effectiveness of the proposed control scheme will be confirmed through numerical simulations.

Consider the following SISO affine nonlinear system:

$$\begin{aligned} \dot{y} &= a(y,\boldsymbol{\eta}) + b(y,\boldsymbol{\eta})u + f_1 \, , \\ \dot{\boldsymbol{\eta}} &= \boldsymbol{q}(y,\eta) + \boldsymbol{f}_2 \, , \end{aligned} \tag{54}$$

$$a(y,\boldsymbol{\eta}) = y + \sin\eta_2 + \cos\eta_2,$$

$$b(y,\boldsymbol{\eta}) = \exp(y + \eta_1),$$

$$q(y,\boldsymbol{\eta}) = \begin{bmatrix} -\eta_1 + \eta_3 \sin y \\ -\eta_2 - y \\ -\eta_3 + \eta_2 \sin y \end{bmatrix},$$

$$f_1 = 3\exp y + \sin\eta_1 \cos^2 \eta_3,$$

$$\boldsymbol{f}_2 = \left[\cos^2 y, \sin\eta_1 \cos y, \ -y^3 \cos\eta_2\right]^T.$$

The controlled system given in (54) has a relative degree of one and is exponentially minimum phase. In this simulation, it is supposed that we have a priori information about the controlled system such that nonlinearities $a(y,\boldsymbol{\eta})$ and $\boldsymbol{q}(y,\boldsymbol{\eta})$ are Lipshitz in $(y,\boldsymbol{\eta})$ and that nonlinear functions f_1 and f_2 are not Lipshitz in $\boldsymbol{q}(y,\boldsymbol{\eta})$ but can be evaluated by

$$|f_1| \le d_1 \exp(y) + d_0, \tag{55}$$

$$\|f_2\| \le g_1 |y|^3 + g_0. \tag{56}$$

It is also assumed that nonlinearity in the control input term $b(y,\boldsymbol{\eta})$ is unknown. Note that $b(y,\boldsymbol{\eta}) = \exp(y+\eta_1)$ is an unbounded nonlinear function with respect to $(y,\boldsymbol{\eta})$.

We consider the following desired trajectory that the output $y(t)$ is required to follow.

$$y^*(t) = \begin{cases} \dfrac{1}{0.2\delta+1}[r] & \text{for } 0 \le t < 8 \\ r(t) & \text{for } 8 \le t \end{cases}$$
$$r(t) = \begin{cases} 0.3 & 0 \le t < 2 \\ 0 & 2 \le t < 4 \\ 0.6 & 4 \le t < 6 \\ 0 & 6 \le t < 8 \\ 0.3\sin(\pi(t-8)) & 8 \le t < 15. \end{cases} \tag{57}$$

The design parameters of the controller are set as follows:

$$\gamma_I = 100, \ \gamma_p = 100, \ \gamma_d = 10, \quad \sigma_I = 0.1, \ \sigma_d = 0.1, \ \varepsilon_f = 1e^{-3}. \tag{58}$$

Fig.1 shows the simulation results of the proposed method. To illustrate the effectiveness of the proposed method, the simulation results for the controller without robust control input term k_p and μ_{fi} against f_1 and f_2, i.e., a controller with only high gain adaptive output feedback, are shown in Fig.2. It is clear that performance of the proposed control system is better than that of the conventional high gain output adaptive control system.

5. CONCLUSIONS

In this paper, we proposed a robust high-gain adaptive output feedback control for a class of nonlinear systems with uncertain nonlinearities. It was shown that using the robust high gain adaptive output feedback control method, one can design a stable adaptive output feedback control system even if the controlled system has uncertainty in the control input term. It was also confirmed that the appropriate choice of design parameters ensures the tracking error be small, i.e., that the tracking error be less than δ for any given $\delta > 0$. The effectiveness of the proposed method was borne out through the numerical simulations.

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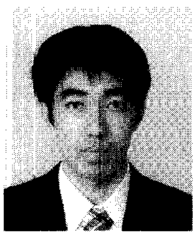

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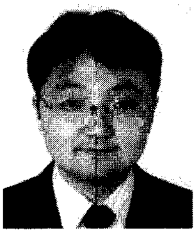

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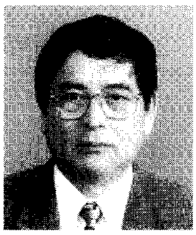

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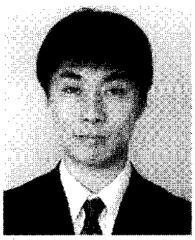

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