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NECESSARY AND SUFFICIENT CONDITIONS FOR CONVERGENCE OF ISHIKAWA ITERATIVE SCHEMES WITH ERRORS TO ϕ-HEMICONTRACTIVE MAPPINGS

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Abstract. The purpose of this paper is to establish the necessary and sufficient conditions which ensure the strong convergence of the Ishikawa iterative schemes with errors to the unique fixed point of a ϕ-hemicontractive mapping defined on a nonempty convex subset of a normed linear space. The results of this paper extend substantially most of the recent results.

1. Introduction

Let X be a normed linear space, X^* its dual space and $J : X \to 2^{X^*}$ the normalized duality mapping defined by

$$J(x) = \{f \in X^* \ : \ Re\langle x, f\rangle = \|x\|\|f\|, \quad \|x\| = \|f\|\}, \quad x \in X,$$

where $\langle\cdot,\cdot\rangle$ denotes the generalized duality pairing. The symbols $D(T)$, $R(T)$ and $F(T)$ stand for the domain, the range and the set of fixed points of T, respectively. Let us recall the following concepts due to Chidume [4], Chidume-Osilike [9], Osilike [17], Mann [16], Ishikawa [12] and Xu [21], respectively.

Definition 1.1. Let $T : D(T) \subseteq X \to X$ be an operator.

(i) T is said to be *strongly pseudocontractive* if there exists $t > 1$ such that for each $x, y \in D(T)$, there exists $j(x-y) \in J(x-y)$

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satisfying

$$Re\langle Tx - Ty, j(x-y)\rangle \leq \frac{1}{t}\|x-y\|^2;$$

(ii) T is said to be *strictly hemicontractive* if $F(T) \neq \emptyset$ and if there exists $t > 1$ such that for each $x \in D(T)$ and $q \in F(T)$, there exists $j(x-y) \in J(x-y)$ satisfying

$$Re\langle Tx - q, j(x-q)\rangle \leq \frac{1}{t}\|x-q\|^2;$$

(iii) T is said to be ϕ-strongly *pseudocontractive* if there exists a strictly increasing function $\phi : [0,\infty) \to [0,\infty)$ with $\phi(0) = 0$ such that for each $x, y \in D(T)$, there exists $j(x-y) \in J(x-y)$ satisfying

$$Re\langle Tx - Ty, j(x-y)\rangle \leq \|x-y\|^2 - \phi(\|x-y\|)\|x-y\|;$$

(iv) T is said to be ϕ*-hemicontractive* if $F(T) \neq \emptyset$ and if there exists a strictly increasing function $\phi : [0,\infty) \to [0,\infty)$ with $\phi(0) = 0$ such that for each $x \in D(T)$ and $q \in F(T)$, there exists $j(x-y) \in J(x-y)$ satisfying

$$Re\langle Tx - q, j(x-q)\rangle \leq \|x-q\|^2 - \phi(\|x-q\|)\|x-q\|.$$

Clearly, each strictly hemicontractive operator is ϕ-hemicontractive. It was shown in [9, 17] that the classes of strongly pseudocontractive (ϕ-strongly pseudocontractive, resp.) operators with fixed points are proper subclasses of the classes of strictly hemicontractive (ϕ-hemicontractive, resp.) operators.

DEFINITION 1.2. Let K be a nonempty convex subset of X and let $T : K \to K$ be an operator.

(i) for any given $x_0 \in K$ the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$\begin{cases} x_{n+1} = (1-a_n)x_n + a_nTy_n, \\ y_n = (1-b_n)x_n + b_nTx_n, \qquad n \geq 0, \end{cases}$$

is called the *Ishikawa iterative sequence*, where $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ are real sequences in [0,1] satisfying appropriate conditions.

(ii) if $b_n = 0$ for all $n \geq 0$ in (i), then the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$\begin{cases} x_0 \in K, \\ x_{n+1} = (1 - a_n)x_n + a_n T x_n, \qquad n \geq 0, \end{cases}$$

is called the *Mann iterative sequence.*

(iii) For any given $x_0 \in K$ the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$\begin{cases} x_{n+1} = a_n x_n + b_n T y_n + c_n u_n, \\ y_n = a'_n x_n + b'_n T x_n + c'_n v_n, \qquad n \geq 0, \end{cases}$$

where $\{u_n\}_{n=0}^{\infty}$ and $\{v_n\}_{n=0}^{\infty}$ are arbitrary bounded sequences in K and $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$ and $\{c'_n\}_{n=0}^{\infty}$ are real sequences in [0, 1] such that $a_n + b_n + c_n = a'_n + b'_n + c'_n = 1$ for all $n \geq 0$ is called the *Ishikawa iterative sequence with errors.*

(iv) If $b'_n = c'_n = 0$ for all $n \geq 0$ in (iii), then the sequence $\{x_n\}_{n=0}^{\infty}$ now defined by

$$\begin{cases} x_0 \in K, \\ x_{n+1} = a_n x_n + b_n T x_n + c_n u_n, \qquad n \geq 0, \end{cases}$$

is called the *Mann iterative sequence with errors.*

It is clear that the Mann and Ishikawa iterative sequences are all special cases of the Ishikawa iterative sequences with errors.

Chidume [3] established that the Mann iterative scheme can be used to approximate the unique fixed point of a Lipshitz strongly pseudocontractive operator $T : X \to X$, where K is a nonempty bounded closed convex subset of a L_p (or l_p) space with $p \geq 2$. The result of Chidume have been generalized and extended in several directions by many authors (see, [1, 2, 4–11, 14, 15, 17–21]).

The aim of this paper is to characterize conditions for the convergence of the Ishikawa iterative schemes with errors to the unique fixed point of a ϕ-hemicontractive mapping in a nonempty convex subset of a normed linear space. Our results improve and generalize most results in recent literature.

The following result plays an important role in proving our main results.

LEMMA 1.1. ([21]) *Let X be a normed linear space. Then for all $x, y \in X$ and $j(x+y) \in J(x-y)$,*

$$\|x + y\|^2 \leq \|x\|^2 + 2Re\langle y, j(x+y)\rangle.$$

2. Main results

THEOREM 2.1. *Let K be a nonempty convex subset of a normed linear space X and let $T : K \to K$ be a uniformly continuous and ϕ-hemicontractive mapping. Suppose that $\{u_n\}_{n=0}^{\infty}$ and $\{v_n\}_{n=0}^{\infty}$ are bounded sequences in K and $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$, $\{c'_n\}_{n=0}^{\infty}$ and $\{r_n\}_{n=0}^{\infty}$ are sequences in $[0,1]$ satisfying*

$$a_n + b_n + c_n = a'_n + b'_n + c'_n = 0, \qquad n \geq 0; \tag{2.1}$$

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} r_n = \lim_{n\to\infty} b'_n = \lim_{n\to\infty} c'_n = 0; \tag{2.2}$$

$$c_n(1 - r_n) = r_n b_n, \qquad n \geq 0; \tag{2.3}$$

$$\sum_{n=0}^{\infty} b_n = \infty. \tag{2.4}$$

For any $x_0 \in K$, define $\{x_n\}_{n=0}^{\infty}$ inductively as follows:

$$\begin{cases} y_n = a'_n x_n + b'_n T x_n + c'_n v_n, \\ x_{n+1} = a_n x_n + b_n T y_n + c_n u_n, \qquad n \geq 0. \end{cases} \tag{2.5}$$

Then the following conditions are equivalent:

(i) *$\{x_n\}_{n=0}^{\infty}$ converges strongly to the unique fixed point q of T;*
(ii) *$\lim_{n\to\infty} T y_n = q$;*
(iii) *$\{T y_n\}_{n=0}^{\infty}$ is bounded.*

PROOF. Set $d_n = b_n + c_n$ and $d'_n = b'_n + c'_n$ for each $n \geq 0$. Since T is ϕ-hemicontractive, it follows that $F(T)$ is a singleton. Let $F(T) = \{q\}$ for some $q \in K$.

Suppose that $\lim_{n\to\infty} x_n = q$. Then (2.1), (2.2), (2.5) and the uniform continuity of T yield that

$$\lim_{n\to\infty} y_n = \lim_{n\to\infty} [(1 - d'_n)x_n + b'_n T x_n + c'_n v_n] = q,$$

which implies that $\lim_{n\to\infty} T y_n = q$. Therefore $\{T y_n\}_{n=0}^{\infty}$ is bounded.

Now suppose that $\{T y_n\}_{n=0}^{\infty}$ is bounded. Put $A = \|x_0 - q\| + \sup\{\|T y_n - q\| : n \geq 0\} + \sup\{\|u_n - T y_n\| : n \geq 0\}$. Then A is bounded by the boundedness of $\{u_n\}_{n=0}^{\infty}$. By induction on $n \geq 0$, we show that

$$\|x_n - q\| \leq A, \qquad n \geq 0. \tag{2.6}$$

Clearly (2.6) is true for $n = 0$. Suppose that (2.6) holds for some $n \geq 0$. From (2.3) and (2.5), we have

$$\begin{aligned}
\|x_{n+1} - q\| &\leq (1 - d_n)\|x_n - q\| + d_n\|Ty_n - q\| + c_n\|u_n - Ty_n\| \\
&\leq (1 - d_n)A + d_n\|Ty_n - q\| + r_n d_n\|u_n - Ty_n\| \\
&\leq (1 - d_n)\|x_0 - q\| + (1 - d_n)\sup\{\|Ty_n - q\| : n \geq 0\} \\
&\leq +d_n\|Ty_n - q\| + (1 - d_n)\sup\{\|u_n - Ty_n\| : n \geq 0\} \\
&\quad + d_n\|u_n - Ty_n\| \\
&\leq A.
\end{aligned}$$

That is, (2.6) holds for all $n \geq 0$. Since T is uniformly continuous and $\{\|x_n - q\|\}_{n=0}^{\infty}$ is bounded, $\{\|Tx_n - q\|\}_{n=0}^{\infty}$ and $\{\|y_n - q\|\}_{n=0}^{\infty}$ are also bounded. Thus there is a constant $D > 0$ satisfying

$$\begin{aligned}
\sup\{&\|x_n - q\|, \|Tx_n - q\|, \|y_n - q\|, \|Ty_n - q\|, \\
&\|u_n\|, \|v_n\| : n \geq 0\} \leq D.
\end{aligned} \tag{2.7}$$

Let $s_n = \|Ty_n - Tx_{n+1}\|$ for each $n \geq 0$. The uniform continuity of T ensures that

$$\lim_{n\to\infty} s_n = 0, \tag{2.8}$$

because

$$\begin{aligned}
\|y_n - x_{n+1}\| &\leq d_n\|x_n - Ty_n\| + d'_n\|x_n - Tx_n\| + c_n\|u_n - Ty_n\| \\
&\quad + c'_n\|v_n - Tx_n\| \\
&\leq 2D(d_n + d'_n + c_n + c'_n) \\
&\to 0
\end{aligned}$$

as $n \to \infty$. By virtue of Lemma 1.1, (2.1), (2.5) and (2.7), we infer that

$$\begin{aligned}
\|x_{n+1} - q\|^2 &= \|(1 - d_n)(x_n - q) + d_n(Ty_n - q) + c_n(u_n - Ty_n)\|^2 \\
&\leq (1 - d_n)^2\|x_n - q\|^2 \\
&\quad + 2d_n Re\langle Ty_n - q, j(x_{n+1} - q)\rangle + 4Dc_n \\
&\leq (1 - d_n)^2\|x_n - q\|^2 \\
&\quad + 2d_n Re\langle Ty_n - Tx_{n+1}, j(x_{n+1} - q)\rangle + 2d_n\|x_{n+1} - q\|^2 \\
&\quad - 2d_n\phi(\|x_{n+1} - q\|)\|x_{n+1} - q\| + 4Dc_n \\
&\leq (1 - d_n)^2\|x_n - q\|^2 + 2Dd_n s_n + 2d_n\|x_{n+1} - q\|^2 \\
&\quad - 2d_n\phi(\|x_{n+1} - q\|)\|x_{n+1} - q\| + 4Dc_n
\end{aligned}$$

for any $n \geq 0$. (2.2) and (2.3) mean that there exists a positive integer N_0 such that $d_n < \frac{1}{2}$ for each $n \geq N_0$. It follows from (2.2), (2.3) and (2.7)-(2.9) that for each $n \geq N_0$,

$$\begin{aligned}
\|x_{n+1} - q\|^2 &\leq \frac{(1-d_n)^2}{1-2d_n}\|x_n - q\|^2 + \frac{2Dd_ns_n + 4Dc_n}{1-2d_n} \\
&\quad - \frac{2d_n}{1-2d_n}\phi(\|x_{n+1} - q\|)\|x_{n+1} - q\| \\
&\leq \|x_n - q\|^2 + \frac{d_n(D^2d_n + 2Ds_n + 4Dr_n)}{1-2d_n} \\
&\quad - \frac{2d_n}{1-2d_n}\phi(\|x_{n+1} - q\|)\|x_{n+1} - q\| \\
&\leq \|x_n - q\|^2 + \frac{d_nt_n}{1-2d_n} \\
&\quad - \frac{2d_n}{1-2d_n}\phi(\|x_{n+1} - q\|)\|x_{n+1} - q\|,
\end{aligned} \tag{2.10}$$

where

$$t_n = D^2d_n + 2Ds_n + 4Dr_n \to 0 \tag{2.11}$$

as $n \to \infty$. Let $r = \inf\{\|x_{n+1} - q\| : n \geq 0\}$. We claim that $r = 0$. Otherwise $r > 0$. Thus (2.11) implies that there exists a positive integer $N_1 > N_0$ such that $t_n < \phi(r)r$ for each $n \geq N_1$. In view of (2.10), we conclude that

$$\|x_{n+1} - q\|^2 \leq \|x_n - q\|^2 - \phi(r)r\frac{d_n}{1-2d_n}, \qquad n \geq N_1,$$

which implies that

$$\phi(r)r\sum_{n=N_1}^{\infty} d_n \leq \|x_{N_1} - q\|^2,$$

which contradicts to (2.4). Therefore $r = 0$. Thus there exists a subsequence $\{x_{n_i+1}\}_{i=0}^{\infty}$ of $\{x_n\}_{n=0}^{\infty}$ such that

$$\lim_{i\to\infty} x_{n_i+1} = q. \tag{2.12}$$

Let $\varepsilon > 0$ be a fixed number. By virtue of (2.11) and (2.12), we can select a positive integer $i_0 > N_1$ such that

$$\|x_{n_{i_0}+1} - q\| < \varepsilon, \quad t_n < \phi(\varepsilon)\varepsilon, \qquad n \geq n_{i_0}. \tag{2.13}$$

Let $p = n_{i_0}$. By induction, we show that

$$\|x_{p+m} - q\| < \varepsilon, \qquad m \geq 1. \tag{2.14}$$

Observe that (2.13) means that (2.14) is true for $m = 1$. Suppose that (2.14) is true for some $m \geq 1$. If $\|x_{p+m+1} - q\| \geq \varepsilon$, by (2.10) and (2.13) we know that

$$\begin{aligned}
\varepsilon^2 &\leq \|x_{p+m+1} - q\|^2 \\
&\leq \|x_{p+m+1} - q\|^2 + \frac{d_{p+m}t_{p+m}}{1 - 2d_{p+m}} \\
&\quad - \frac{2d_{p+m}}{1 - 2d_{p+m}}\phi(\|x_{p+m+1} - q\|)\|x_{p+m+1} - q\| \\
&< \varepsilon^2 + \frac{d_{p+m}\phi(\varepsilon)\varepsilon}{1 - 2d_{p+m}} - \frac{2d_{p+m}\phi(\varepsilon)\varepsilon}{1 - 2d_{p+m}} \\
&< \varepsilon^2,
\end{aligned}$$

which is impossible. Hence $\|x_{p+m+1} - \varepsilon\| < \varepsilon$. That is, (2.14) holds for all $m \geq 1$. Thus (2.14) ensures that $\lim_{n\to\infty} x_n = q$. This completes the proof. □

REMARK 2.1. Theorem 2.1 extends, improves and unifies Theorem 3.4 in [1], Theorem 3.4 in [2], Theorem in [3], Theorem 2 in [4], Theorem 2 in [5], Theorem 4 in [6], Theorem 4 and Theorem 13 in [7], Theorem 1 in [8], Theorem 2 in [9], Theorem 4 in [10], Theorem 1 in [11], Theorem 1 in [15], Theorem 2 in [18] and Theorem 4 in [19] in the following directions:

(a) The Mann iterative schemes in [3, 4, 10, 15] and the Ishikawa iterative schemes in [1, 2, 5-7, 9, 11, 18, 19] are replaced by the more general Ishikawa iterative schemes with errors;

(b) The Lipschitz strongly pseudocontractive mappings in [3-7, 9, 10, 15, 18, 19] and the uniformly continuous and strongly pseudocontractive mappings in [1, 2, 7, 8, 11] are replaced by the more general uniformly continuous and ϕ-hemicontractive mappings;

(c) Theorem 2.1 holds in arbitrary normed linear spaces whereas the results of [1-12, 15, 18, 19] have been proved in the restricted in L_p (or l_p) spaces, q-uniformly smooth Banach spaces, real uniformly smooth Banach spaces, real smooth Banach spaces, real Banach spaces of type (U, λ, $m+1$, m) and Banach spaces;

(d) The assumptions $\alpha_n \leq \beta_n$ in [5, 7] and $\sum_{n=0}^{\infty} \alpha_n b(\alpha_n) < \infty$ in [5] are superfluous;

(e) The boundedness requirement imposed on $\{Ty_n\}_{n=0}^{\infty}$ in Theorem 2.1 is weaker than the boundedness assumption of subsets K in [1-11, 15, 19].

REMARK 2.2. The following example proves that Theorem 2.1 extends substantially the corresponding results in [1-11, 15, 18, 19].

EXAMPLE 2.1. Let $X = (-\infty, \infty)$ with the usual norm and let $K = [0, \infty)$. Define $T : K \to K$ by $Tx = \frac{x}{1+2x}$ for all $x \in K$. Then $F(T) = \{0\}$, $R(T) = [0, \frac{1}{2})$ and

$$\begin{aligned} \|Tx - Ty\| &= \left\| \frac{x-y}{(1+2x)(1+2y)} \right\| \\ &\leq \|x - y\|, \qquad x, y \in K. \end{aligned}$$

Define $\phi : [0, \infty) \to [0, \infty)$ by $\phi(t) = \frac{t^2}{1+2t}$ for any $t \in [0, \infty)$. Clearly $\phi(0) = 0$, $\phi(t)$ is strictly increasing in $[0, \infty)$ and

$$\begin{aligned} \langle Tx, j(x) \rangle &= \left\| \frac{x^2}{1+2x} \right\| \\ &\leq \left\| \frac{x^2 + x^3}{1+2x} \right\| \\ &= \|x\|^2 - \phi(\|x\|)\|x\|, \qquad x \in K. \end{aligned}$$

Hence T is ϕ-hemicontractive. Observe that for given $t > 1$, there exists $x = \frac{t-1}{3} \in K$ such that

$$\langle Tx, j(x) \rangle > \frac{1}{t} \|x\|^2.$$

Therefore T is not strictly hemicontractive. Set

$$a_n = 1 - (1+n)^{-\frac{1}{2}} - (1+n)^{-1}, \quad b_n = (1+n)^{-\frac{1}{2}}, \quad c_n = (1+n)^{-1},$$
$$a'_n = 1 - (1+n)^{-1}, \quad b'_n = c'_n = (2+2n)^{-1}, \quad r_n = [1 + (1+n)^{\frac{1}{2}}]^{-1}$$

for each $n \geq 0$. Then the conditions of Theorem 2.1 are satisfied. But Theorem 3.4 in [1], Theorem 3.4 in [2], Theorem in [3], Theorem 2 in [4], Theorem 2 in [5], Theorem 4 in [6], Theorem 4 and Theorem 13 in [7], Theorem 1 in [8], Theorem 2 in [9], Theorem 4 in [10], Theorem 1 in [11], Theorem 1 in [15], Theorem 2 in [18] and Theorem 4 in [19] are not applicable since T is not strongly pseudocontractive.

Using the method of proof in Theorem 2.1, we have the following theorem.

THEOREM 2.2. *Let X, K, T, $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$, $\{x_n\}_{n=0}^{\infty}$ and $\{y_n\}_{n=0}^{\infty}$ be as in Theorem 2.1. Suppose that $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$ and $\{c'_n\}_{n=0}^{\infty}$ are sequences in $[0,1]$ satisfying (2.1), (2.4) and*

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} b'_n = \lim_{n\to\infty} c'_n = 0; \tag{2.15}$$

$$\sum_{n=0}^{\infty} c_n < \infty. \tag{2.16}$$

Then the conclusion of Theorem 2.1 holds.

REMARK 2.3. Theorem 2.2 generalizes Theorem 1 of Chidume [8] from real Banach spaces to normed linear spaces and from strongly pseudocontractive mappings to ϕ-hemicontractive mappings.

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