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Determination of Critical State Parameters in Sandy Soils from Standard Triaxial Testing (I) : Review and Application

표준삼축시험으로부터 사질토에서의 한계상태정수 결정에 관한 연구 (I) : 고찰 및 적용

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요 지

한계상태 토질역학을 사질토의 전단거동에 대한 적용을 용이하게 하기위해서, 표준삼축시험으로부터 사질토에서의 한계상태정수 결정에 관하여 종합적인 고찰을 실시하였다. 첫째로, 문헌에 있는 어휘론적인 차이점들을 명확히 함으로써, 한계상태는 대변형에서의 궁극적인 최종상태를 나타냄을 추론하였다. 둘째로, 한계상태정수의 특성들에 관하여 검토하였고, 초기상태, 구조, 하중조건 및 배수조건에 따른 한계상태선의 유일성과 유사한계상태조건의 민감성을 검증하였다. 셋째로, 한계상태정수로부터 액상화후 전단강도 즉 액상화된 흙에서의 신뢰할 수 있는 궁극적인 전단강도를 산정하기위하여 한계상태 토질역학을 예제로서 적용하였다.

Abstract

Comprehensive review on the determination of critical state parameters in sandy soils from standard triaxial testing was performed to facilitate the application of critical state soil mechanics to the shear behavior of sandy soils. First, semantic differences in literature were clarified, inferring that critical state should be considered as the ultimate state at large deformation. Second, the characteristics of critical state parameters were discussed, and also the uniqueness of critical state line and the sensitivity of quasi-steady state condition were verified in relation to initial state, fabric, loading condition, and drainage condition. Third, as an example, the critical state soil mechanics was applied to evaluate the post-liquefaction shear strength, i.e. the reliable ultimate shear strength in liquified soils, in terms of critical state parameters.

Keywords : Critical state, Critical state parameters, Friction angle, Post-liquefaction, Sandy soil, Shear strength, Steady state

1. Introduction

Coulomb in the eighteenth century understood that the strength of freshly remolded soils is of frictional nature, hence, stress dependent (Heyman, 1997; Schofield, 1998). Reynolds (1885) highlighted the tendency of granular materials to change volume when sheared, a fact that was well known by grain dealers at the time. Casagrande (1936) recognized that a critical density divides the tendency of volume change into contractive and dilative behaviors. Later, Taylor (1948) showed experimentally that dilatancy is stress-dependent, and Taylor (1948) and Bishop (1950) expressed the shear strength in terms of friction and dilatancy components.

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Finally, Roscoe, Schofield, and Wroth (1958), and Schofield and Wroth (1968) brought together stress-dependent strength and dilatancy in the unifying structure of critical state soil mechanics, within the framework of plasticity theory.

Meanwhile, Casagrande postulated the existence of a "flow structure" after the liquefaction failure at the Ft. Peck dam in September 1938. This structure leads to a minimum frictional resistance and explains the phenomenon of flow liquefaction (Casagrande 1965; 1971; 1975). Later, Castro (1969) showed the existence of the flow structure with dead-load tests, and found that the critical void ratio line from the dead-load tests is different compared with the critical void ratio line from the strain-controlled tests. Finally, Castro (1975), Casagrande (1975), and Poulos (1981; et al., 1985) developed an undrained test-based design procedure for the evaluation of flow liquefaction, which has been called steady state approach.

Since the concept of critical/steady state captures the large-strain behavior of soils in terms of shear stress, effective stress and volume, it has been applied to various engineering designs such as foundations, embankments, landslides, retaining walls and liquefaction. However, its application has been undermined by difficulties, starting with conceptual differences among leading researchers, and including other issues such as the inherent limitations in standard testing devices (e.g. loading path, compliance restrictions, efficiency). This situation has been aggravated by the limited understanding of the underlying physical processes.

Thus, the purpose of this paper is to clarify semantic differences in literature, to review the theory of critical state soil mechanics for sandy soils, to discuss the characteristics and uniqueness of critical state parameters, and to explore the application of critical state soil mechanics to the determination of post-liquefaction shear strength. Meanwhile, in the following paper, underlying physical processes and inherent limitations are identified through experimental tests and a proper procedure is suggested to determine the critical state parameters from standard triaxial testing.

2. Semantic Difference

2.1 Critical State and the Steady State

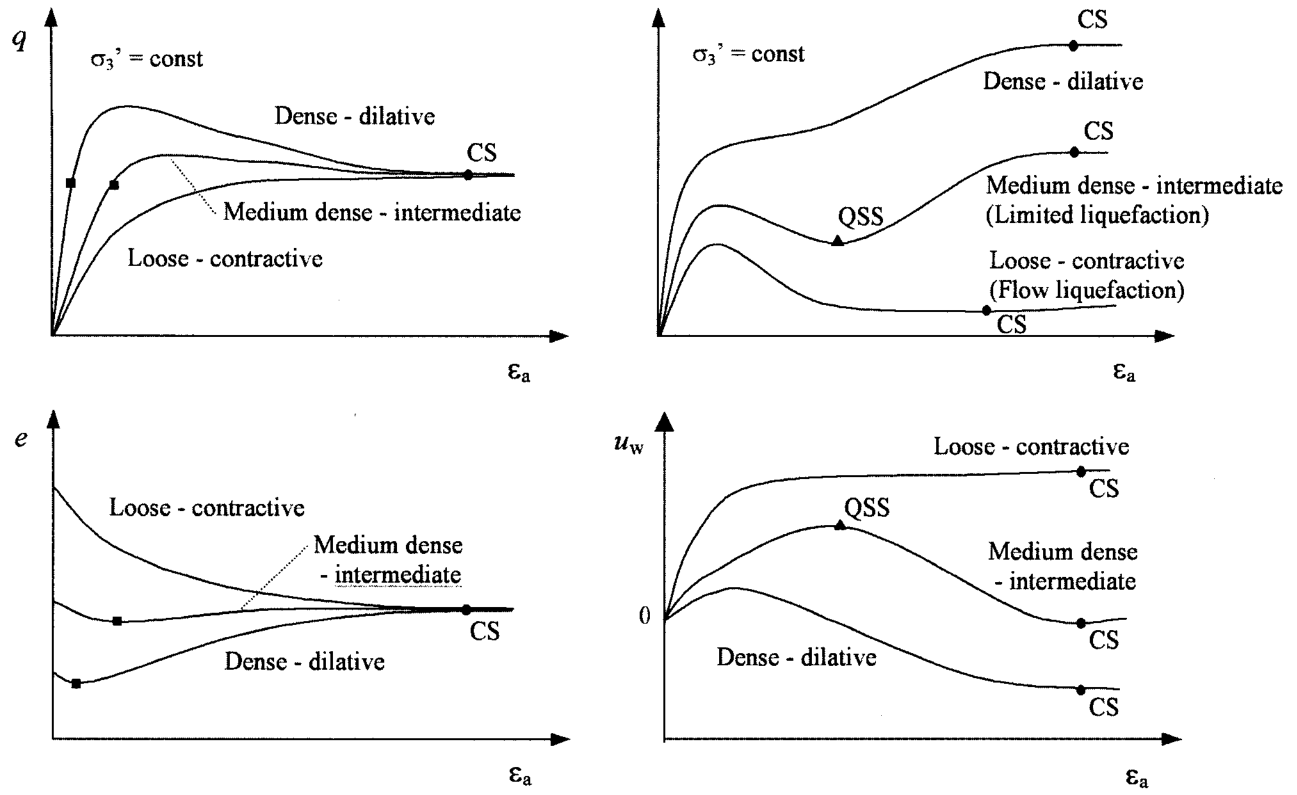
Roscoe et al. (1958) defined the critical state as the state at which the soil continues to deform at constant stress and constant void ratio. This definition follows from the critical density concept in drained tests by Casagrande (1936) and the critical stress concept in undrained tests by Taylor (1948). Critical state can be achieved during shear deformation either by changing volume and effective stress in drained tests or by changing the effective stress at a constant volume in undrained tests. It is hypothesized that a unique line exists in the e-p'-q space for the ultimate states in drained and undrained tests, and this line is termed critical state line (CSL) (Roscoe et al., 1958).

Poulos (1981) defined the steady state as the state at which the soil continues to deform at constant volume, constant normal effective stress, constant shear stress, and constant velocity, after particle breakage ends. The term particle breakage can be generalized to imply the breakage of soil conglomerates within the fabric. The locus of steady states for undrained tests on e-p' space is called steady state line (SSL).

Researchers in critical state (CS) soil behavior have relied on drained, strain-rate-controlled tests on dilatant specimens to determine the critical state line because the critical state in terms of stresses can be achieved at a relatively low global strain level (Been et al., 1991; Lee, 1995). On the other hand, researchers in liquefaction have centered their efforts on loose-contractive specimens tested under undrained conditions to determine the steady state line (Castro, 1969; Poulos, 1981; Poulos et al., 1985; Vaid and Chern, 1985; Alarcon-Guzman et al., 1988; Konrad, 1990b; Ishihara, 1993; Riemer and Seed, 1997).

2.2 General Features of Soil Behavior Under Quasi-static Loading

When a sandy soil is subjected to drained shearing, the volume of the soil changes, and either dilative, intermediate, or contractive behavior can be observed depending



(a) Drained behavior

(b) Undrained behavior

Fig. 1. Drained and undrained responses for a sandy soil subjected to quasi-static loading

Notation: Circle denotes critical, steady, or ultimate state (CS). Triangle denotes minimum, phase transformation, or quasi-steady state (QSS). Square shows the characteristic state.

upon initial void ratios and effective confinements (Fig. 1a). For medium dense and dense specimens, the volume decreases to a minimum transient value prior to the peak strength and then increases approaching critical state. The transient void ratio is called the characteristic state (Luong, 1980). For a loose specimen, the volume decreases continuously towards the critical state value.

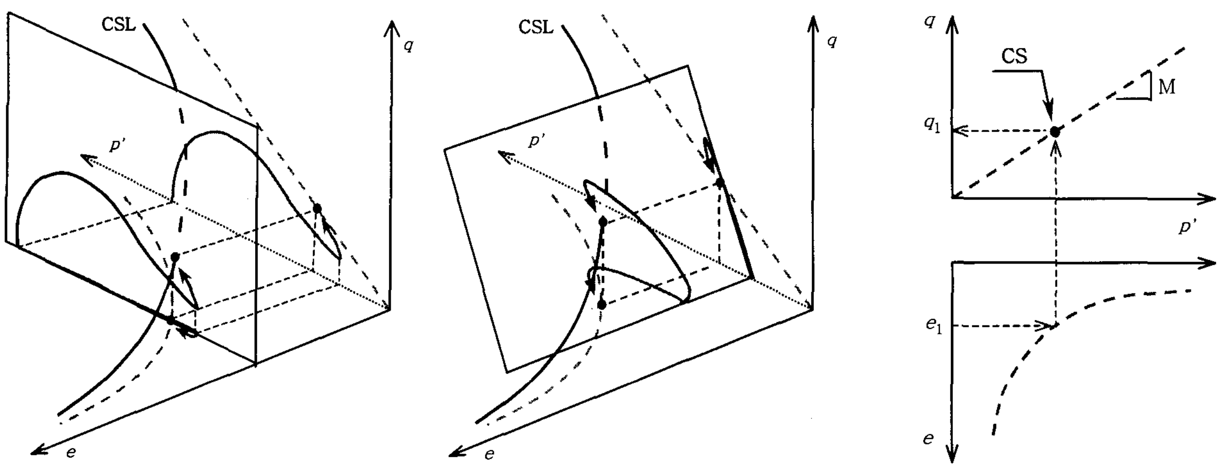
When a sandy soil is subjected to undrained shearing, the pore water pressure and the effective confining stress change at a constant void ratio. Once again, different types of stress-strain response can be observed depending upon the initial void ratio (Fig. 1b). Loose specimens with contractive tendency display post-peak behavior and suffer from flow liquefaction due to low ultimate strength. Medium dense specimens experience a state of minimum strength called the state of phase transformation by Ishihara et al. (1975) or quasi-steady state (QSS) by Alarcon-Guzman et al. (1988).

The response of the soil can be captured in the 3-D space, in terms of void ratio (e), mean principal stress (p'), and deviator stress (q) (Fig. 2a and b). Here, p' is $(\sigma_1' + 2\sigma_3')/3$ and q is $(\sigma_1 - \sigma_3)$. For convenience, two 2-D projections of the 3-D space are commonly used: the p'-q space and the e-p' space (Fig. 2c). The projection of critical state line on p'-q space corresponds to the Coulomb strength criterion. In fact, the strength parameter for axisymmetric, axial compression tests is:

$$\mathrm{M} = \left(\frac{q}{p'}\right)_{cs} = \frac{6\sin\phi_{cs}}{3-\sin\phi_{cs}} \tag{1}$$

where ϕ_{cs} is the Coulomb's "friction angle" or the angle of internal shear strength at critical state. On the other hand, the projection of the critical state line on the e-log p' space is expressed in terms of the intercept Γ and the slope λ as follows:

$$e_{cs} = \Gamma - \lambda \log\left(\frac{p'_{cs}}{1kPa}\right) \tag{2}$$



(a) Undrained response in e-p'-q space (b) Drained response in e-p'-q space (c) Projections of CSL on two 2-D spaces

Fig. 2. General feature of critical state line

where e_{cs}, p'_{cs} and q_{cs} are the void ratio, the mean principal stress, and the deviator stress at critical state, respectively. Thus, the critical state of a soil can be represented by the three parameters M, λ and Γ.

2.3 Strain Level and the Identification of the Critical State

While relatively low strains are needed to alter the network of interparticle forces ("elastic threshold strain," γ_{el}), a micro-scale conceptualization of the problem suggests that strains in excess of 100% are needed so that particles have high probability of exchanging neighbors to attain a unique fabric condition that corresponds to critical state (Santamarina et al., 2001). Direct shear test data for Ottawa standard sand (specimen thickness t = 10.4 mm) obtained by Taylor (1948) showed that the deformation required to reach critical state is about δ = 5.1 mm. Thus, the average strain level required to reach critical state is $\gamma_{cs} \approx \delta/t \approx 50\%$. However, if strain localization is assumed in a region of thickness, $t^* \approx 10\ D_{50}$, then the required "local" strain level is much greater, in fact, it exceeds $\gamma \approx 100\%$. Such strain levels are not achievable in standard triaxial testing. Indeed, drained tests on loose contractive specimens clearly show that critical state is not reached at the standard 20% strain limit.

On the other hand, Poulos et al. (1988) insisted that only uniform clean sands and highly contractive specimens in undrained tests approach the steady state in the laboratory. Been et al. (1991) mentioned that "even at 20% strain in triaxial tests where a continuous steady state is reached, it is not known whether this is a true ultimate state or whether further changes would have occurred at larger strains." Nonetheless, some researchers working on flow liquefaction phenomena consider the minimum strength or the quasi-steady state at low strain level as critical state for conservative analysis (Fig. 1). As indicated by Bishop (1971), Chu (1995), Verdugo and Ishihara (1996), and Been (1999; et al., 1991), the ultimate state at large deformations should be considered as the critical state.

3. Characteristics of Critical State Parameters

3.1 Friction Angle at Critical State

The internal friction angle reflects the contributions of interparticle friction, dilatancy (i.e., packing effect), rotation (i.e., rolling and frustration), fabric rearrangement, and particle crushing (Bishop, 1950; Lee and Seed, 1967; Bolton, 1986; Santamarina et al., 2001). The angle of peak friction is related to the critical state friction angle ϕ_{cs} and the peak angle of dilation ψ (Bolton 1986):

$$\phi_{peak} = \phi_{cs} + 0.8\,\psi \tag{3}$$

The friction angle at critical state ϕ_{cs} (i.e., constant volume, statistically stable fabric) is greatly affected by the interparticle friction angle ϕ_{μ}, which depends on the microscale features of particles such as roundness and sphericity, and the ability of particles to rotate. The following relations have been theoretically suggested:

$$\tan\phi_{cs} = \frac{1}{2}\pi\tan\phi_{\mu}$$

(Caquot, 1934; from Horne, 1964) (4)

$$\sin\phi_{cs} = \frac{15\tan\phi_{\mu}}{10+3\tan\phi_{\mu}} \quad \text{(Bishop, 1954)} \tag{5}$$

For a given soil, the critical state friction angle is unique irrespective of drainage condition, initial density, confining stress, and strain rate (Cornforth, 1973; Frossard, 1979; Vaid and Chern, 1985; Negussey et al., 1988; Verdugo and Ishihara, 1996).

An alternative relation is developed herein, starting from energy considerations. The energy-based analysis proposed by Taylor (1948) and Bishop (1950; 1954) supports the Mohr-Coulomb criterion. For axisymmetric compression tests (Feda, 1982):

$$\frac{\sigma'_1}{\sigma'_3} = \frac{1-\dot{\varepsilon}_v/\dot{\varepsilon}_a}{1-\sin\phi_{cs}} + \frac{\sin\phi_{cs}}{1-\sin\phi_{cs}} \quad \text{at any stress ratio} \tag{6a}$$

then,

$$\left(\frac{\sigma'_1}{\sigma'_3}\right)_{cs} = \tan^2\left(\frac{\pi}{4}+\frac{\phi_{cs}}{2}\right) \quad \text{at critical state} \quad \dot{\varepsilon}_v \cong 0 \tag{6b}$$

where $\dot{\varepsilon}_v$ and σ'_1 are the increments in volumetric and axial strains, and $(\sigma'_1)_{cs}$ and $(\sigma'_3)_{cs}$ are the effective axial and confining stresses at critical state, respectively. Meanwhile, the evolution of the state of stress is related to the evolution of volume and the mobilization of interparticle friction. Based on the minimum energy criterion, Rowe (1962; 1963) suggested a stress dilatancy relation for axisymmetric compression tests,

$$\frac{\sigma'_1}{\sigma'_3} = (1-\dot{\varepsilon}_v/\dot{\varepsilon}_a)\tan^2\left(\frac{\pi}{4}+\frac{\phi_{\mu}}{2}\right) \tag{7}$$

At small strains, theory of elasticity predicts that the ratio of strain increments $\dot{\varepsilon}_v/\dot{\varepsilon}_a$ is equal to 1-2υ, where υ is the Poisson's ratio. Then, Equation (7) becomes

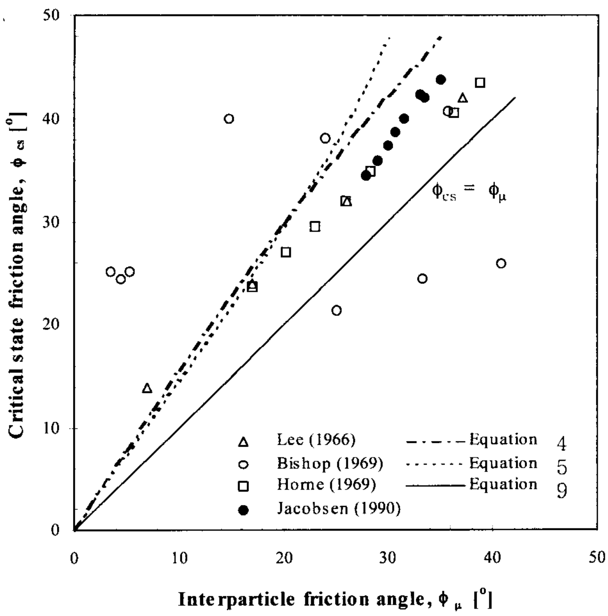


Fig. 3. Experimental and theoretical relations between interparticle and critical state friction angles

$$\frac{\sigma'_1}{\sigma'_3} = 2\upsilon\tan^2\left(\frac{\pi}{4}+\frac{\phi_{\mu}}{2}\right) \tag{8}$$

Equation (6) (i.e. energy dissipation) and Equation (8) (i.e. stress-dilatancy) are in terms of the effective stress ratio. Combining Equations (6b) and (8) at critical state ($\dot{\varepsilon}_v = 0$, $\upsilon = 0.5$), the relation between the critical state friction angle and the interparticle friction angle becomes:

$$\tan^2\left(\frac{\pi}{4}+\frac{\phi_{cs}}{2}\right) = \tan^2\left(\frac{\pi}{4}+\frac{\phi_{\mu}}{2}\right) \tag{9}$$

Fig. 3 shows experimental test results and mathematical relations between interparticle and critical state friction angles.

3.2 Curvature of the Critical State Line

Some experimental test results show a non-linear or a bi-linear projection of the critical state line on e-log p' space. It appears that this break reflects particle crushing and other particle level processes at high confining stresses (Been et al., 1991; Verdugo and Ishihara, 1996; Riemer and Seed, 1997). Contact crushing and the tensile splitting of particles depend on the mineralogy, particle shape, and formation history (Hardin, 1987).

Li and Wang (1998) suggested the use of the e-$(p'/p_a)^{\alpha}$

plot instead, where p_a is the atmospheric pressure and α is the material parameter (for instance, α = 0.7 for Toyoura Sand). However, Been et al. (1991) presented that the linearization on the e-log p' space in the stress range from 10 to 500 kPa is a reasonable approximation for sub-angular or subrounded quartz sands. This observation can be generalized for hard grain sands. In general, liquefaction takes place at depths ranging between 3 to 30 m (i.e. $\sigma' \approx$ 30 to 300 kPa). Then, it is adequate to consider the critical state line projection on the e-log p' space as a straight line.

3.3 Uniqueness of the CS Line and Sensitivity of the QSS Condition

The uniqueness of the critical state line on e-p' space has been argued in relation to the initial state (i.e. void ratio, loading history, over-consolidated ratio OCR, and induced anisotropy), fabric or sample preparation method (i.e. inherent anisotropy), loading condition and stress path, drainage condition, and presence of fines. These effects are discussed next. For completeness, the effect of these parameters on quasi-steady state is also discussed. The potential effects of localization, limited shear strain level, and particle crushing are briefly mentioned here.

3.3.1 Initial State

Table 1 summarizes the effect of the initial state. There is a strong effect of the initial state on the quasi-steady state: as the confining stress increases, the minimum or quasi-steady state strength increases. On the other hand, the critical state line is unique regardless of initial state.

Fig. 4 shows undrained axisymmetric test results, obtained as part of this study, with two medium dense specimens prepared with blasting sand and confined at different initial effective confining stresses (material and test details are discussed in Cho, 2001). There is initial localization in the specimen confined at $(\sigma'_3)_0$ = 320 kPa, but this is a transient effect: as shearing continues, the dilative tendency takes over, the response becomes

Table 1. Effect of initial state on critical state and quasi-steady state lines

Reference	Soil used	Referred state	Test condition	Initial state	Conclusion
Konrad (1990a)	Dune sand	QSS	Stress-controlled undrained on contractive specimen	Various confining stresses	Strong effect. (not unique)
Konrad (1990a)	Dune sand	QSS	Stress-controlled undrained on contractive specimen	Different OCRs and anisotropy conditions	No effect.
Ishihara (1993), Yoshimine et al. (1999)	Toyoura sand	QSS	Undrained on various densities. Strain-controlled test	Various confining stresses	Strong effect. (proportional)
Vaid and Thomas (1995) Vaid and Sivathayalan (1999)	Fraser river sand	QSS	Undrained on loose and medium specimen	Various confining stresses	Strong effect. (proportional)
Negussey and Islam (1994)	Mine tailings	QSS	Strain-controlled undrained on contractive specimen	Different anisotropy conditions	No effect.
Riemer and Seed (1997)	Monterey #0 sand	QSS	Strain-controlled undrained compression and extension on contractive specimen	Various confining stresses	Strong effect. (proportional)
Dobry et al. (1985)	Sand A	CS	Stress-controlled undrained on contractive specimen	Different anisotropy conditions	No effect.
Been et al. (1991)	Erksak sand	CS	Drained on dilative specimen and undrained on contractive specimen	Various void ratios and confining stresses	No effect.
Chu (1995)	Sydney sand	CS	Drained and undrained on contractive specimen	Various void ratios and confining stresses	No effect.
Ishihara (1993), Verdugo and Ishihara (1996), Yoshimine et al. (1999)	Toyoura sand	CS	Undrained on various densities. Strain-controlled test	Various confining stresses	No effect.
Chen and Liao (1999)	Mailiao sand	CS	Undrained on contractive and medium loose specimen.	Different OCRs	No effect.

Notation: CS is the critical state and QSS is the quasi-steady state.

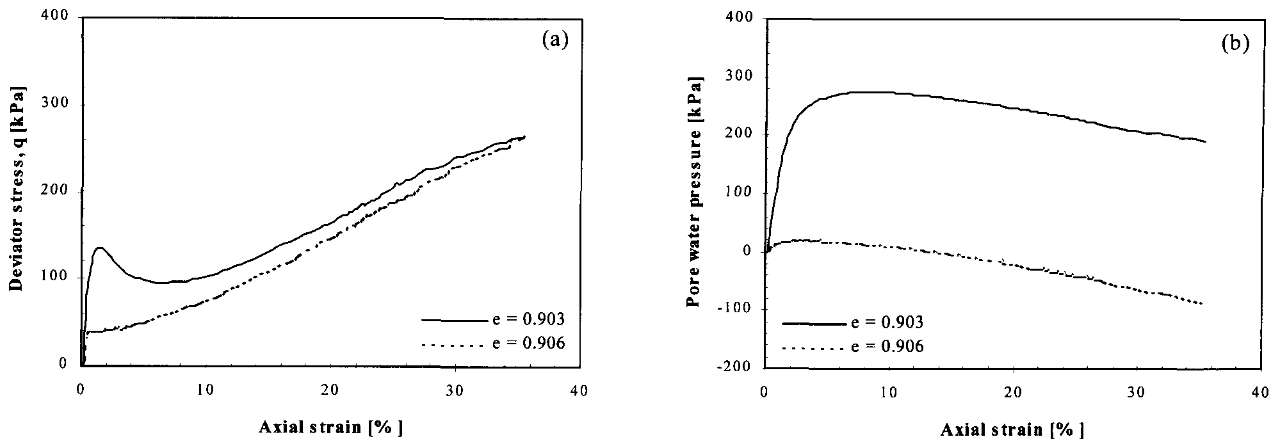


Fig. 4. Undrained triaxial test results for blasting sand at different initial effective confining stresses

strain hardening and the specimen evolves towards homogeneous deformation. Hence, the transient quasi-steady state strength depends on the confining stress, however, the critical state strength is not dependent on the initial state of stress.

3.3.2 Fabric or Sample Preparation Method

Previous studies summarized in Table 2 show that the initial fabric affects the peak strength, and it has a strong effect on the quasi-steady state; however, the critical state is unique irrespective of the initial fabric.

3.3.3 Stress Path

The results from biaxial tests (plane strain condition, $\varepsilon_2 = 0$) for dense specimens render a peak friction angle 2~5 degrees higher than values obtained in triaxial tests (Cornforth, 1973; Feda, 1982); the difference wherein is attributed to boundary effects. Furthermore, no difference is found for critical state friction angles determined with loose specimens (Cornforth, 1973; Feda, 1982).

Table 2. Effect of sample preparation (fabric) on critical state and quasi-steady state lines

Reference	Soil used	Referred state	Test condition	Sample preparation	Conclusion
DeGregorio (1990)	F-70 Ottawa banding sand	QSS	Undrained stress-controlled on loose specimen	Dry pluviation, moist tamping, and moist vibration	Strong effect. (e_{MT} and e_{MV} > e_{DP})
Ishihara (1993)	Toyoura sand	QSS	Undrained strain-controlled on various density	Moist placement and dry deposition	Strong effect. (e_{MP} > e_{DD})
Negussey and Islam (1994)	Mine tailings	QSS	Undrained strain controlled compression and extension	Different bedding orientation by water pluviation	No effect.
Vaid et al. (1999)	Syncrude sand	QSS	Undrained compression.	Water pluviated, air pluviated, and moist tamped	Strong effect. (e_{WP} > e_{AP} > e_{MP})
Poulos et al. (1988)	Syncrude tailings	CS	Undrained strain controlled on various densities	Compacted and deposited as slurry	No effect.
Been et al. (1991)	Erksak sand	CS	Drained on dense specimen and undrained on loose specimen	Air pluviation and moist compaction	No effect.
Ishihara (1993) Verdugo et al. (1995)	Toyoura sand	CS	Undrained strain-controlled on various density	Moist placement and dry deposition	No effect.
Ishihara et al. (1998)	Masado sand	CS	Undrained on loose specimen	Water sedimentation and wet tamping	No effect.
Tsukamoto et al. (1998)	Reclaimed deposits in Kobe	CS	Undrained compression and extension tests on loose and medium dense	Moist tamping and water sedimentation	No effect.

Notation: e_{MT}, e_{MV}, e_{DP}, e_{DD}, e_{WP}, and e_{AP} are state lines from specimens prepared with moist tamping, moist vibration, dry pluviation, dry deposition, wet pluviation, and air pluviation methods respectively.

Table 3. Effect of stress path on critical state and quasi-steady state lines

Reference	Soil used	Referred state	Test condition	Stress path	Conclusion
Vaid et al. (1990)	Ottawa sand	QSS	Undrained on contractive specimen	Compression and extension tests	Strong effect.
Negussey and Islam (1994)	Mine tailings	QSS	Undrained on contractive specimen	Compression and extension tests	Strong effect (e_C > e_E).
Vaid and Thomas (1995)	Fraser river sand	QSS	Undrained loose and medium dense specimen	Compression and extension tests	Strong effect (e_C > e_E).
Finno et al. (1996)	Masonry sand	QSS	Undrained loose and medium dense specimen	Triaxial compression and plane strain tests	Strong effect (e_P > e_C).
Riemer and Seed (1997)	Monterey #0 sand	QSS	Undrained contractive specimen	Compression, extension, and simple shear tests	Strong effect. (e_C > e_E > e_S)
Yoshimine et al. (1999)	Toyoura sand	QSS	Undrained loose and medium dense sepcimen	Compression, simple shear, and extension tests	Strong effect. (e_C > e_S > e_E)
Been et al. (1991)	Erksak sand Toyoura sand	CS	Undrained on contractive specimen	Compression and extension tests	No effect.
Lee (1995)	Likan sand (quartz and mica)	CS	Undrained on contractive specimen	Compression and extension tests	Strong effect (e_E > e_C).
Ishihara et al. (1998)	Masado soil	CS	Undrained on contractive specimen	Compression and extension tests	No effect.
Tsukamoto et al. (1998)	Reclaimed deposits in Kobe	CS	Undrained loose and medium dense specimen	Compression and extension tests	No effect.
Chen and Liao (1999)	Mailiao sand	CS	Undrained on loose and medium dense specimen	Compression and extension tests	Some effect (e_C > e_E).

Notation: e_C, e_E, e_P, and e_S are state lines from compression, extension, plane strain, and simple shear tests respectively.

Table 3 summarizes previous studies on the effect of the stress path on the projection of the critical state line on *e-p'* space. The stress path has a strong effect on the quasi-steady state. In general, this does not seem to be the case for the critical state line. However, Lee (1995) observed a strong effect on the critical state line, probably because the Likan sand he tested consists of quartz and mica: since mica is platy, the bedding direction may interact with the loading direction. Furthermore, he observed that the critical state from extension tests is higher than that of compression tests; this may be related to the observation by Yamamuro and Lade (1995), showing that the extension test is more prone to strain localization than the compression test.

Table 4. Effect of test type and strain rate on critical state and quasi-steady state lines

Reference	Soil used	Referred state	Test condition	Test type	Conclusion
Castro (1969) Casagrande (1971, 1975)	Banding sand	QSS	Undrained on contractive specimen	Load-controlled (eF) and strain-controlled (eD) tests	Strong effect (e_D > e_F). Existence of flow structure.
Hird and Hassona (1990)	Leighton Buzzard sand	QSS	Undrained on contractive specimen	Load-controlled (eF) and strain-controlled (eD) tests	Strong effect (e_D > e_F). Existence of flow structure.
DeGregorio (1990)	F-70 Ottawa banding sand	QSS	Undrained on contractive loose specimen	Dead-load and hydraulic loading systems	Little effect.
Poulos et al. (1988)	Syncrude tailings	CS	Undrained on contractive specimen	Load-controlled and strain-controlled tests	No effect.
Been et al. (1991)	Erksak sand	CS	Undrained on dilative specimen	Load-controlled and strain-controlled tests	No effect.

Notation: e_F and e_D are state lines from load-controlled and strain-controlled tests.

3.3.4 Loading (strain) Rate

The effect of loading (strain) rate on critical state has been studied by various researches; results are summarized in Table 4. While the loading rate may have some effect on the quasi-steady state, this is not the case for the critical state. In addition, Casagrande (1975), Poulos (1981), and Hird and Hassona (1990) insisted that the flow structure can develop only at high loading rate, resulting in the achievement of low shear strength. So, they recommended using load-controlled tests. However, if the internal time scale during testing is considered, this flow structure might be a case of strain localization due to local high pore water pressure generation and particle inertia.

3.3.5 Drainage Condition

Experimental studies on the effect of drainage condition on the critical state line are summarized in Table 5. Most test results, except those by Castro (1969), showed that drainage condition has no effect on the critical state line. Problems with localization are discussed next.

3.3.6 Localization - Heterogeneity

Mooney et al. (1998) performed plane-strain compression drained tests on dense specimens and measured the local void ratio in shear bands using a stereophotogrammetry technique. They showed that the critical void ratio is non-unique for a given confining stress. However, E.R. Cole showed in 1970's that the critical state line obtained from simple shear tests is unique when the local void ratio in the shearing zone is used rather than the average void ratio of the specimen (McRoberts and Sladen, 1992). Therefore, critical state values should not be inferred from specimens prone to localization, that is, contractive soils in undrained shear or dilative soils in drained shear.

Fig. 5 shows the effect of heterogeneity in the undrained response of Ottawa 20-30 sand (material and test details are discussed in Cho, 2001), for two specimens prepared at the same global void ratio and initial effective confining stress. The heterogeneous specimen has three layers, and the top and bottom layers have low void ratio while the middle layer has high void ratio. The measured stress-stain behavior is determined by the loose layer in the heterogeneous specimen: while the homogeneous specimen shows strain-hardening behavior, the heterogeneous specimen displays post-peak response and localization affecting the determination of critical state parameters. Therefore, the spatial variability of void ratio may alter critical state parameters in as much as it promotes non-

Table 5. Effect of drainage condition on critical state and quasi-steady state lines

Reference	Soil used	Referred state	Drainage condition: Drained	Drainage condition: Undrained	Conclusion
Castro (1969), Casagrande (1971), Alarcon-Guzman et al. (1988)	Banding sand	QSS	Contractive specimen	Contractive specimen	Strong effect (edrain > eund). Flow structure or Structural collapse in undrained shear.
Riemer and Seed (1997)	Monterey #0 sand	QSS	Contractive specimen	Contractive specimen	No effect of drainage.
Poulos et al. (1988)	Syncrude tailings	CS	Partly dilative specimen	Contractive specimen	No effect of drainage.
Been et al. (1991)	Erksak sand	CS	Dilative specimen	Contractive specimen	No effect of drainage.
Chu (1995)	Sydney sand	CS	Contractive specimen	Contractive specimen	No effect of drainage.
Lee (1995)	Likan sand	CS	Dilative specimen	Contractive specimen	No effect of drainage.
Verdugo and Ishihara (1996)	Toyoura sand	CS	Loose and medium dense specimen	Loose and medium dense specimen	No effect of drainage.
Yamamuro and Lade (1998)	Nevada sand (silty sand)	CS	Contractive and medium dense.	Medium dense specimen.	No effect of drainage at higher pressures but diverge at lower pressure of SSL.

Notation: edrain and eund are state lines from drained test and undrained test.

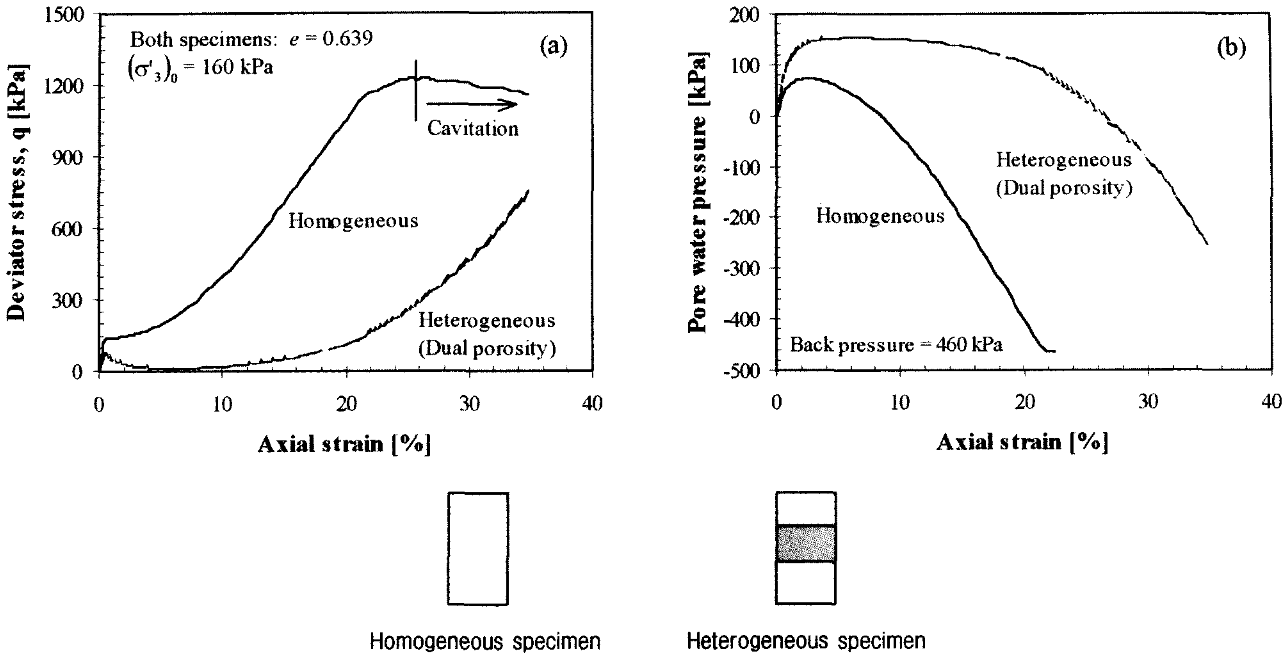


Fig. 5. Effect of heterogeneity on undrained response for Ottawa 20-30 sand. Note: Specimens have the same global void ratio and are subjected to the same initial effective confining stress

homogeneous strain fields.

3.3.7 Fines Content

The presence of fines in soil alters gradation and drainage condition (i.e., reduce permeability), increases interparticle forces, changes brittleness (i.e., threshold strain), and increases the potential range of void ratio. Hence, the critical state and quasi-steady state strengths are necessarily affected by the amount of fines content.

Yamamuro and Lade (1997; 1998) showed the non-uniqueness of the critical state line for Nevada silty sand (specimens consolidated at higher confining stress show higher dilatancy, leading to greater strength). In contrast, Been and Jefferies (1985), Sladen et al. (1985), Dobry et al. (1985), Ishihara (1993), and Chen and Liao (1999) showed that the critical state line is unique for a given fines content. The evidence also suggests that small amount of fines content does not affect the strength parameter but may affect both the intercept and slope of the critical state line projection on e-log p' space. For instance, Dobry et al. (1985) showed that the slope decreases with the increase of fines content.

Table 6. Effect of parameters on critical state and quasi-steady state lines

Parameters	Effect	
	Quasi-steady state line	Critical state line
Initial void ratio (relative density)	Strong effect	No effect (unless localization develops)
Confining stress	Strong effect	No effect
Initial stress ratio (induced anisotropy)	Some effect	No effect
Fabric (inherent anisotropy)	Strong effect	No effect May trigger localization
Prestraining (and aging)	Strong effect	No effect
Stress path (mode of loading)	Strong effect	No effect May trigger localization
Fines content	Strong effect	Strong effect

3.3.8 Summary

Table 6 summarizes the effect of different parameters on the quasi-steady state line and the critical state line. It is concluded that the quasi-steady state line is very sensitive to initial conditions and loading history. However, the critical state line is very robust, displaying "uniqueness." Furthermore, testing conditions must avoid localization, allow large strains, and must not cause particle crushing.

4. Application to Post-liquefaction Shear Strength

Once critical state parameters M, λ and Γ are known for a given soil, the post-liquefaction shear strength (i.e. reliable ultimate shear strength at large deformation) can be evaluated following the critical state concept. For a void ratio e_0, the deviator stress at critical state q_{cs} is:

Table 7. Estimation of relative density or void ratio from in-situ tests

In-situ tests	Empirical relationships		Reference
SPT	$D_r(\%)=100\cdot[(N_1)_{60}/60]^{1/2}$		Terzaghi and Peck (1948)
	General expression $D_r(\%)=100\cdot\left[\frac{N}{(a+b\cdot\sigma_{vo}')}\right]^{1/2}$	a=27 and b=28.	Skempton (1986)
		a=16 and b=23.	Gibbs and Holtz (1957), Meyerhof (1956)
	$D_r(\%)=12.2+0.75\left[\begin{matrix}222\cdot N+2311-711\cdot OCR\\-779\cdot(\sigma_{vo}'/p_a)-50\cdot C_u^2\end{matrix}\right]^{1/2}$		Marcuson and Bieganousky (1977; from Kulhawy and Mayne, 1990)
	$D_r(\%)=100\cdot[(N_1)_{60}/(60\cdot OCR^{0.2})]^{1/2}$		Skempton (1986)
	$D_r(\%)=25\cdot(\sigma_{vo}')^{-0.12}\cdot(N_1)_{60}{}^{0.46}$		Yoshida (1988)
	$D_r(\%)=100\cdot\left[\frac{(N_1)_{60}}{60+25\cdot\log(D_{50})}\right]^{1/2}$ for NC, unaged sands		Kulhawy and Mayne (1990)
CPT	$D_r(\%)=\frac{100}{C_2}\cdot\ln\left[\frac{q_c}{C_0(\sigma_{vo}')^{C_1}}\right]$ For K_o = 0.45 and N.C., moderately compressible, uncemented, unaged Quartz sands. Here, C_0=157, C_1=0.55, C_2=2.41 for Ticino sand, and σ_{vo}' and q_c in kPa.		Baldi et al. (1986; from Robertson and Powell, 1997)
	General expression $D_r(\%)=A+B\cdot\log\left(\frac{q_c}{[\sigma_{vo}']^{1/2}}\right)$ q_c and σ_{vo}' in t/m^2	A = 98 and B = 66 for N.C. uncemented, unaged, predominantly Quartz sands.	Jamiolkowski et al. (1988)
		A = 85 and B = 76.	Tatsuoka et al. (1990; from Ishihara, 1993)
	$D_r(\%)=100\cdot[q_{c1}/(305\cdot OCR^{0.2})]^{1/2}$		Kulhawy and Mayne (1990)
	for clean quartzitic sands $e_o=1.159-0.230\cdot\log(q_{c1})$ for NC $e_o=1.232-0.245\cdot\log(q_{c1})$ for OC $e_o=1.152-0.233\cdot\log(q_{c1})+0.043\cdot\log(OCR)$ for all sands		Mayne (1995)
V_s measurement	For isotropically consolidated sand $e=\frac{1}{m_2}\left(m_1-\frac{V_s}{(\sigma_m')^{1/4}}\right)$ where m_1 = 111 and m_2 = 51 when stress is in kPa and V_s in m/s.		Hardin and Richart (1963)
	For anisotropically consolidated sand $e=\frac{1}{b}\left[a-V_s\left(\frac{P_a}{\sigma_a'}\right)^m\left(\frac{P_a}{\sigma_p'}\right)^n\right]$ where m = n = 1/8; a, b and V_s are in m/s. a = 381; b = 259 for Ottawa sand a = 307; b = 167 for Alaska sand (fines content 31.7%) a = 311; b = 188 for Syncrude sand (fines content 12.5%)		Rosler (1979) Cunning et al. (1995)

Table 7. (Continued)

In-situ tests	Empirical relationships	Reference
Resistivity measurement	Archie (1942) from electrical resistivity measurements Porosity: $n=\left(A\cdot\frac{R_f}{R_{sb}}\right)^{1/m}$ A ≅ 1 for unconsolidated soils and m ≅ 1.5 for sands.	Campanella and Kokan (1993)
Permittivity measurement	For S_s = 0 to 100 m²/g, $\kappa'=3.03+9.3n+146.0n^2-76.7n^3$ at the frequency of MHz to GHz.	Topp et al. (1980)

Remarks

$(N_1)_{60} = N_{60}/(\sigma_{vo}'/p_a)^{1/2}$ = energy-corrected N-value at normalized stress level

$q_{c1} = (q_1/p_a)/(\sigma_{vo}'/p_a)^{1/2}$ = normalized cone tip resistance

p_a = reference stress = 1 atm ≈ 1 tsf ≈ 1 kg/cm² ≈ 100 kPa ≈ 14.7 psi

C_u = coefficient of uniformity

n = porosity

σ_m' = effective mean normal stress (kPa)

a and b = constants for a given sand, both in m/s

σ_a' = effective stress in the direction of wave propagation (kPa)

σ_p' = effective stress in the direction of particle motion (kPa)

R_{sb} = bulk resistivity of the soil (Ω)

R_f = resistivity of the pore fluid (Ω)

A and m = constants which can be found by laboratory calibration

κ' = permittivity

Table 8. Relative density of sand versus penetration resistance

Relative Density	D_r (%)	Angle of friction ϕ, degrees	N Value (blows/ft or 305 mm)	Cone Tip Resistance (q_c/p_a)	Cyclic stress ratio causing liquefaction
Very loose	0 ~ 15 (20)	< 30	0 ~ 4	0 ~ 20	0 ~ 0.04
Loose	15 (20) ~ 35 (40)	30 ~ 35	4 ~ 10	20 ~ 40	0.04 ~ 0.11
Medium	35 (40) ~ 65 (60)	35 ~ 40	10 ~ 30	40 ~ 120	0.11 ~ 0.35
Dense	65 (60) ~ 85 (80)	40 ~ 45	30 ~ 50	120 ~ 200	over 0.35
Very dense	85 (80) ~ 100	> 45	over 50	over 200	-

Note: ( ) used by Meyerhof (1956); friction angle measured on the peak strength.
Sources: Terzaghi and Peck (1967), Lambe and Whitman (1969), Meyerhof (1956), Seed (1979).

$$q_{cs} = M\cdot 10^{\frac{\Gamma-e_0}{\lambda}}\ (1kPa) \tag{10}$$

Thus, the post-liquefaction shear strength τ_{cs} is estimated as:

$$\tau_{cs} = \frac{q_{cs}}{2}\cdot\cos\phi_{cs} \tag{11}$$

Liquefaction design analyses with this approach requires knowing the in-situ void ratio. Various empirical relationships suggested to estimate the in-situ void ratio or relative density are summarized in Table 7 and Table 8.

5. Final Remarks

Comprehensive review on the determination of critical state parameters in sandy soils from standard triaxial testing was performed to facilitate the application of critical state soil mechanics to the shear behavior of sandy soils. The main findings are as follows:

- When a soil is subjected to shearing, its ultimate state in the 3-D space can be expressed in terms of three critical state parameters M, λ and Γ.
- The critical state should be considered as the ultimate state at large strains.

- The critical state friction angle is directly related to the interparticle friction angle.
- For a given soil, the critical state friction angle is unique irrespective of drainage condition, initial density, confining stress, and strain rate.
- The projection of the critical state line on the *e*-log p' space can be reasonably considered as a straight line in the stress range from 10 to 500 kPa for hard grain sands.
- The quasi-steady state line is very sensitive to fabric or sample preparation method, drainage condition, the initial state of stress, and stress path. However, the critical state line on *e*-log p' space is unique for a given soil regardless of those conditions.
- The post-liquefaction shear strength in field can be effectively estimated from critical state parameters and in-situ void ratio.

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References

1. Alarcon-Guzman, A., Leonards, G.A., and Chameau, J.L. (1988), "Undrained monotonic and cyclic strength of sand", *Journal of Geotechnical Engineering*, ASCE, Vol.114, No.2, pp.1089-1109.
2. Baldi, G., Bellotti, R., Ghionna, V., Jamiolkowski, M., and Pasqualini, E. (1986), "Interpretation of CPT's and CPTU's, 2nd Part: Drained penetration of sands", *4th International Geotechnical Seminar*, Nenying Technological Institute, Singapore, Field Instrument and In Situ Measurements, pp.143-162.
3. Been, K. and Jefferies, M.G. (1985), "A state parameter for sands", *Géotechnique*, Vol.35, No.2, pp.99-112.
4. Been, K., Jefferies, M.G., and Hachey, J. (1991), "The critical state of sands", *Géotechnique*, Vol.41, No.3, pp.365-381.
5. Been, K. (1999), "The critical state line and its application to soil liquefaction", *Physics and Mechanics of Soil Liquefaction*, P.V. Lade and J.A. Yamamuro (eds.), Rotterdam, pp.195-204.
6. Bishop, A.W. (1950), "Discussion: Measurement of the shear strength of soils", *Géotechnique*, Vol.2, No.1, pp.113-116.
7. Bishop, A.W. (1954), "Correspondence on shear characteristics of a saturated silt measured in triaxial compression", *Géotechnique*, Vol.4, No.1, pp.43-45.
8. Bishop, A.W. (1971), "Shear strength parameters for undisturbed and remoulded soil specimens", *Stress - strain behaviour of soils*, Edited by R.H.G. Parry, Proceedings of the Roscoe Memorial Symposium, Cambridge University, 29-31, March, 1971, G.T. Foulis & Co LTD., pp.3-58.
9. Bolton, M.D. (1986), "The strength and dilatancy of sands", *Geotechnique*, Vol.36, No.1, pp.65-78.
10. Campanella, R.G. and Kokan, M.J. (1993), "A new approach to measuring dilatancy in saturated sands", *Geotechnical Testing Journal*, ASTM, Vol.16, No.4, pp.485-495.
11. Caquot, A. (1934), "Equililibre des Massifs Frottement Interne", *Stabilit des Terres Pulvrents et Cohrentes*, Paris: Gauthier Villars, French.
12. Casagrande, A., (1936), "Characteristics of cohesionless soils affecting the stability of slopes and earth fills," *Journal of the Boston Society of Civil Engineers*, January; reprinted in Contributions to Soil Mechanics (1925-1940), the Boston Society of Civil Engineers, 1948, pp.257-276.
13. Casagrande, A. (1965), "The Terzaghi Lecture: Role of the "Calculated risk" in earthwork and foundation engineering", *Journal of Soil Mechanics and Foundations*, Vol.91, No.SM 4, pp.1-40.
14. Casagrande, A. (1971), "On liquefaction phenomena: report of lecture", *Géotechnique*, Vol.21, No.3, pp.197-202.
15. Casagrande, A. (1975), "Liquefaction and cyclic deformation of sands a critical review", *The 5th Panamerican conference on Soil Mechanics and Foundation Engineering*, Vol.5, pp.81-133.
16. Castro, G. (1969), *Liquefaction of sands*, Ph.D. thesis, Harvard Soil Mechanics Series, No.81, Harvard University, Cambridge, MA.
17. Castro, G. (1975), "Liquefaction and cyclic mobility of saturated sands", *Journal of Geotechnical Engineering*, Division, ASCE, Vol.101, No.6, pp.551-569.
18. Chen, Y.C. and Liao, T.S. (1999), "Studies of the state parameter and liquefaction resistance of sands", *Earthquake Geotechnical Engineering*, Seco e Pinto (eds.), pp.513-518.
19. Cho, G.C. (2001), *Unsaturated Soil Stiffness and Post-Liquefaction Shear Strength*, Ph.D. Thesis, Georgia Institute of Technology, Atlanta, USA, p.288.
20. Chu, B. (1995), "An experimental examination of the critical state and other similar concepts for granular soils", *Canadian Geotechnical Journal*, Vol.32, pp.1065-1075.
21. Cornforth, D.H. (1973), "Prediction of drained strength of sands from relative density measurements", *Evaluation of relative density and its role in geotechnical projects involving cohesionless soils*, ASTM STP 523, American Society for Testing and Materials, Philadelphia, pp.281-303.
22. Cunning, J.C., Robertson, P.K. and Sego, D.C. (1995), "Shear Wave Velocity to Evaluate in situ State of Cohesionless Soils", *Canadian Geotechnical Journal*, Vol.32, pp.848-858.
23. DeGregorio, V.B. (1990), "Loading systems, sample penetration, and liquefaction", *Journal of Geotechnical Engineering*, Vol.116, No.5, pp.805-821.
24. Desrues, J., Chambon, R., Mokni, M., and Mazerolle, F. (1996), "Void ratio evolution inside shear bands in triaxial sand specimens studied by computed tomography", *Geotechnique*, Vol.46, No.3, pp.529-546.
25. Dobry, R., Vasquez-Herrera, A., Mohamad, R., and Vucetic, M. (1985), "Liquefaction flow failure of silty sand by torsional cyclic tests", *Advances in the art of testing soils under cyclic conditions*, Proceedings of a session sponsored by the Geotechnical Engineering Division in conjunction with the ASCE Convention in Detroit,

Michigan, Khosla (ed.), pp.29-50.
26. Feda, J. (1982), *Mechanics of particulate materials*, Developments in Geotechnical Engineering No.30, Elsevier.
27. Finno, R.J., Harris, W.W., Mooney, M.A., and Viggiani, G. (1996), "Strain localization and undrained steady state of sand", *Journal of Geotechnical Engineering*, ASCE, Vol.122, No.6, pp.462-473.
28. Frossard, E. (1979), "Effect of sand grain shape on interparticle friction; indirect measurements by Rowe's stress dilatancy theory", *Géotechnique*, Vol.29, No.3, pp.341-350.
29. Gibbs, H.J., and Holtz, W.G. (1957), "Research on determining the density of sands by spoon penetration testing", *Proceedings of 4th International Conference on Soil Mechanics and Foundation Engineering*, London, Vol.1, pp.35-39.
30. Hardin, B.O. (1987), "1-D strain in normally consolidated cohesionless soils", *Journal of Geotechnical Engineering*, Vol.113, No.12, December, pp.1449-1467.
31. Hardin, B.O. and Richart, F.E. Jr. (1963), "Elastic wave velocities in granular soils", *Journal of Soil Mechanics and Foundations*, ASCE, Vol.89, No.1, pp.33-65.
32. Heyman, J. (1997), *Coulomb's Memoir on Statics*, An Essay in the History of Civil Engineering, Imperial College Press.
33. Hird, C.C., and Hassona, F.A.K. (1990), "Some factors affecting the liquefaction and flow of saturated sands in laboratory tests", *Engineering Geology*, Vol.28, pp.149-170.
34. Horne, M.R. (1964), "The behavior of an assembly of rotund, rigid, cohesionless particles. I", *Proceedings of Royal Society*, A286, pp.62-78.
35. Ishihara, K. (1993), "The Rankine Lecture: Liquefaction and flow failure during earthquakes", *Géotechnique*, Vol.43, No.3, pp.351-415.
36. Ishihara, K., Tatsuoka, F., and Yasuda, S. (1975), "Undrained deformation and liquefaction of sand under cyclic stress", *Soils and Foundations*, Vol.15, pp.29-44.
37. Ishihara, K., Cubrinovski, M., and Nonaka, T. (1998), "Characterization of undrained behavior of soils in the reclaimed area of Kobe", *Special Issue of Soils and Foundations*, Japanese Geotechnical Society, September, pp.33-46.
38. Jamiolkowski, M., Chionna, V.N., Lancellotta, R., and Pasqualini (1988), "New correlations of penetration test for design practice", *Invited Lecture, ISOPT-1*, Disney World, March, Balkema Publication, pp.263-296.
39. Konrad, J.M. (1990a), "Minimum undrained strength of two sands", Journal of Geotechnical Engineering, Vol. 116, No. 6, pp. 932-947.
40. Konrad, J.M. (1990b), "Minimum undrained strength versus steady-state strength of sands", *Journal of Geotechnical Engineering*, Vol.116, No.6, pp.948-963.
41. Kulhawy, F.H. and Mayne, P.W. (1990), *EPRI Manual on Estimating Soil Properties for Foundation Design*, Report EL-6800, Electric Power Research Institute, Palo Alto, CA, page 306.
42. Lambe, T.W. and Whitman, R.V. (1969), *Soil Mechanics*, John Wiley and Sons, New York, p.553.
43. Lee, C.J. (1995), "Static shear and liquefaction potential of sand", *Proceedings of 3rd International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics*, St. Louis, Missouri, April 2-7, Vol.1, pp.115-118.
44. Lee, K.L. and Seed, H.B. (1967), "Drained strength characteristics of sands", *Journal of Soil Mechanics and Foundations Division*, Vol.93, No.SM6, November, pp.117-141.
45. Li, X.S. and Wang, Y. (1998), "Linear representation of steady-state line for sand", *Journal of Geotechnical and Geoenvironmental Engineering*, Vol.124, No.12, December, pp.1215-1217.
46. Luong, M.P. (1980), "Stress-strain aspects of cohesionless soils under cyclic and transient loading", *International Symposium on Soils under Cyclic and Transient Loading*, Swansea, pp.315-324.
47. Marcuson,W. F. III, and Bieganousky, W.A. (1977), "SPT and relative density in coarse sands", *Journal of Geotechnical Engineering*, ASCE, Vol.103, No.11, pp.1295-1309.
48. Mayne, P.W. (1995), "CPT determination of overconsolidation ratio and lateral stresses in clean quartz sands", *Proceedings of International Symposium on Cone Penetration Testing*, CPT '95, Sweden.
49. McRoberts, E.C., and Sladen, J.A. (1992), "Observations on static and cyclic sand-liquefaction methodologies", *Canadian Geotechnical Journal*, Vol.29, pp.650-665.
50. Meyerhof, G.G. (1956), "Penetration tests and bearing capacity of cohesionless soils", *Journal of the Soil Mechanics and Foundations Division*, ASCE, Vol.82, No.SM1, January, pp.1-19.
51. Mooney, M.A., Finno, R.J., and Viggiani, M.G. (1998), "A unique critical state for sand?", *Journal of Geotechnical and Geoenvironmental Engineering*, Vol.124, No.11, November, pp.1100-1108.
52. Negussey, D., Wijewickreme, W.K.D., and Vaid, Y.P. (1988), "Constant-volume friction angle of granular materials", *Canadian Geotechnical Journal*, Vol.25, pp.50-55.
53. Negussey, D., and Islam, M.S. (1994), "Uniqueness of steady state and liquefaction potential", *Canadian Geotechnical Journal*, Vol.31, No.1, pp.132-139.
54. Poulos, S.J. (1981), "The steady state of deformation", *Journal of Geotechnical Engineering*, Vol.107, No.GT5, pp.553-562.
55. Poulos, S.J., Castro, G., and France, J.W. (1985), "Liquefaction evaluation procedure", *Journal of Geotechnical Engineering*, Vol.111, No.6, June, pp.772-792.
56. Poulos, S.J., Castro, G., and France, J.W. (1988), Closure of "Liquefaction evaluation procedure", by S.J. Poulos, G. Castro and J.W. France, *Journal of Geotechnical Engineering*, ASCE, Vol.114, No.1, pp.251-259.
57. Reynolds, O. (1885), "On the dilatancy of media composed of rigid particles in contact. With experimental illustrations", *Philosophical magazine and Journal of Science*, Series 5, Vol.20, No.127, pp.469-481.
58. Riemer, M.F. and Seed, R.B. (1997), "Factors affecting apparent position of steady-state line", *Journal of Geotechnical and Geoenvironmental Engineering*, Vol.123, No.3, pp.281-288.
59. Robertson, P.K., and Powell, J.J.M. (1997), *Cone Penetration Testing in Geotechnical Practice*, Blackie Academic & Professional, p.312.
60. Roscoe, K.H., Schofield, A.N., Wroth, C.P. (1958), "On the yielding of soils", *Géotechnique*, Vol.8, pp.22-53.
61. Rosler, S.K. (1979), "Anisotropic shear wave modulus due to stress anisotropy", *Journal of Geotechnical Engineering*, ASCE, Vol.105, No.7, pp.871-880.
62. Rowe, P.W. (1962), "The stress-dilatancy relation for static equilibrium of an assembly of particles in contact", *Proceedings of Royal Society*, A, 269, pp.500-527.
63. Rowe, P.W. (1963), "Stress-dilatancy, earth pressures, and slopes", *Journal of Soil Mechanics and Foundations*, ASCE, Vol.89, SM 3, pp.37-61.

64. Santamarina, J.C., Klein, K.A., Fam, M.A. (2001), *Soils and Waves Particulate Materials Behavior, Characterization and Process Monitoring*, John Wiley and Sons, LTD, page 488.

65. Schofield, A.N., and Wroth, P. (1968), *Critical State Soil Mechanics*, McGraw-Hill Book Company.

66. Schofield, A.N. (1998), "Don't use the C word", *Ground Engineering*, August, pp.30-32.

67. Seed, H.B., 1979, "Soil liquefaction and cyclic mobility evaluation for level ground during earthquakes", *Journal of the Geotechnical Engineering Division*, ASCE, Vol.105, No.GT2, pp.201-255.

68. Skempton, A.W. (1986), "Standard penetration test procedures and the effects in sands of overburden pressure, relative density, particle size, ageing and overconsolidation", *Géotechnique*, Vol.36, No.3, pp.425-447.

69. Sladen, J.A., D'Hollander, R.D., and Krahn, J. (1985), "The liquefaction of sands, a collapse surface approach", *Canadian Geotechnical Journal*, Vol.22, pp.564-578.

70. Taylor, D.W. (1948), *Fundamentals of Soil Mechanics*, John Wiley & Sons, Inc., Pages 770.

71. Tatsuoka, F., Shibuya, S., Teachavorasinskun, S., and Park C.S. (1990), Discussion to "An experimental and theoretical comparison between static and dynamic torsional soil tests" by Bolton, M.D. and Wilson, J.M.R., *Géotechnique*, Vol.40, No.4, pp.659-664.

72. Terzaghi, K. and Peck, R.B. (1948), *Soil Mechanics in Engineering Practice*, 1st Edition, John Wiley and Sons, New York, p.566.

73. Terzaghi, K. and Peck, R.B. (1967), *Soil Mechanics in Engineering Practice*, 2nd Edition, John Wiley and Sons, New York, p.729.

74. Topp, G.C., Davis, J.L., Bailey, W.G., and Zebchuk, W.D. (1980), "The measurement of soil water content using a portable TDR hand probe", *Canadian Journal of Soil Science*, Vol.64, pp.313-321.

75. Tsukamoto, Y., Ishihara, K., and Nonaka, T. (1998), "Undrained deformation and strength characteristics of soils from reclaimed deposits in Kobe", *Special Issue of Soils and Foundations*, Japanese Geotechnical Society, September, pp.47-55.

76. Vaid, Y.P. and Chern, J.C. (1985), "Effect of static shear on resistance to liquefaction", Discussion, *Soils and Foundations*, Japanese Society of Soil Mechanics and Foundation Engineering, Vol.25, No.3, pp.154-156.

77. Vaid, Y.P., Chung, E.K.F., and Kuerbis, R.H. (1990), "Stress path and steady state", *Canadian Geotechnical Journal*, Vol.27, pp.1-7.

78. Vaid, Y.P. and Thomas, J. (1995), "Liquefaction and Postliquefaction Behavior of Sand", *Journal of Geotechnical Engineering*, Vol.121, No.2, February, pp.163-173.

79. Vaid, Y.P. and Sivathayalan, S. (1999), "Fundamental factors affecting liquefaction susceptibility of sands", *Physics and Mechanics of Soil Liquefaction*, Lade & Yamamuro (eds.), Rotterdam, pp. 105-120.

80. Vaid, Y.P., Sivathayalan, S., and Stedman, D. (1999), "Influence of specimen-reconstituting method on the undrained response of sand", *Geotechnical Testing Journal*, GTJODJ, Vol.22, No.3, September, pp.187-195.

81. Verdugo, R. and Ishihara, K. (1996), "The steady state of sandy soils", *Soils and Foundations*, Vol.36, No.2, pp.81-91.

82. Verdugo, R., P. Castillo and Briceno, L. (1995), "Initial Soil Structure and Steady-State Strength", *First International Conference on Earthquake Geotechnical Engineering*, K. Ishihara (eds.), Balkema, Vol.1, pp.209-214.

83. Yamamuro, J.A., and Lade, P.V. (1995), "Strain localization in extension tests on granular materials", *Journal of Engineering Mechanics*, ASCE, Vol.121, No.7, pp.828-836.

84. Yamamuro, J.A. and Lade, P.V. (1997), "Static Liquefaction of Very Loose Sands", *Canadian Geotechnical Journal*, Vol.34, pp.905-917.

85. Yamamuro, J.A. and Lade, P.V. (1998), "Steady-State Concepts and Static Liquefaction of Silty Sands", *Journal of Geotechnical and Geoenvironmental Engineering*, Vol.124, No.9, September, pp. 868-877.

86. Yoshida, Y. (1988), "Empirical formula of SPT blow counts for gravely soils", *Proceedings of the First International Symposium on Penetration*, A.A. Balkema, Vol.1, pp.381-387.

87. Yoshimine, M., Robertson, P.K., and Wride, C.E. (1999), "Undrained shear strength of clean sands to trigger flow liquefaction", *Canadian Geotechnical Journal*, Vol.36, pp.891-906.

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